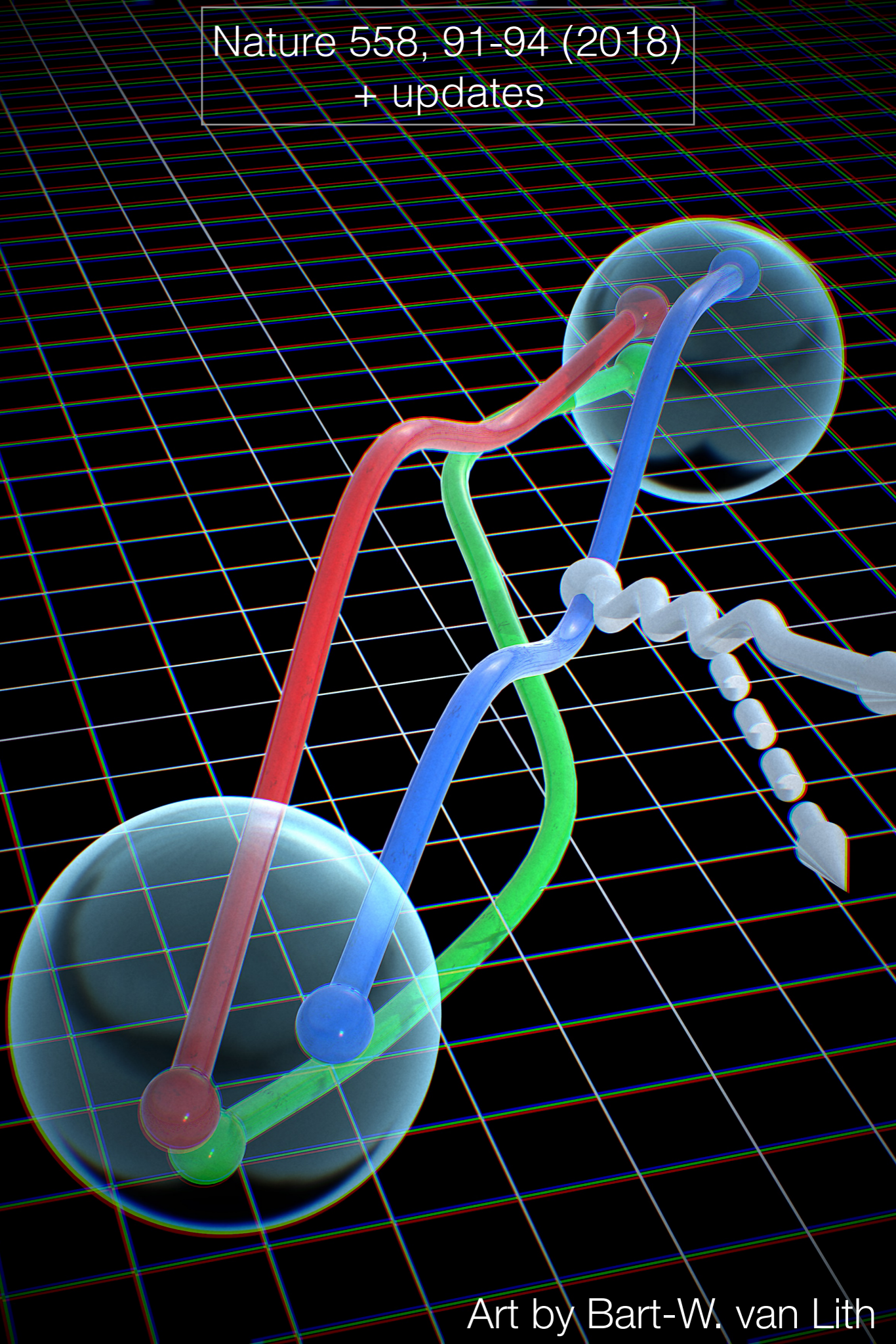


First-principles QCD calculation of the neutron lifetime

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RIKEN BNL Research Center





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Nicolas Garron



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Pavlos Vranas
Arjun Gambhir



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Thorsten Kurth



UNC

Amy Nicholson
Henry Monge Camacho



nVIDIA

nVidia

Kate Clark



Glasgow

Chris Bouchard



INT

Chris Monahan



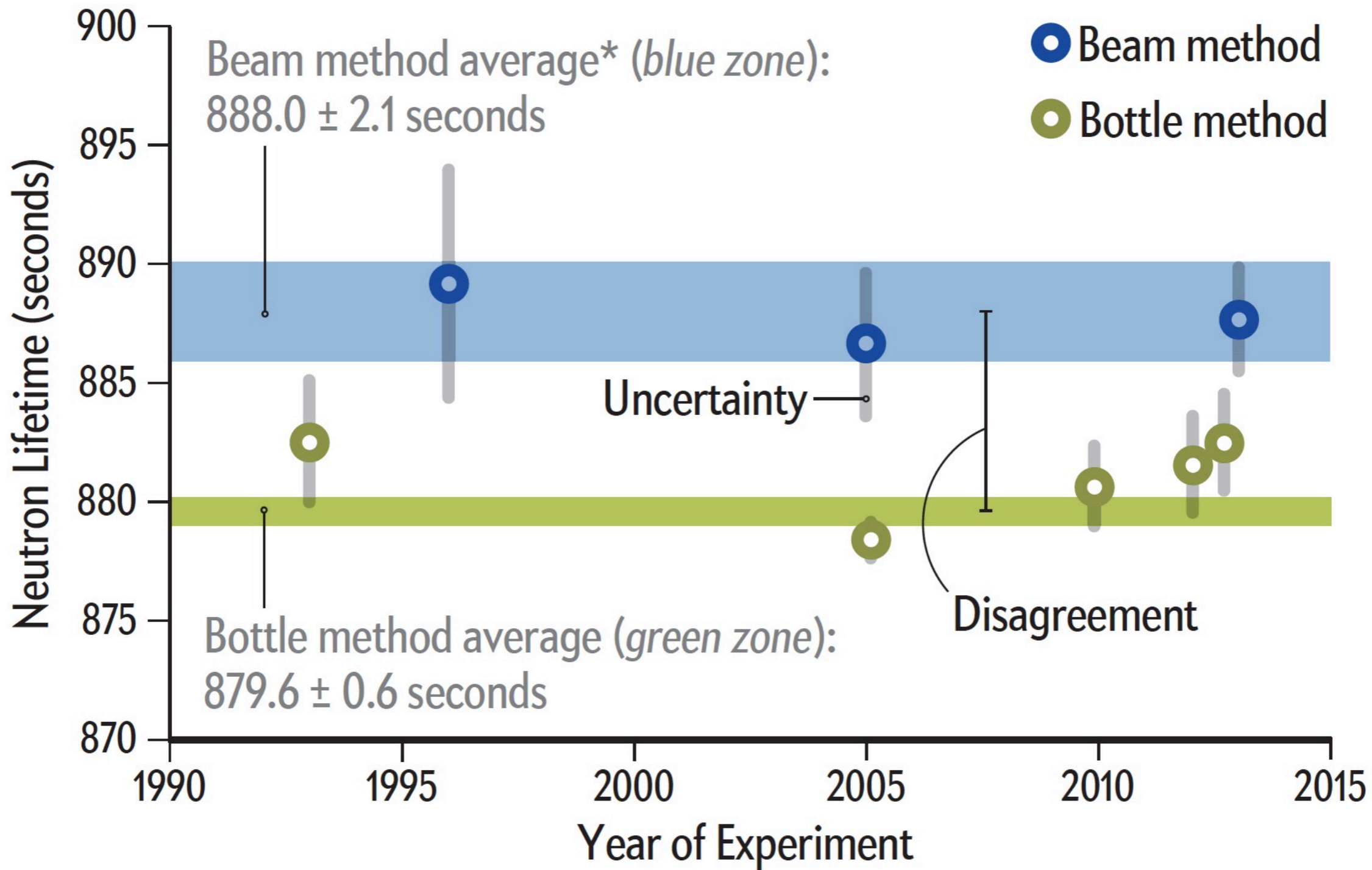
William &
Mary

Kostas Orginos



collaboration
(Cal-ifornia Lat-tice)

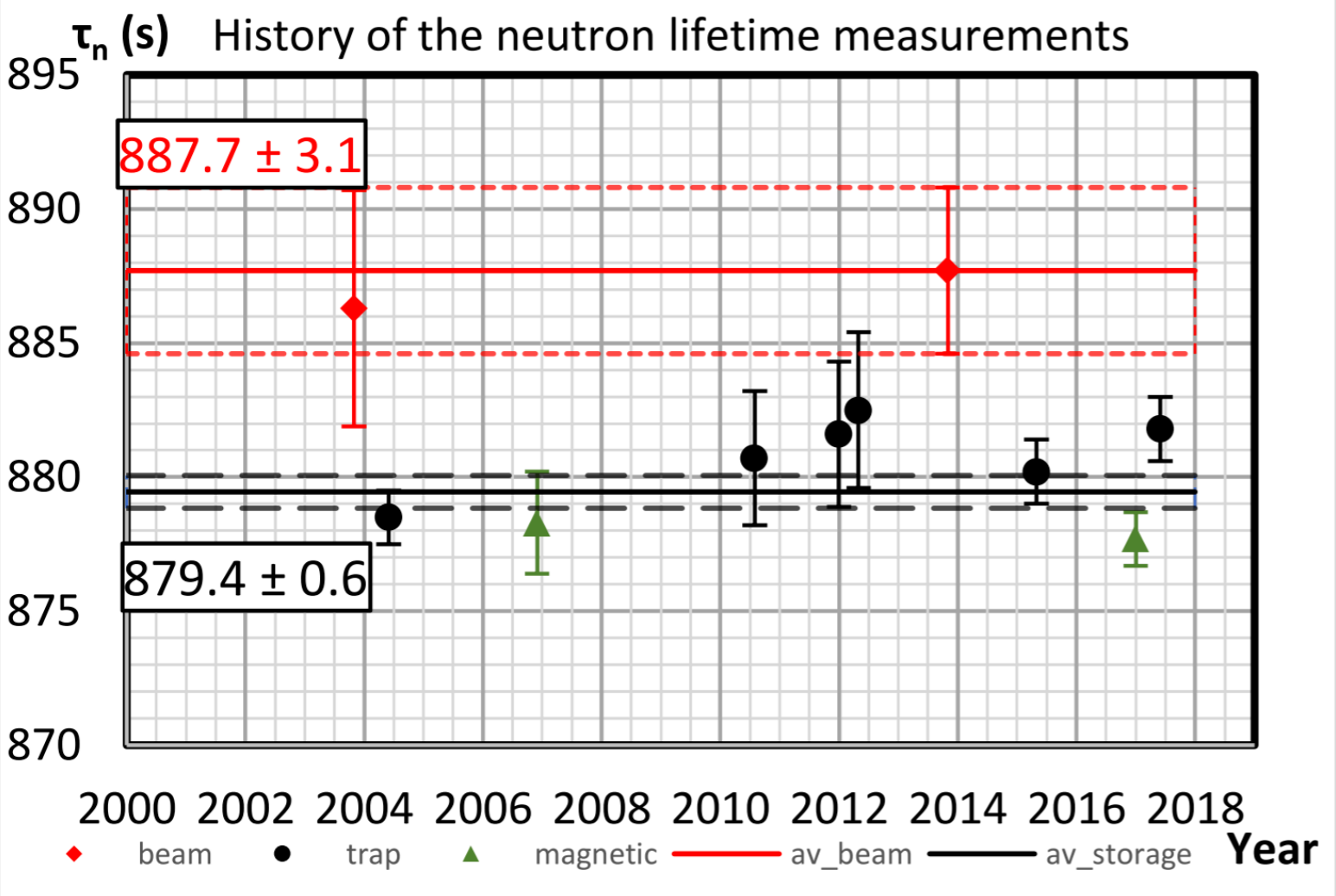
Neutron Lifetime Measurements



Neutron lifetime “puzzle”

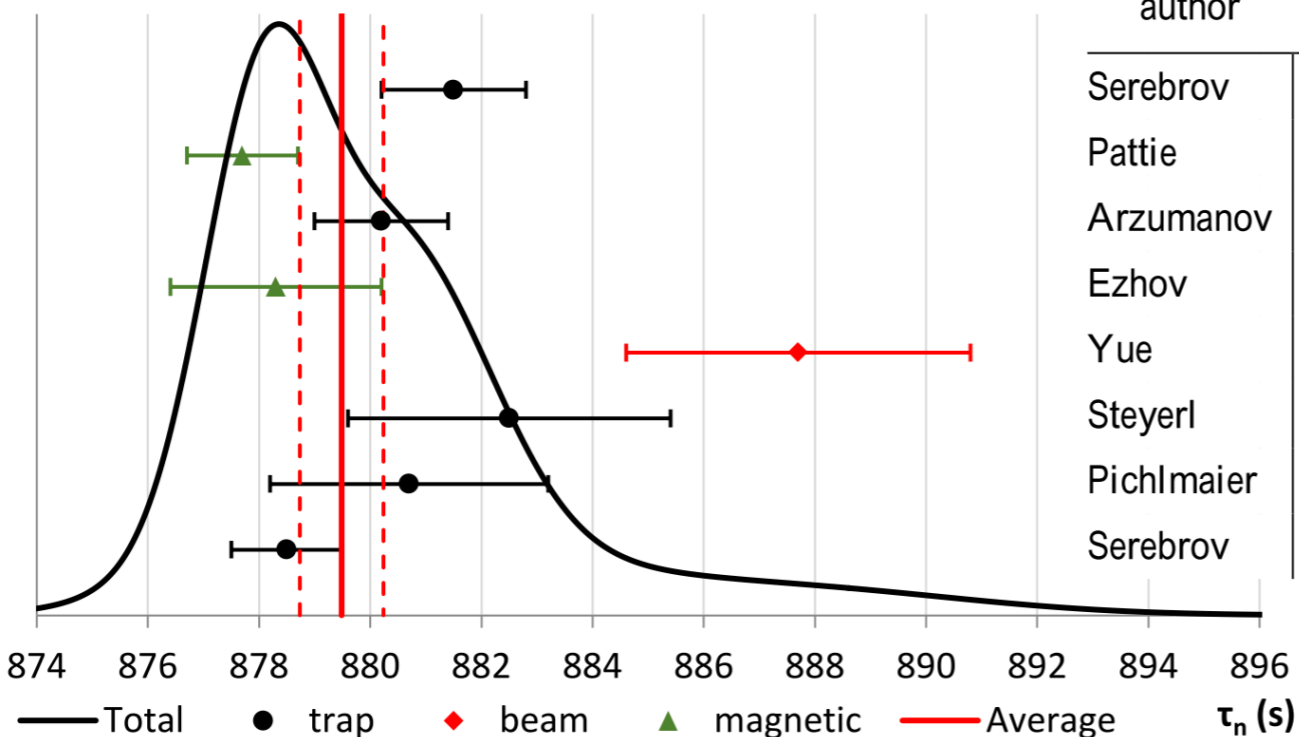
$$\tau_n^{\text{beam}} = 888.0 \pm 2.0 \text{ s}$$

$$\tau_n^{\text{bottle}} = 879.6 \pm 0.6 \text{ s}$$



Serebrov et al. Phys. Rev. C 97 (055503) 2018

Weighted average
879.5 ± 0.8 (error scaled by 1.5)



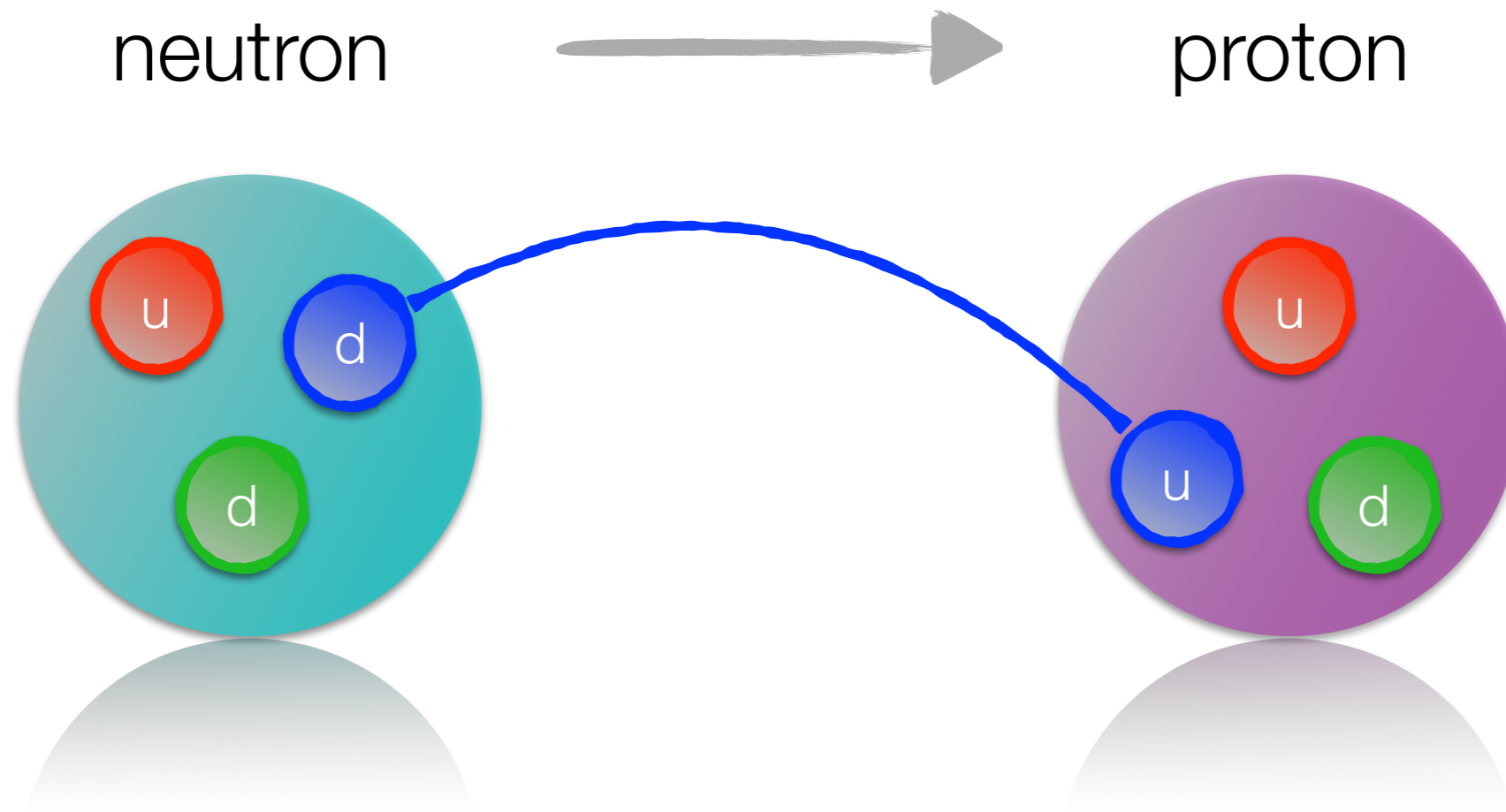
- ❖ The discrepancy of $\sim 4\sigma$ between different methods is still unresolved.
- ❖ Experiments are trying to reduce all their systematics and provide robust estimates for their uncertainties
- ❖ Neutron decays to “dark” or “exotic” particles have been invoked to explain the discrepancy

[Fornal&Grinstein, PRL120(191801)2018]

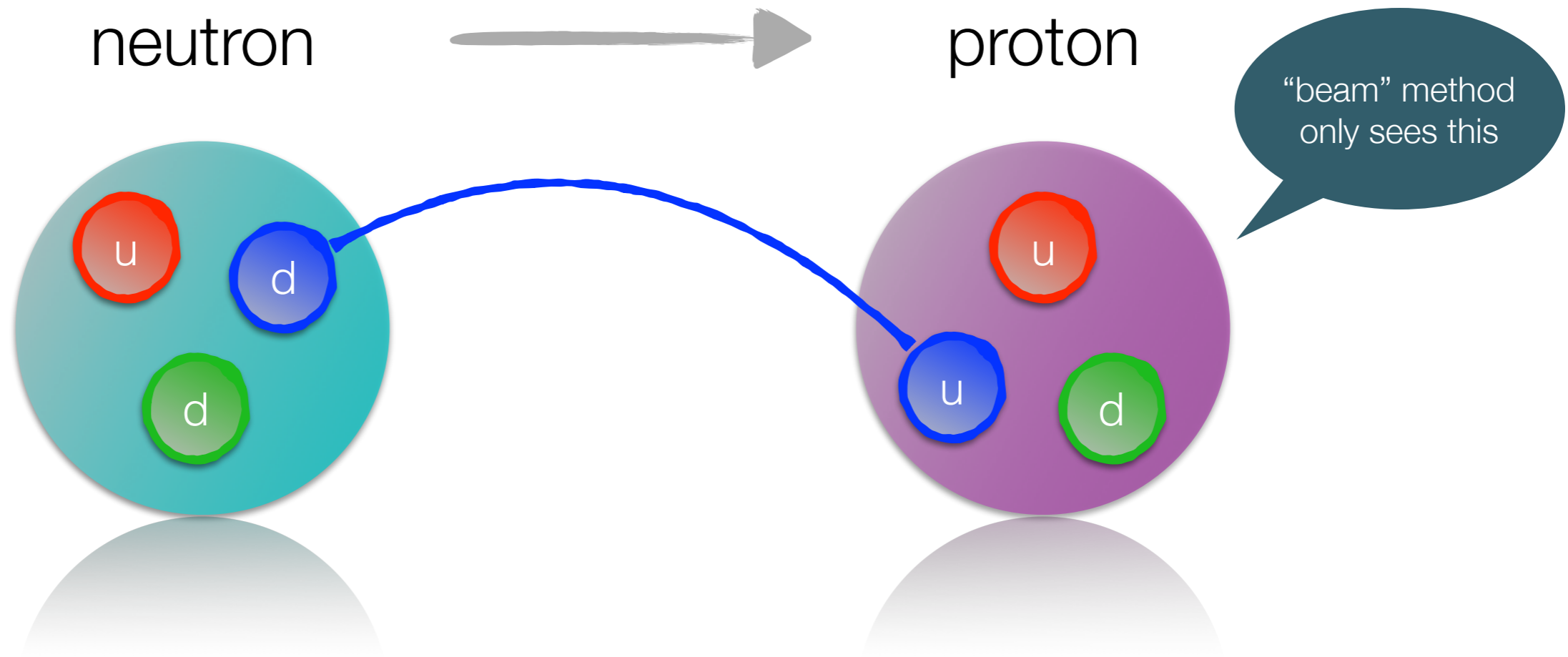
author	year	value	error stat	error sys	Σ	χ^2	Ref
Serebrov	2017	881.5	0.7	0.6	1.3	2.4	—
Pattie	2017	877.7	0.7	0.3	1.0	3.2	[21]
Arzumanov	2015	880.2	1.2		1.2	0.4	[22]
Ezhov	2014	878.3	1.9		1.9	0.4	[23]
Yue	2013	887.7	1.2	1.9	3.1	7.0	[24]
Steyerl	2012	882.5	1.4	1.5	2.9	1.1	[25]
Pichlmaier	2010	880.7	1.3	1.2	2.5	0.2	[26]
Serebrov	2004	878.5	0.7	0.3	1.0	1.0	[15, 16]

Science 11 May 2018:
Vol. 360, Issue 6389, pp. 627-632

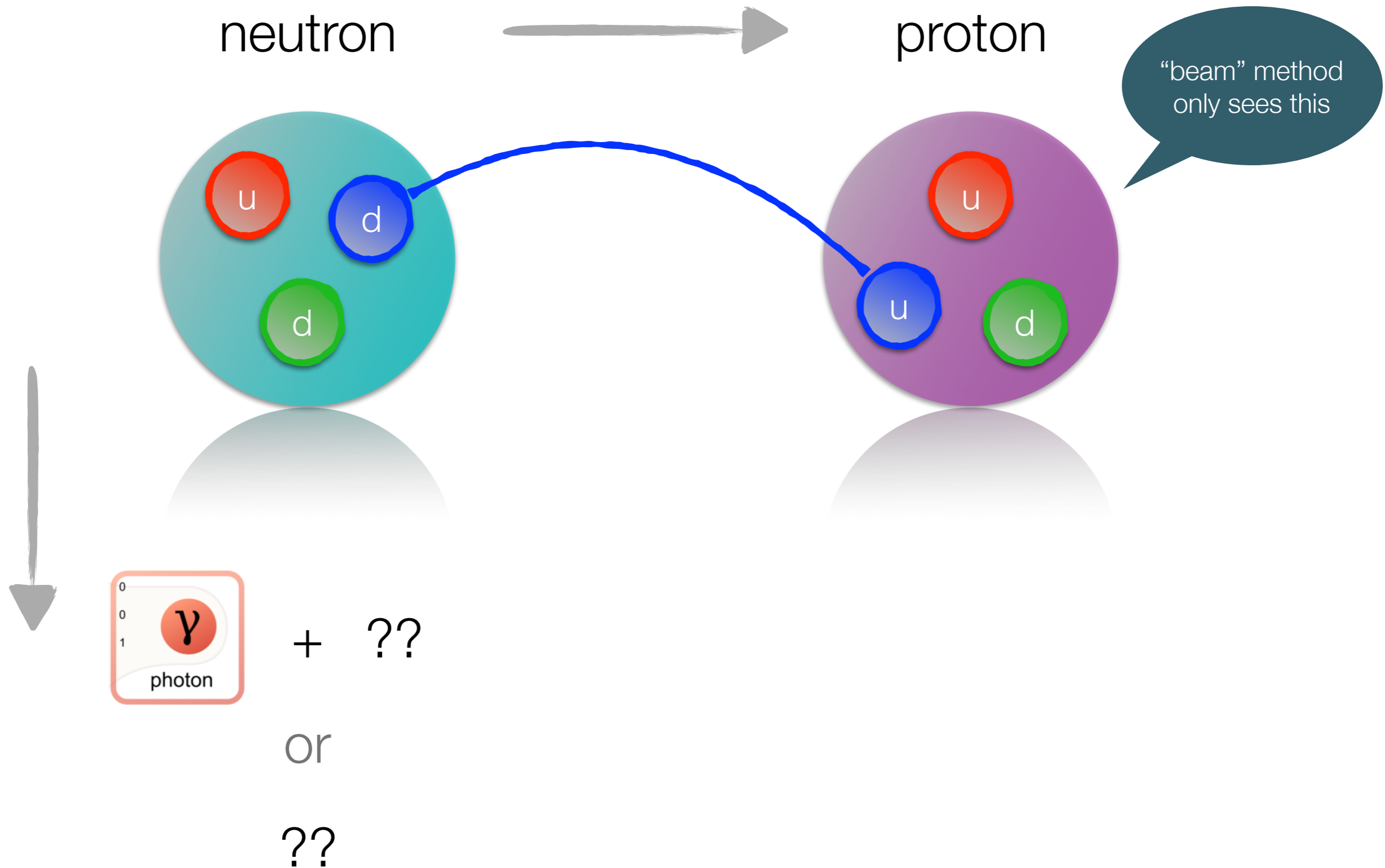
Exotic decays of the neutron



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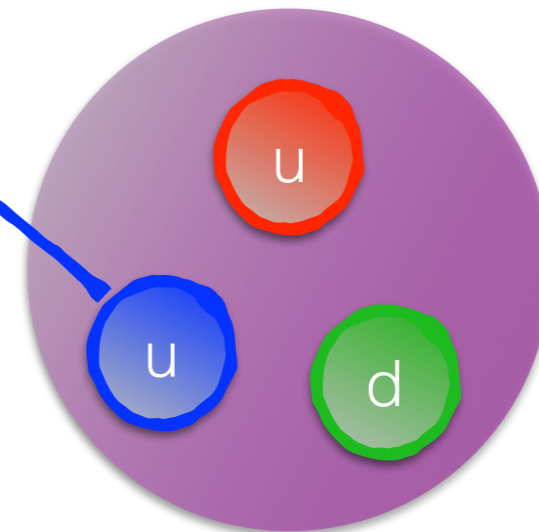
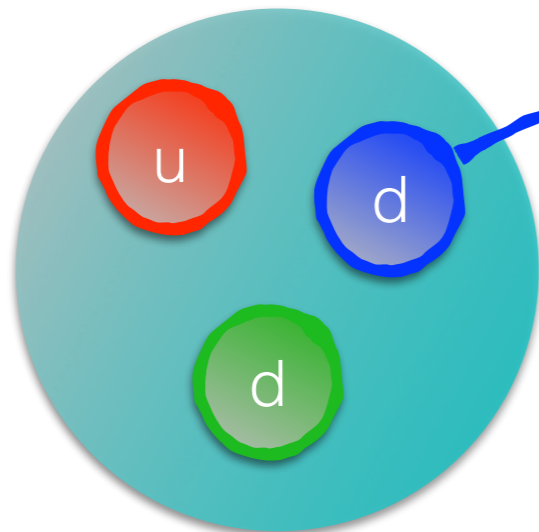


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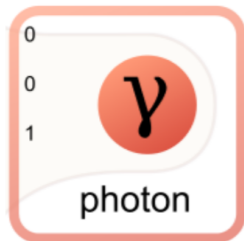
neutron



proton



“beam” method only sees this



+ ??

or

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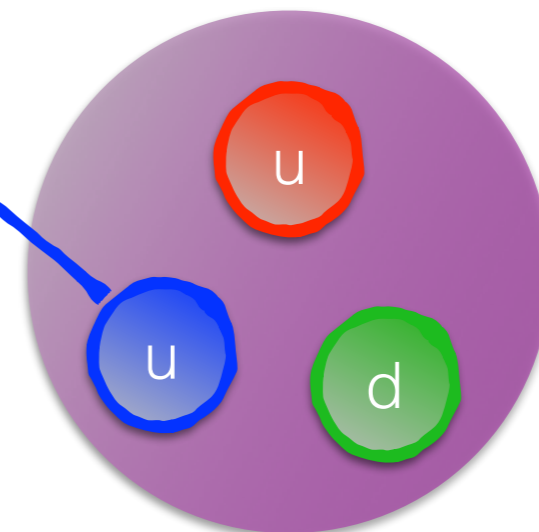
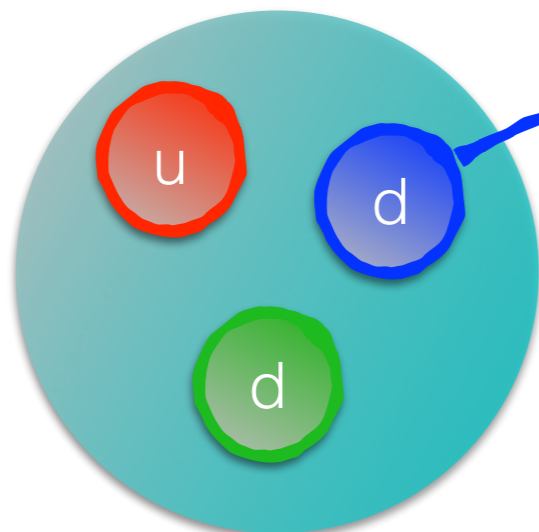
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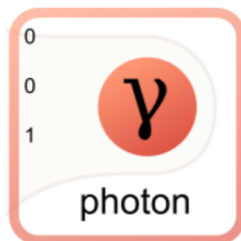
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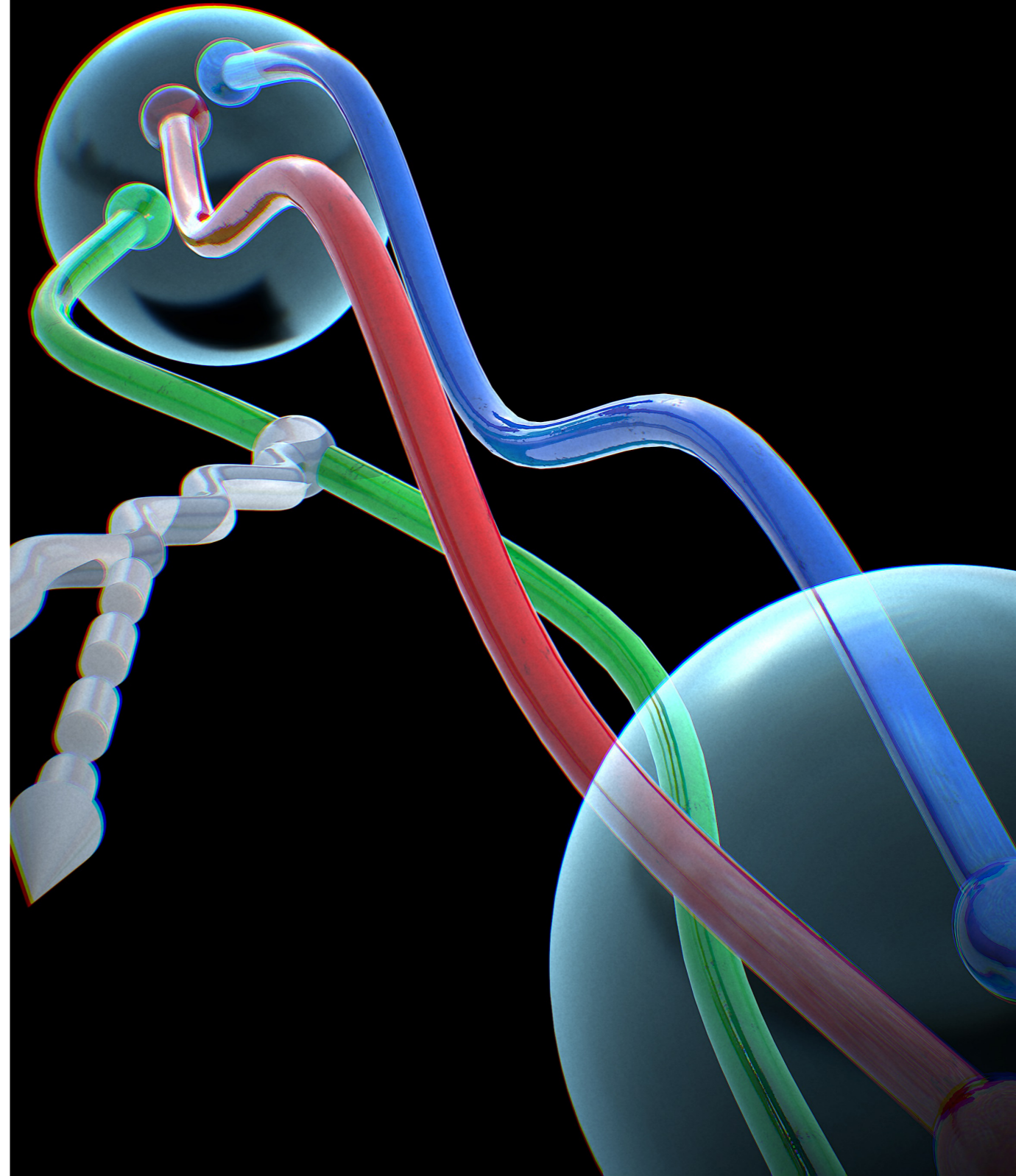
- * Experiments are already putting constraints on decays including photons and invisible particles
- * Theorists are putting bounds on exotic decays by using neutron stars observations
- * What else can we do?

Neutron beta decay

- ◆ In the Standard Model, beta decay is driven by the electroweak sector
- ◆ The master formula includes the **quark mixing matrix element**, the **neutron lifetime** and the **axial coupling**:

$$|V_{ud}|^2 \tau_n (1 + 3g_A^2) = 4908.6(1.9)s$$

[Czarnecki, Marciano and Sirlin, Phys. Rev. Lett. 120, 202002 (2018)]

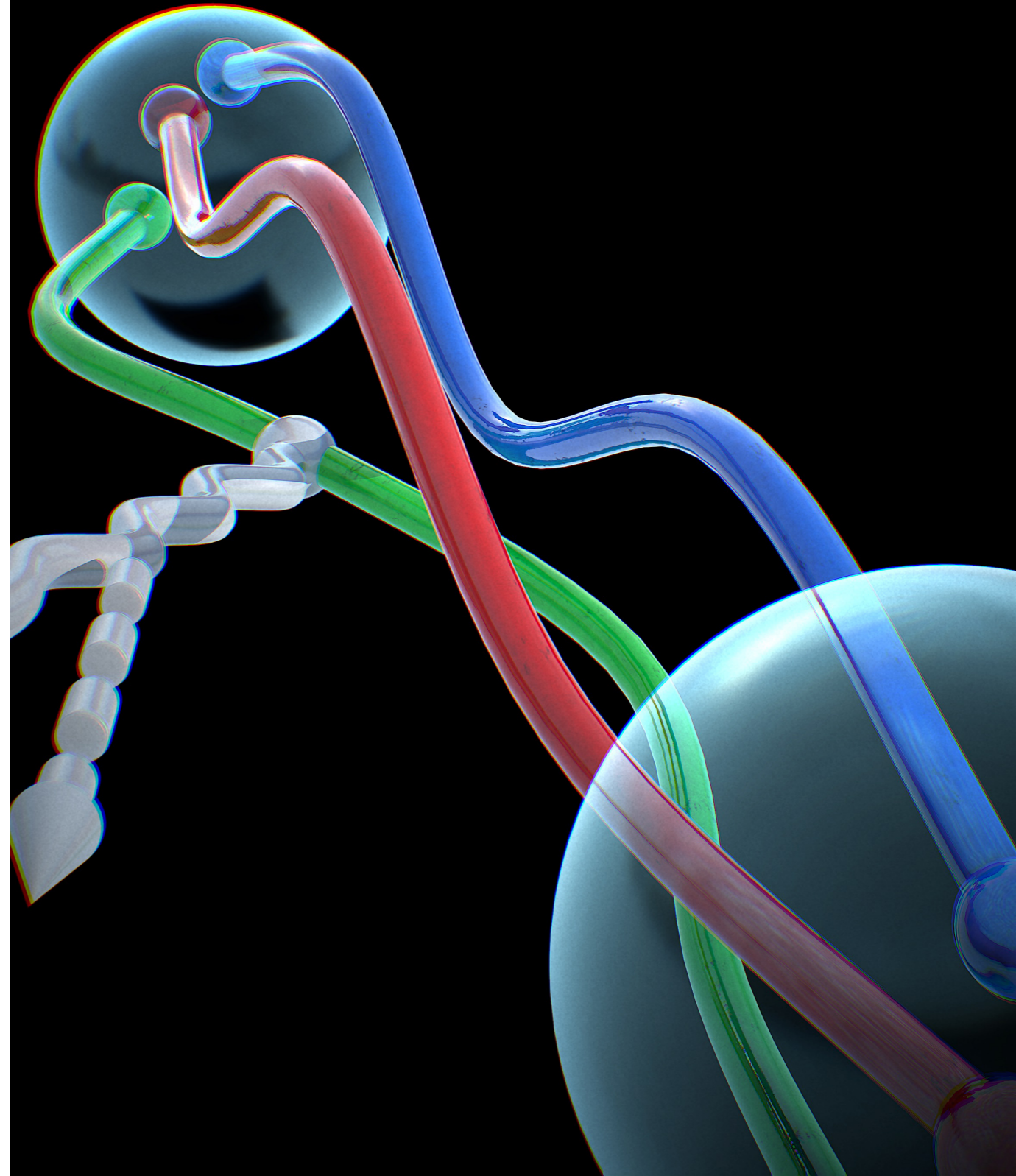
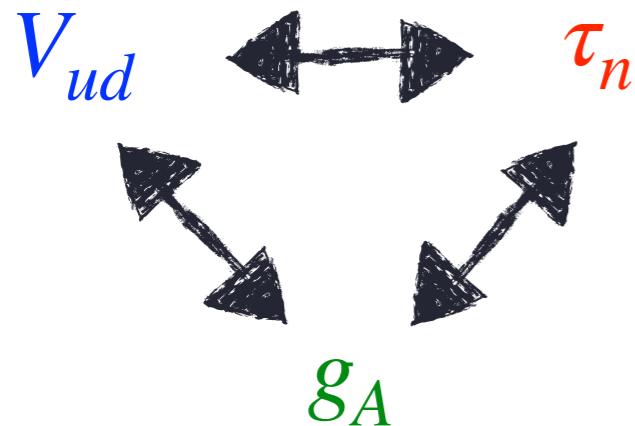


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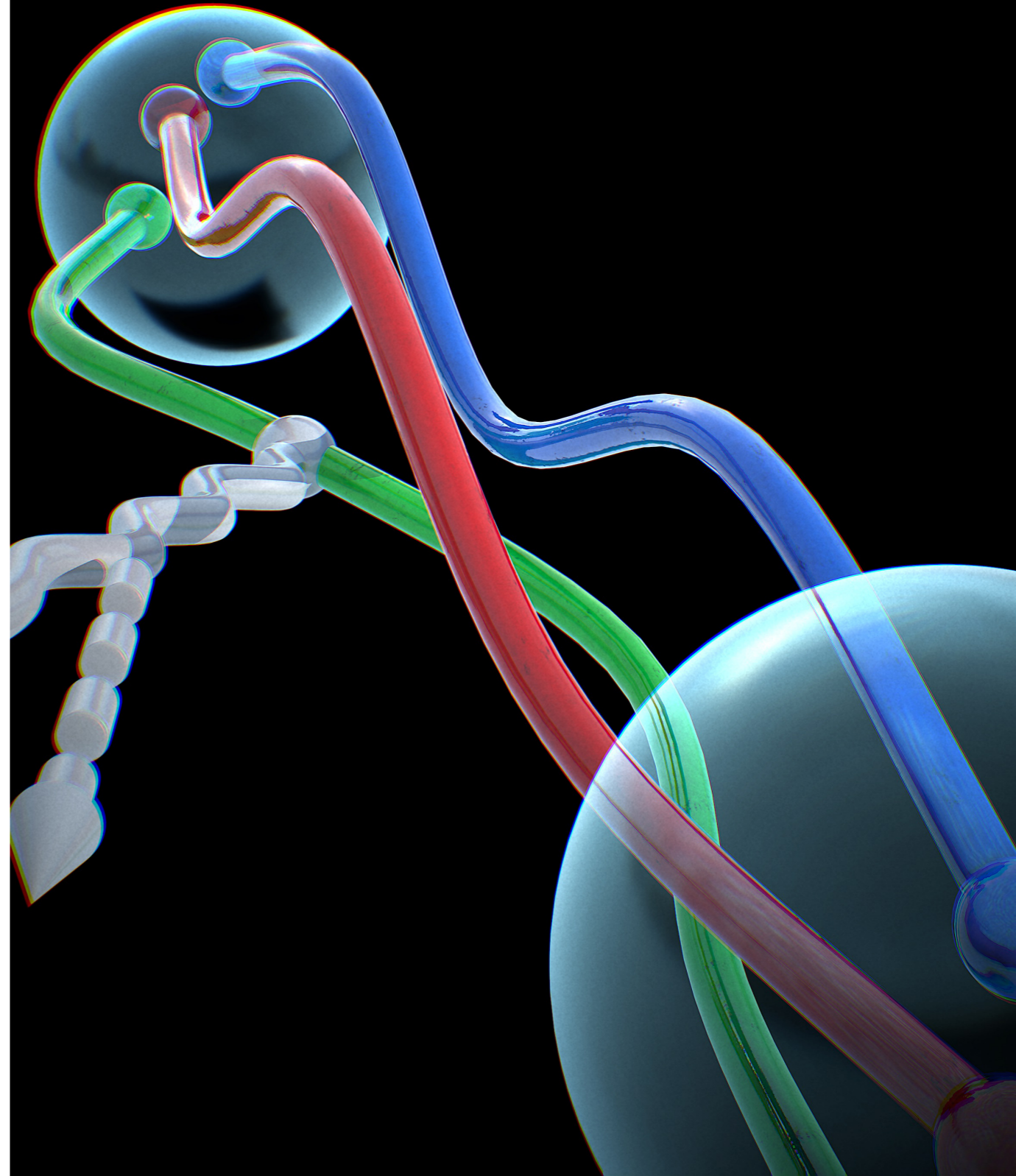
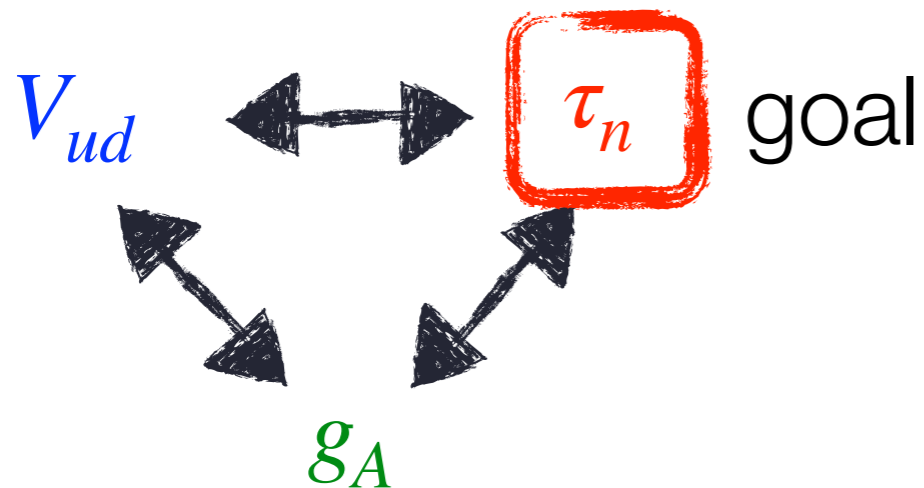


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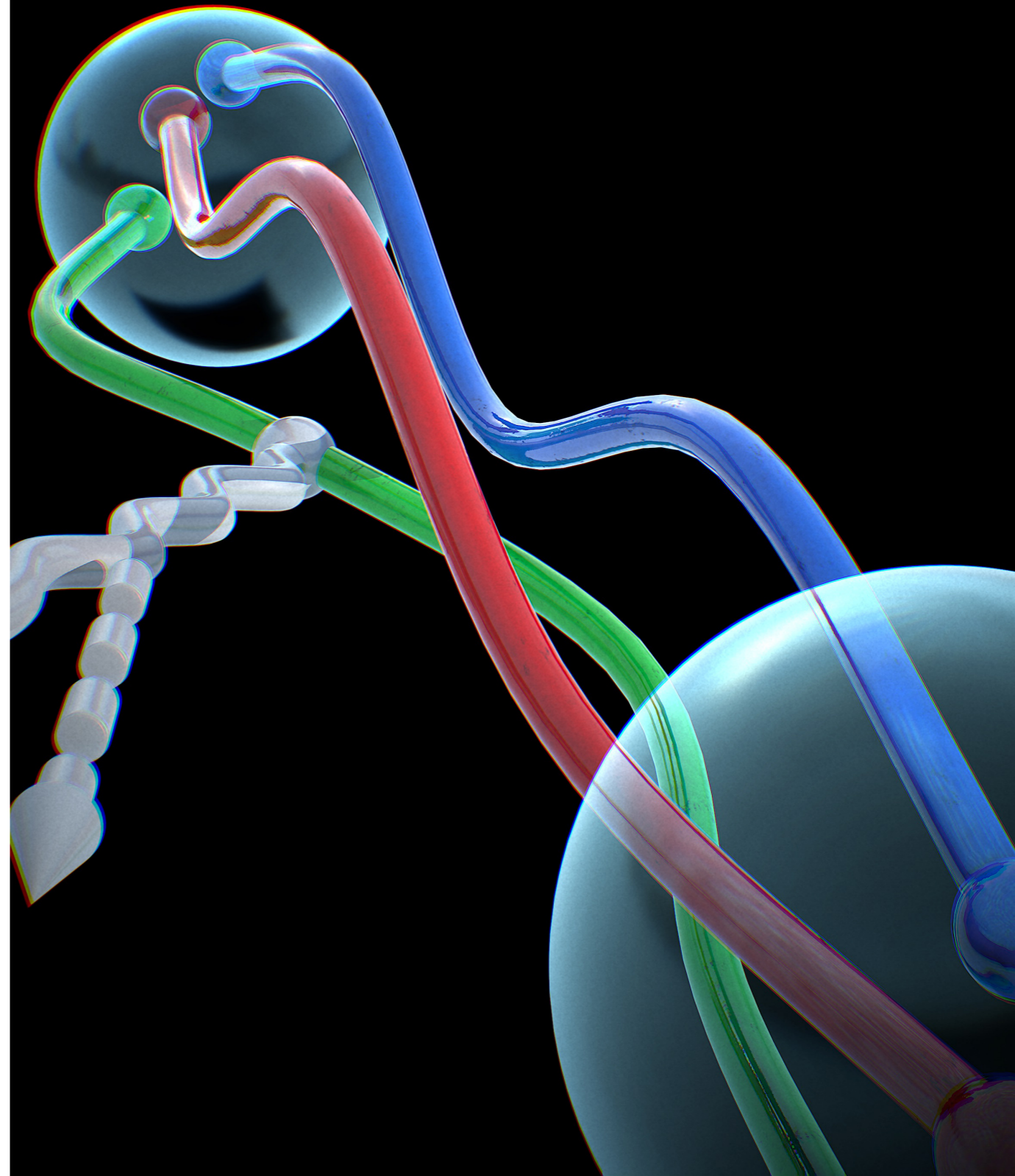
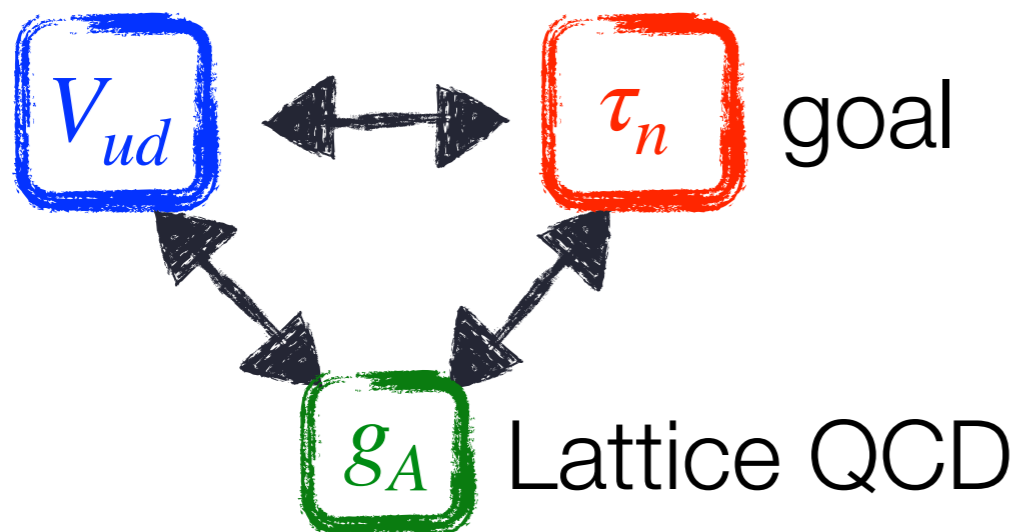


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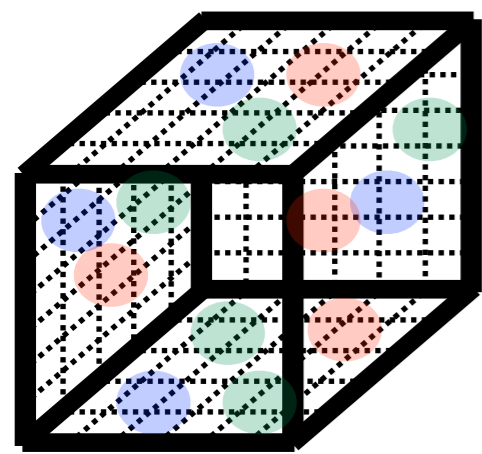
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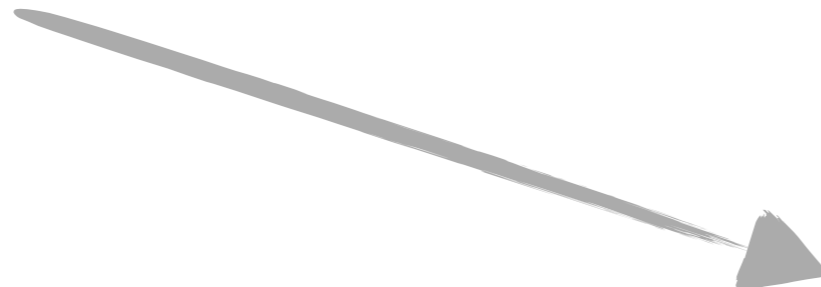
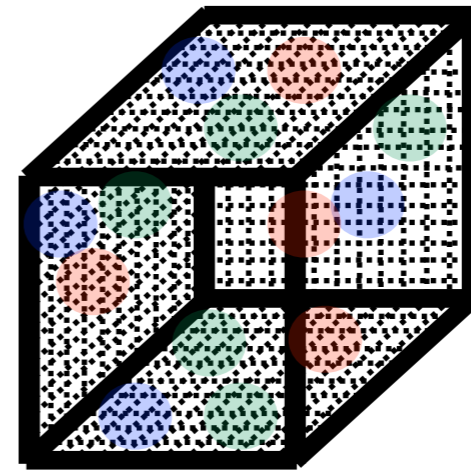
Challenging systematics



$$a \rightarrow 0$$



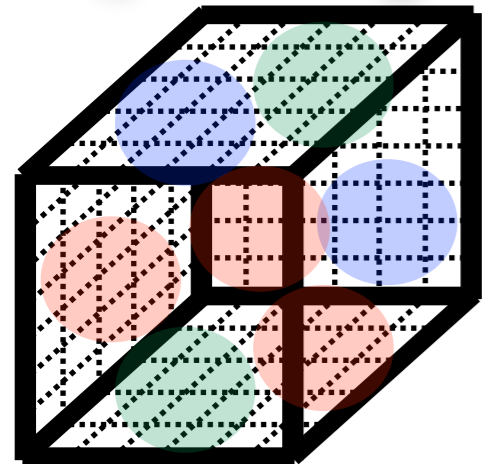
$$t_{comp} \propto \frac{1}{a^6}$$



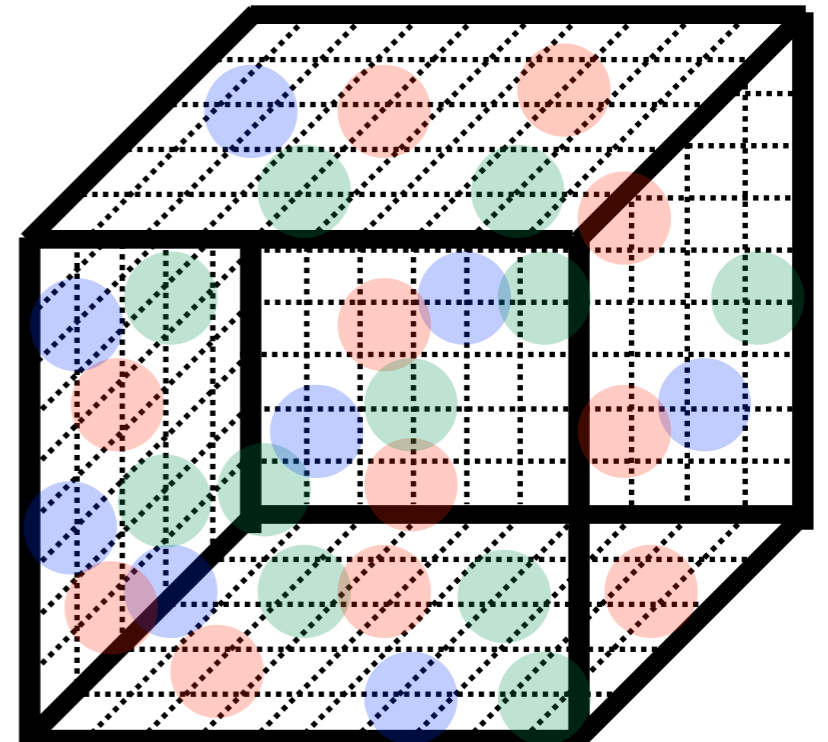
$$m_\pi \rightarrow m_\pi^{\text{phys.}}$$

$$V \rightarrow \infty$$

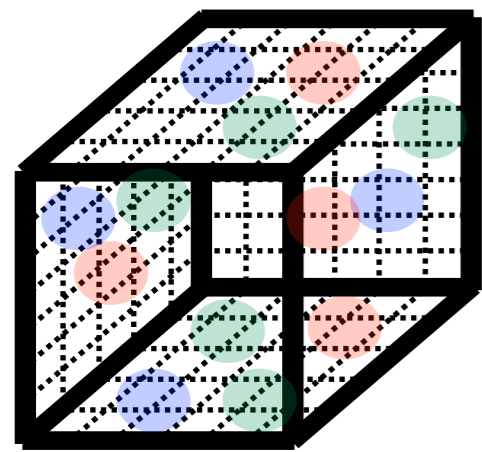
$$t_{comp} \propto V^{5/4}$$



Exponentially bad
signal-to-noise problem



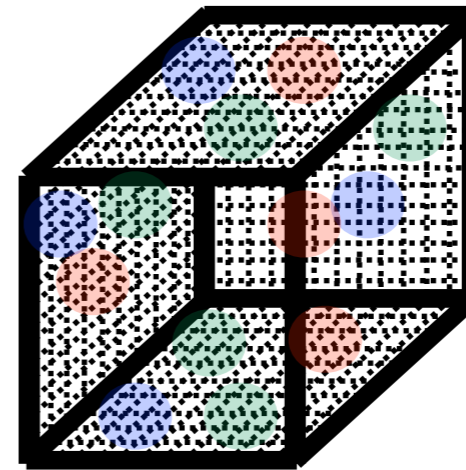
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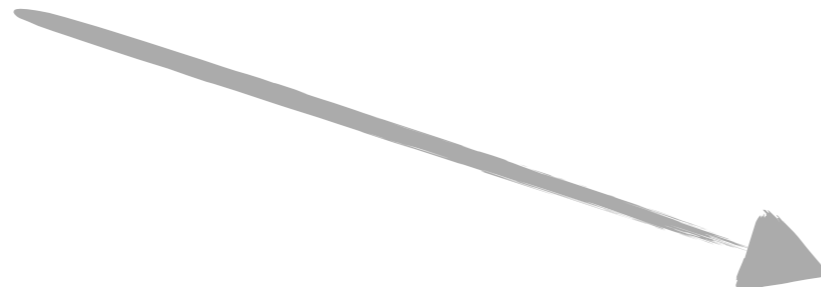
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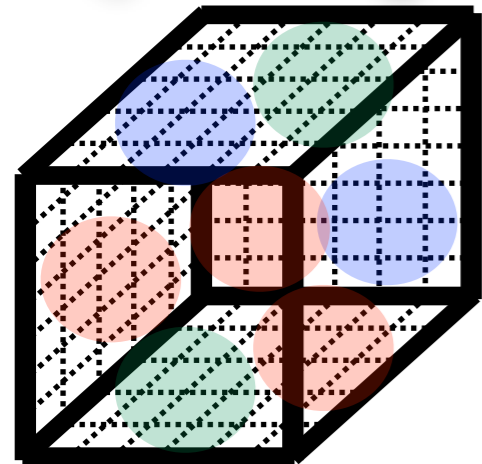
* Complete calculations require multiple "a", "V" and "m_π"
 * Increasing cost requires using "top500" supercomputers



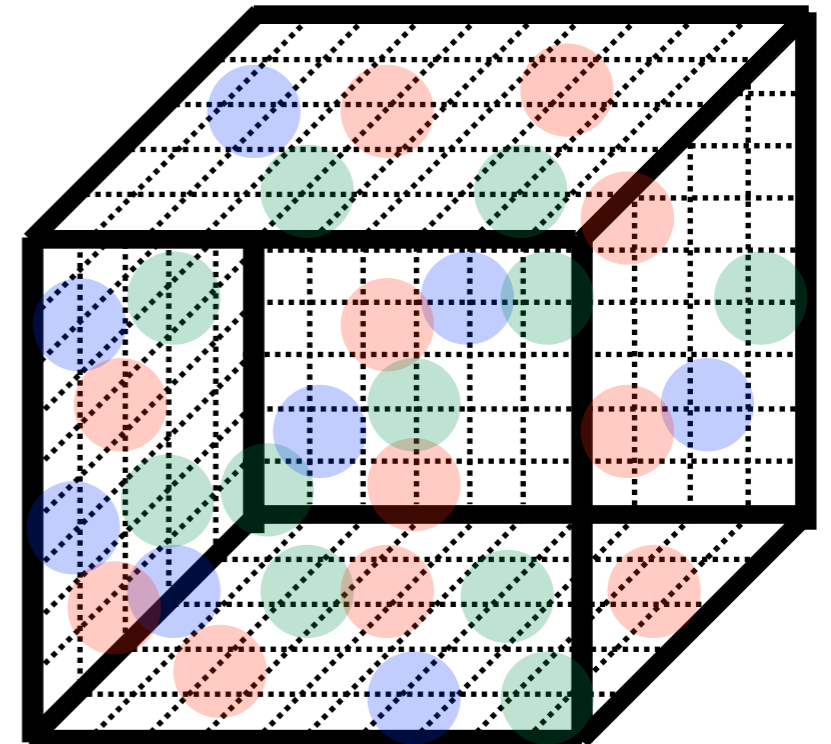
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Lattice QCD gauge configurations

HISQ gauge configuration parameters							valence parameters							
abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{\text{val.}}$	σ_{smr}	N_{smr}
a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
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a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
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abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{\text{val.}}$	σ_{smr}	N_{smr}
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a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

$$a \rightarrow 0 \quad V \rightarrow \infty$$

$$m_\pi \rightarrow m_\pi^{\text{phys.}}$$

Lattice QCD gauge configurations

HISQ gauge configuration parameters							valence parameters							
abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{\text{val.}}$	σ_{smr}	N_{smr}
a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
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a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

- ★ 16 ensembles with $N_f=2+1+1$ Highly Improved Staggered Quarks (HISQ)
- ★ 5 pion masses, 3 lattice spacings, multiple volumes
- ★ High statistics ensembles, publicly available



OAK RIDGE
National Laboratory

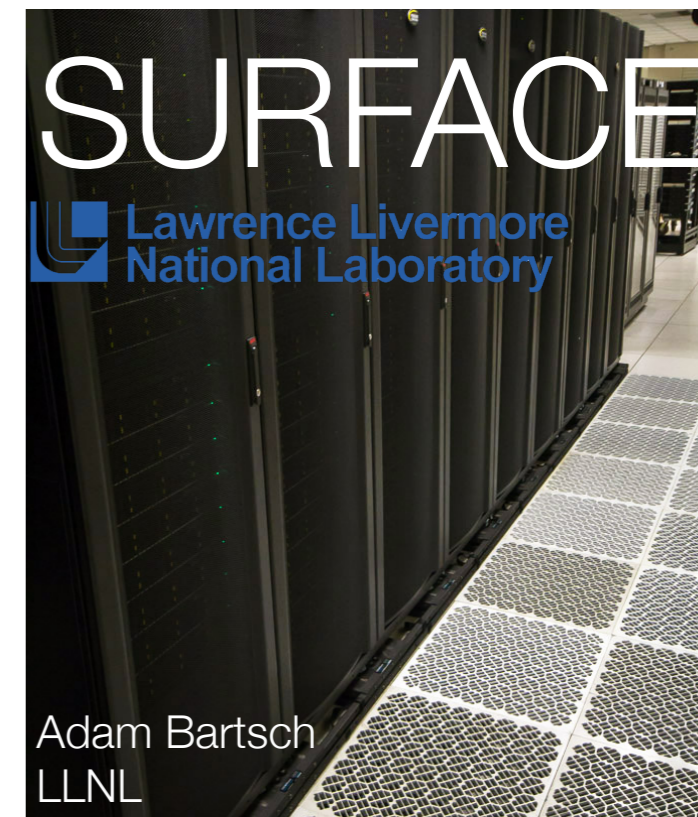
LEADERSHIP
COMPUTING
FACILITY

TITAN



K R G...
LLNL

VULCAN



SURFACE

Lawrence Livermore
National Laboratory

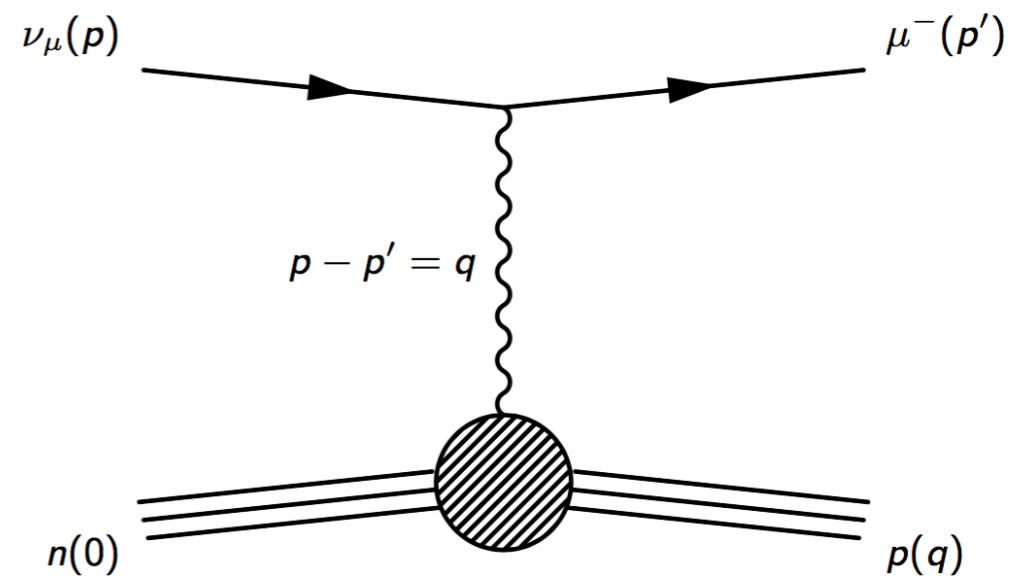
Adam Bartsch
LLNL

g_A from LQCD

g_A from LQCD

- ❖ Matrix element of the axial current between nucleon ground states

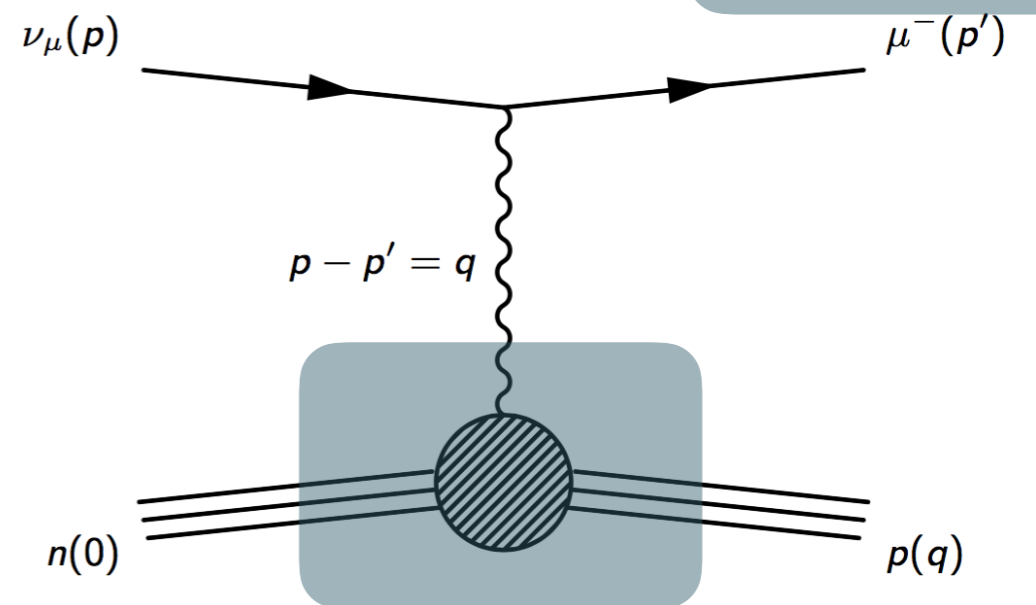
$$\mathcal{M}_{\nu_\mu n \rightarrow \mu p}(p, p') = \langle \mu(p') | (V_\mu - A_\mu) | \nu(p) \rangle \langle p(q) | (V_\mu - A_\mu) | n(0) \rangle$$



g_A from LQCD

- ❖ Matrix element of the axial current between nucleon ground states

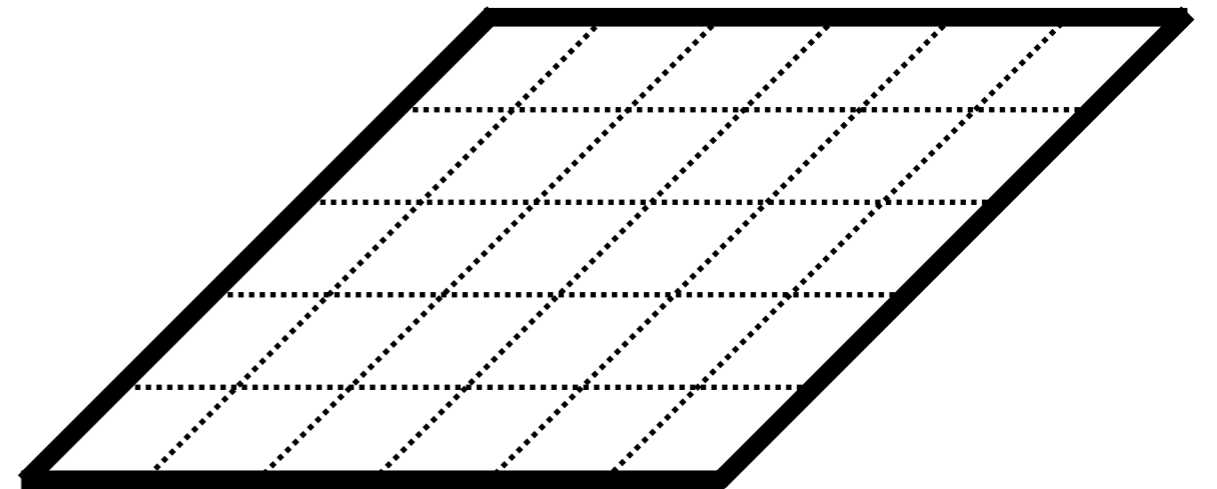
$$\mathcal{M}_{\nu_\mu n \rightarrow \mu p}(p, p') = \langle \mu(p') | (V_\mu - A_\mu) | \nu(p) \rangle \langle p(q) | (V_\mu - A_\mu) | n(0) \rangle$$



g_A from LQCD

- ❖ Matrix element of the axial current between nucleon ground states

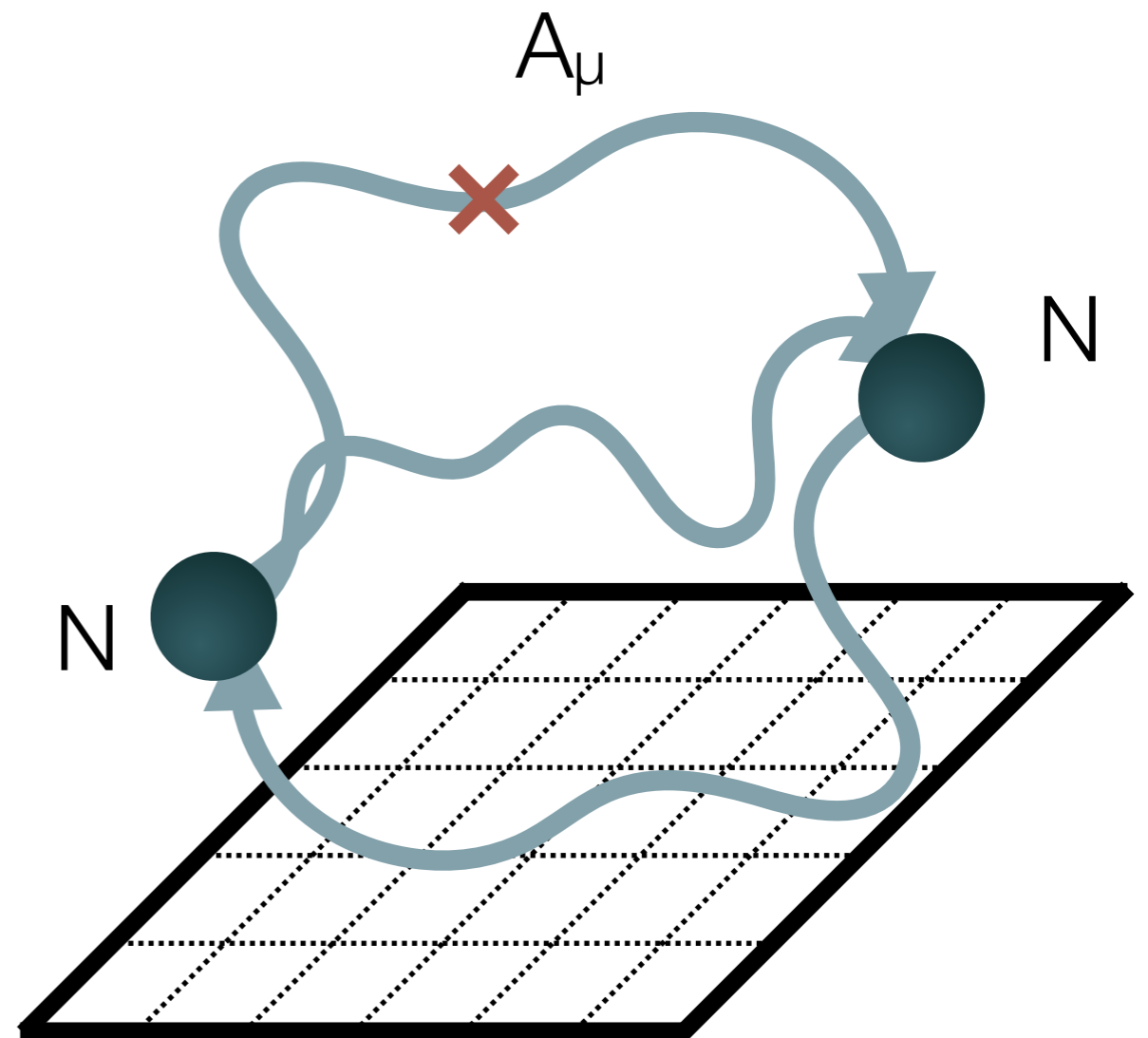
$$\langle 0 | N(t) A_\mu(\tau) N(t') | 0 \rangle$$



g_A from LQCD

- ❖ Matrix element of the axial current between nucleon ground states
- ❖ Calculate a **3-point correlation function**:

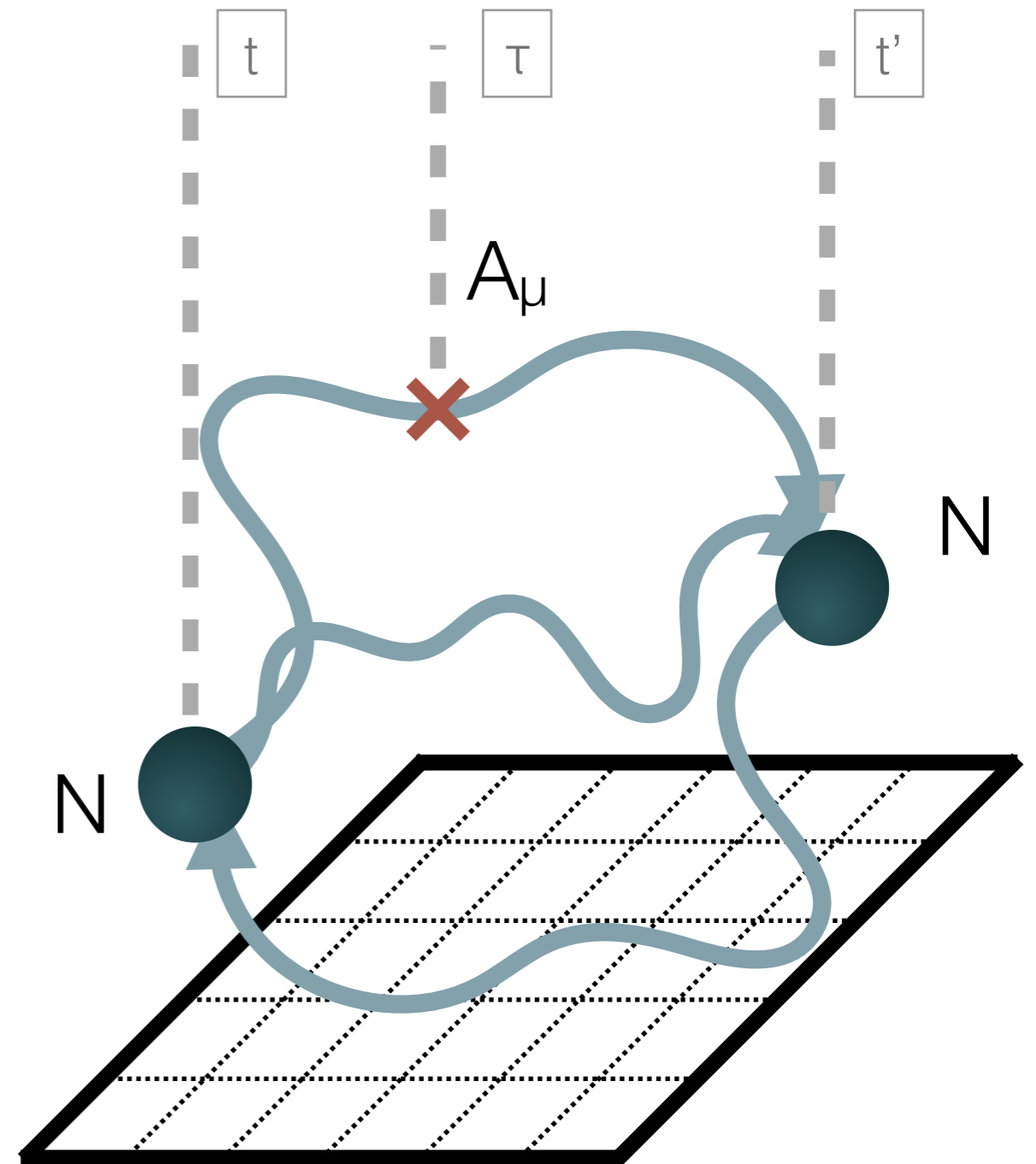
$$\langle 0 | N(t) A_\mu(\tau) N(t') | 0 \rangle$$



g_A from LQCD

- ❖ Matrix element of the axial current between nucleon ground states
- ❖ Calculate a **3-point correlation function**:
- ❖ 2 independent time variables
($t_{\text{sep}} = t' - t, \tau$)

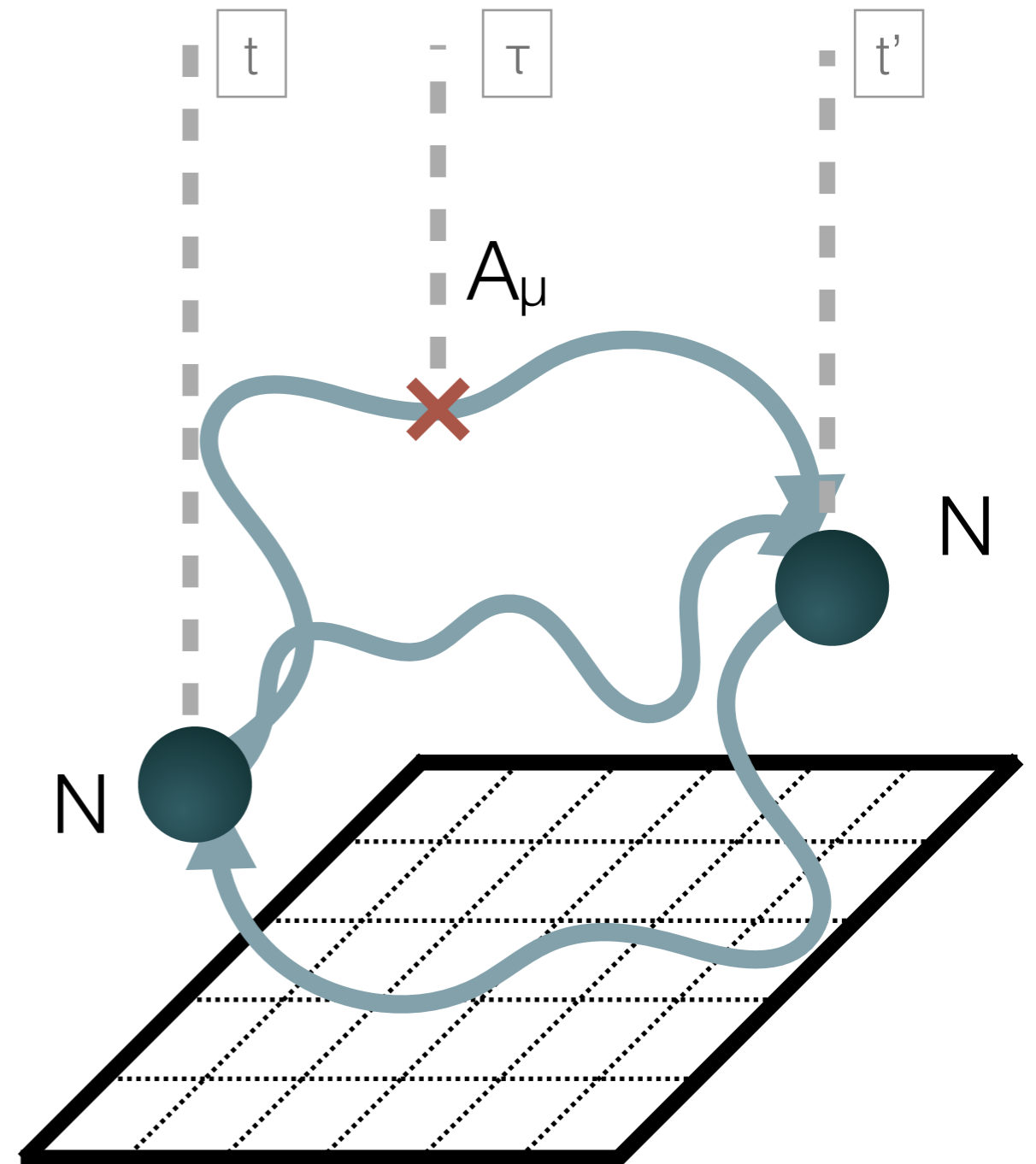
$$\langle 0 | N(t) A_\mu(\tau) N(t') | 0 \rangle$$



g_A from LQCD

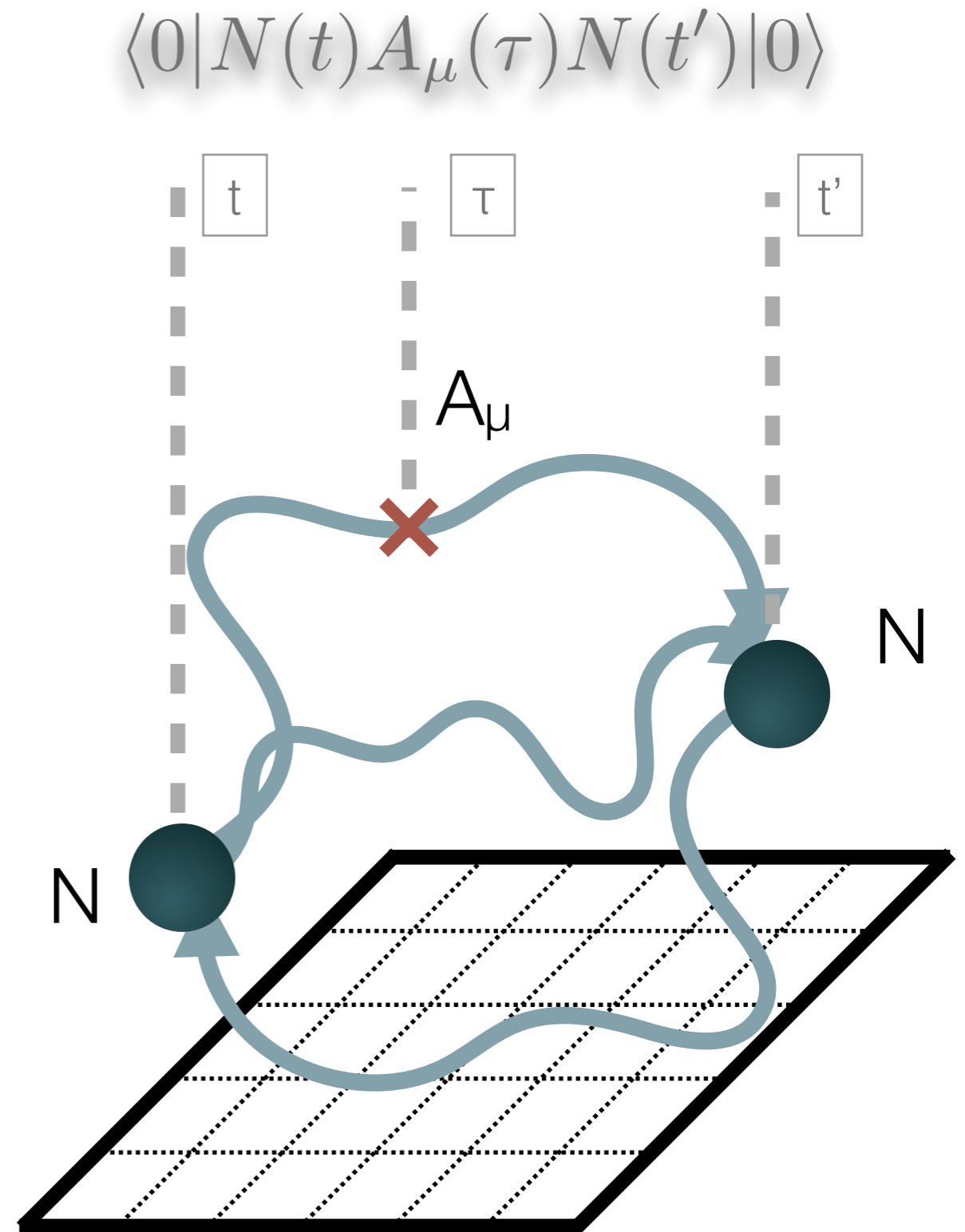
- ❖ Matrix element of the axial current between nucleon ground states
- ❖ Calculate a **3-point correlation function**:
- ❖ 2 independent time variables
($t_{\text{sep}} = t' - t, \tau$)
- ❖ Statistical noise increases *rapidly* with t_{sep}

$$\langle 0 | N(t) A_\mu(\tau) N(t') | 0 \rangle$$



g_A from LQCD

- ❖ Matrix element of the axial current between nucleon ground states
- ❖ Calculate a **3-point correlation function**:
 - ❖ 2 independent time variables
($t_{\text{sep}} = t' - t, \tau$)
 - ❖ Statistical noise increases *rapidly* with t_{sep}
- ❖ Excited states contributions disappear at large (t_{sep}, τ)

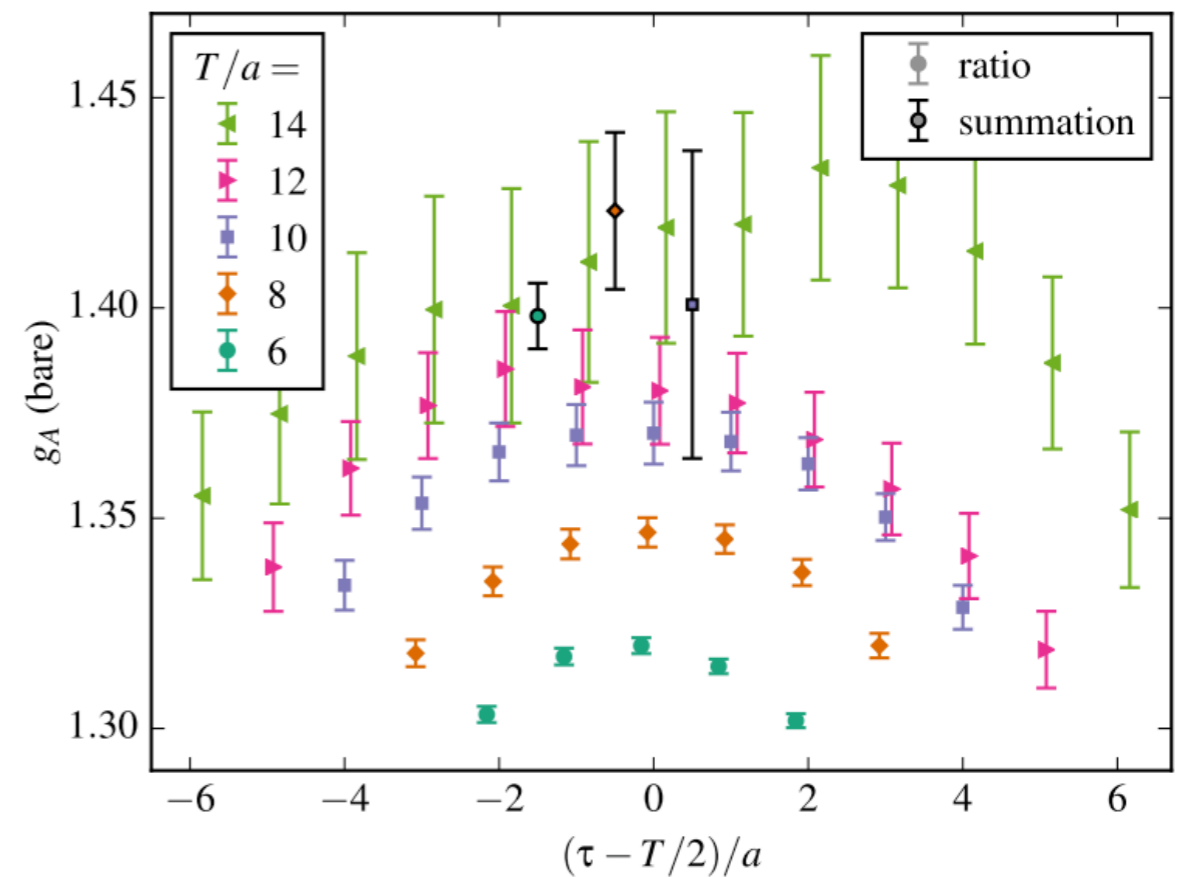
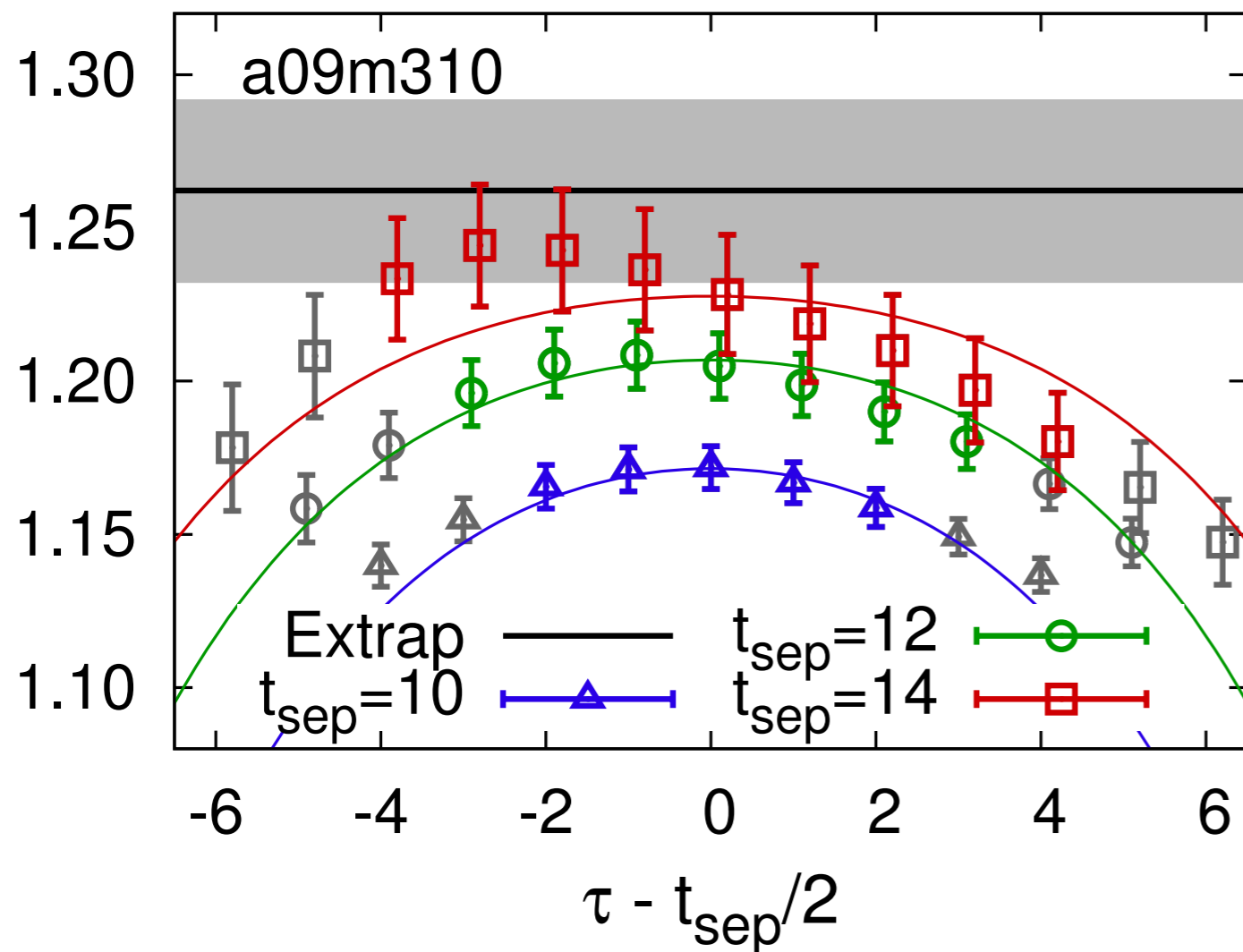


Example of excited states contaminations

Different lattice discretizations and gauge configurations

[PNDME Phys. Rev. D94 (2016) arXiv:1606.07049]

[LHPC arXiv:1703.06703]

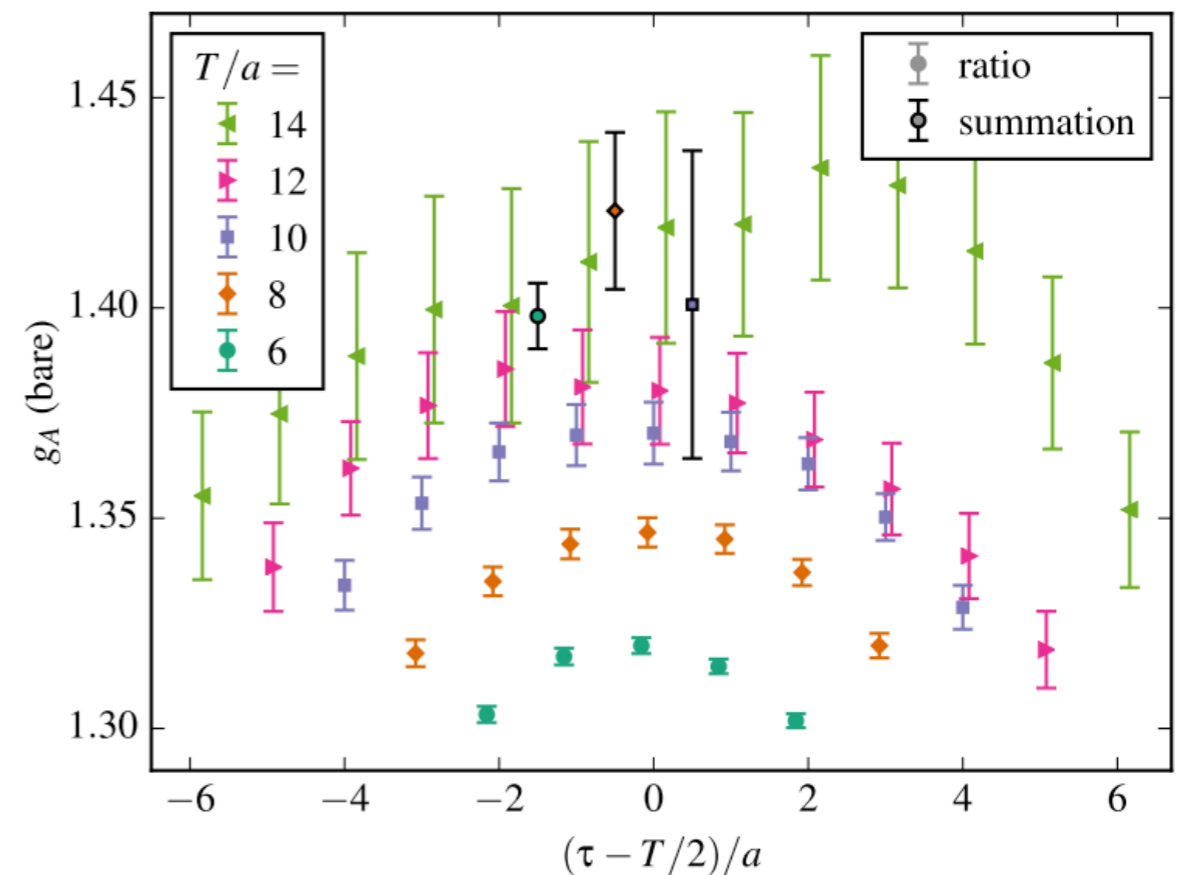
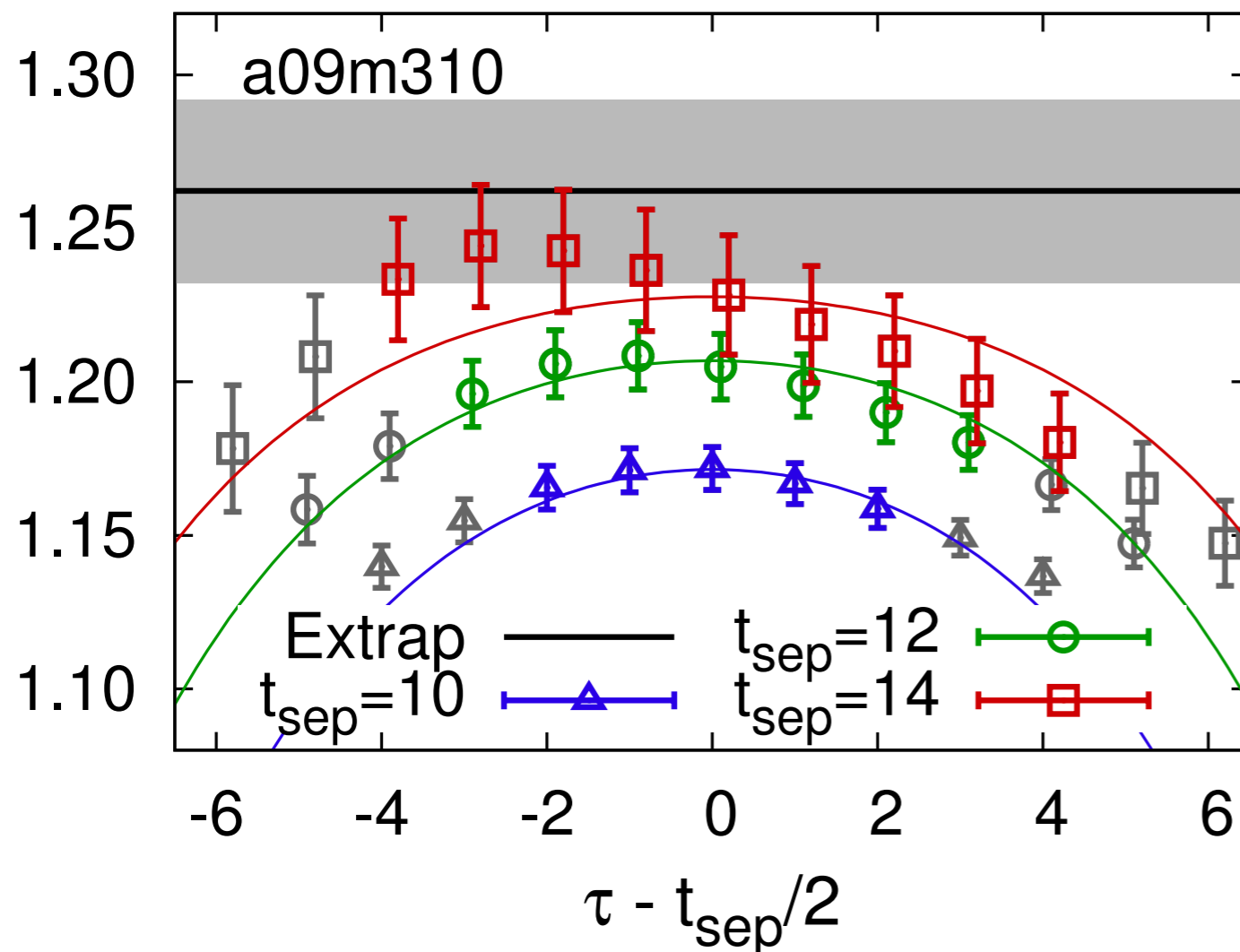


Example of excited states contaminations

Different lattice discretizations and gauge configurations

[PNDME Phys. Rev. D94 (2016) arXiv:1606.07049]

[LHPC arXiv:1703.06703]



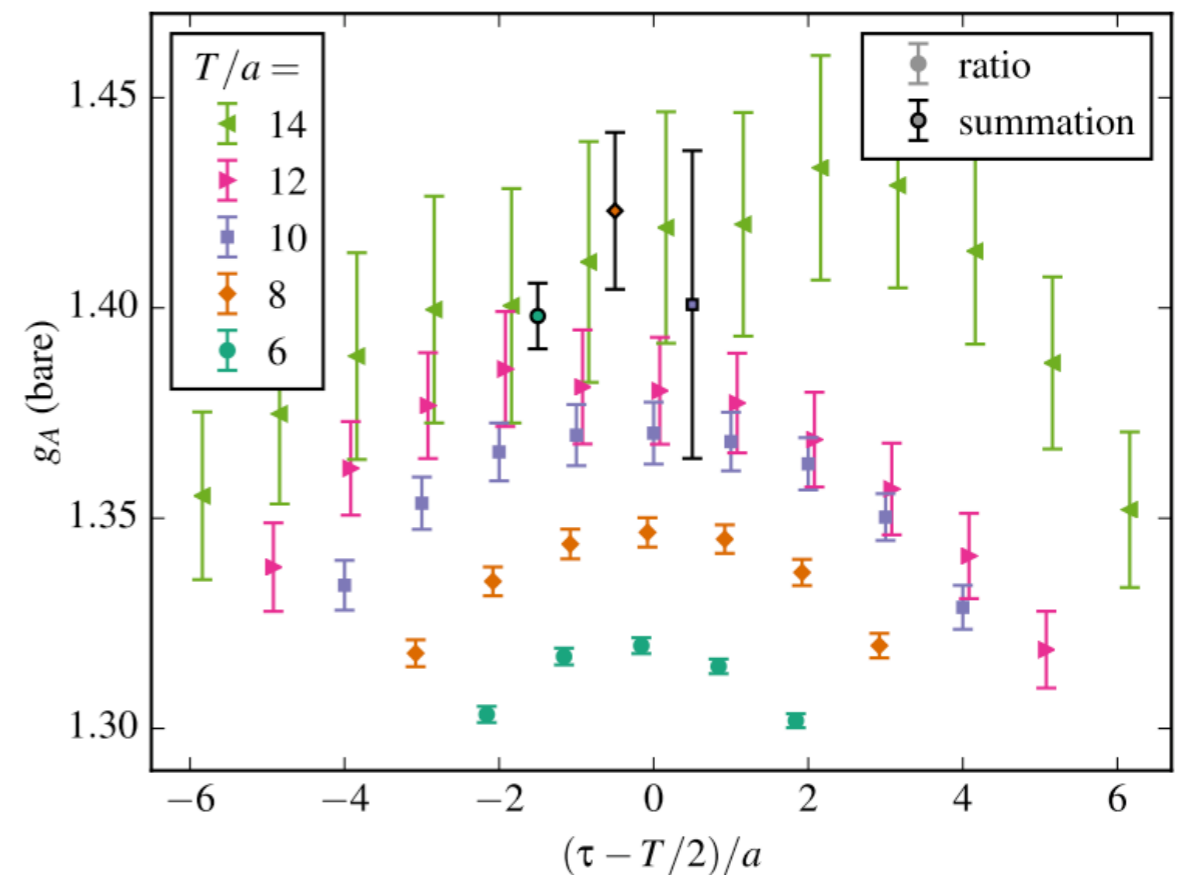
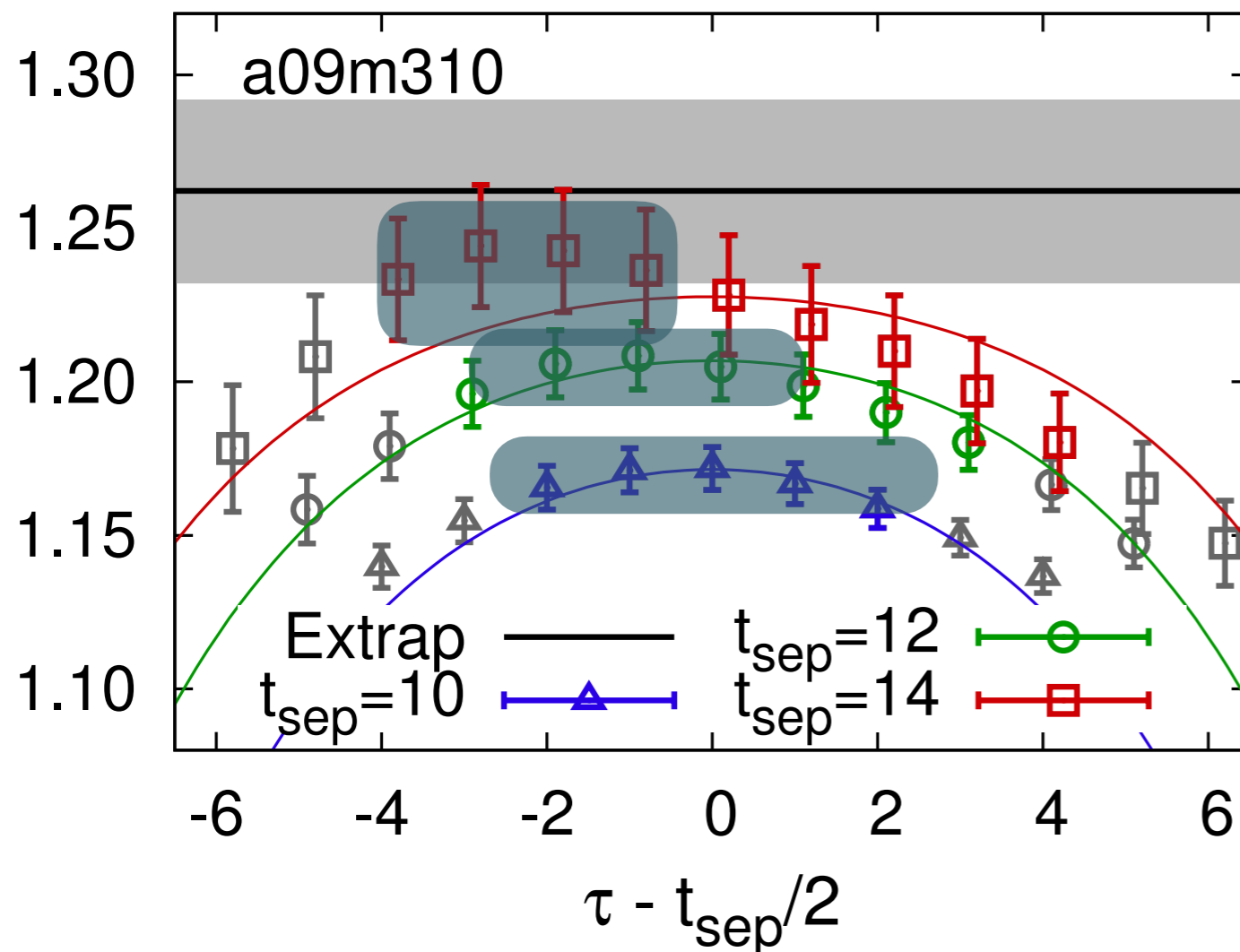
Should have a plateau at each t_{sep} if no exc. states

Example of excited states contaminations

Different lattice discretizations and gauge configurations

[PNDME Phys. Rev. D94 (2016) arXiv:1606.07049]

[LHPC arXiv:1703.06703]



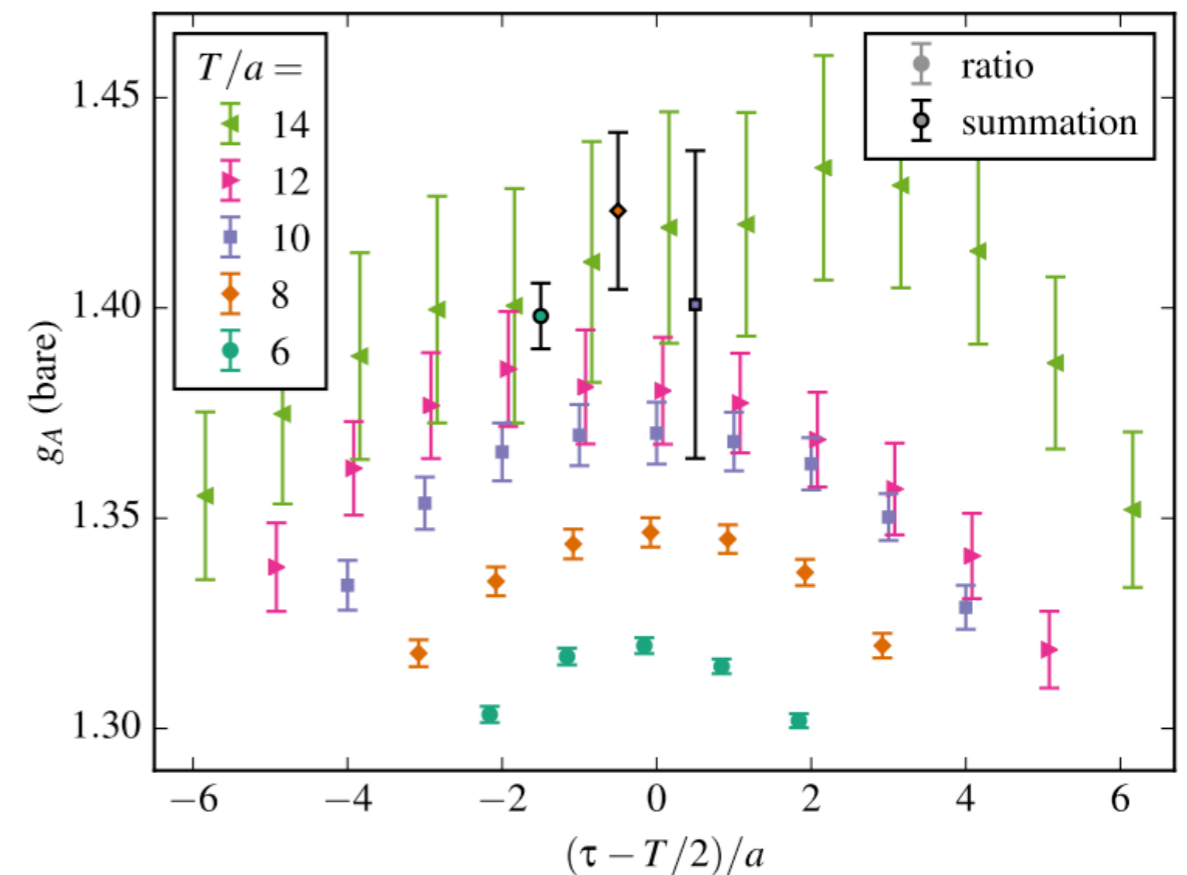
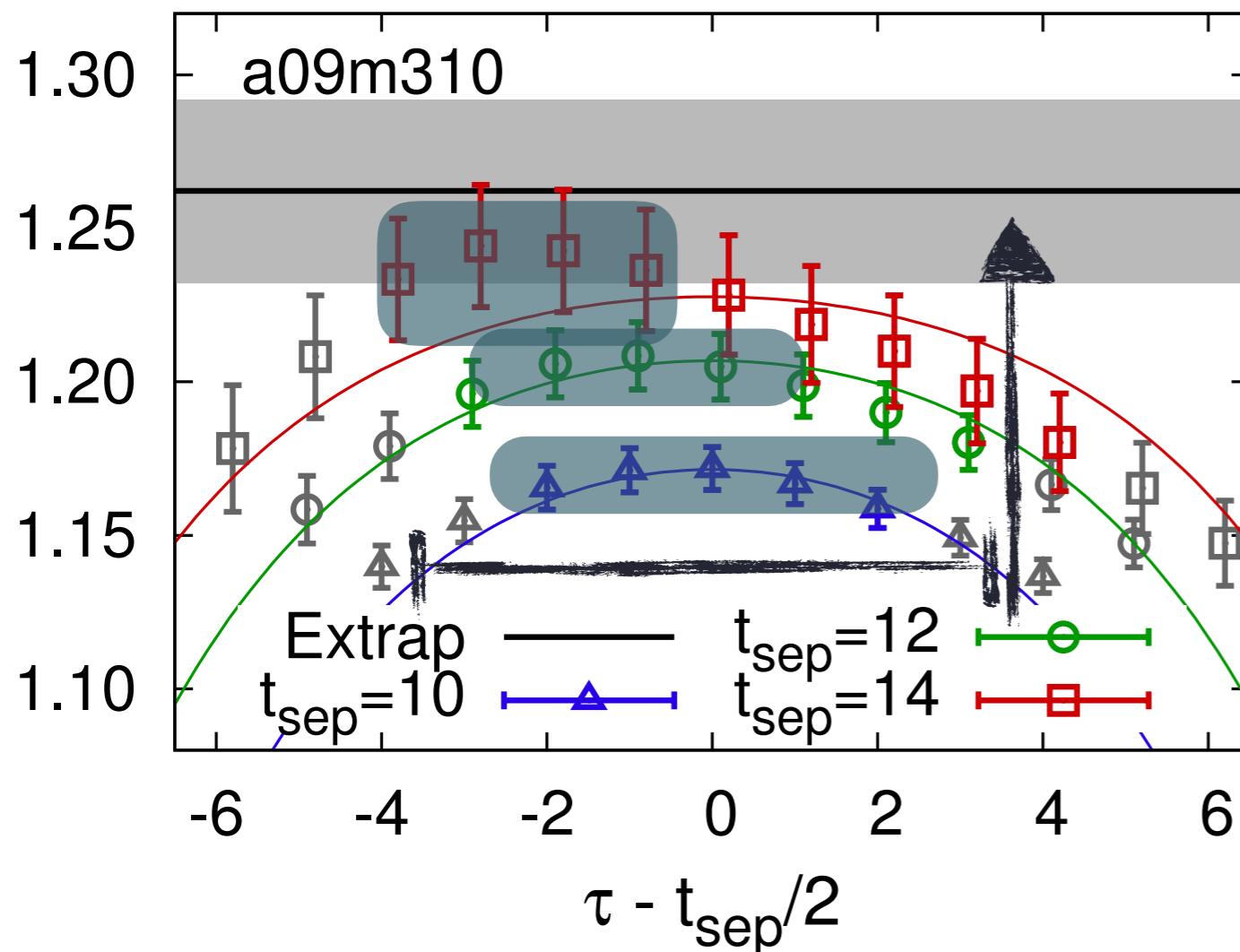
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Example of excited states contaminations

Different lattice discretizations and gauge configurations

[PNDME Phys. Rev. D94 (2016) arXiv:1606.07049]

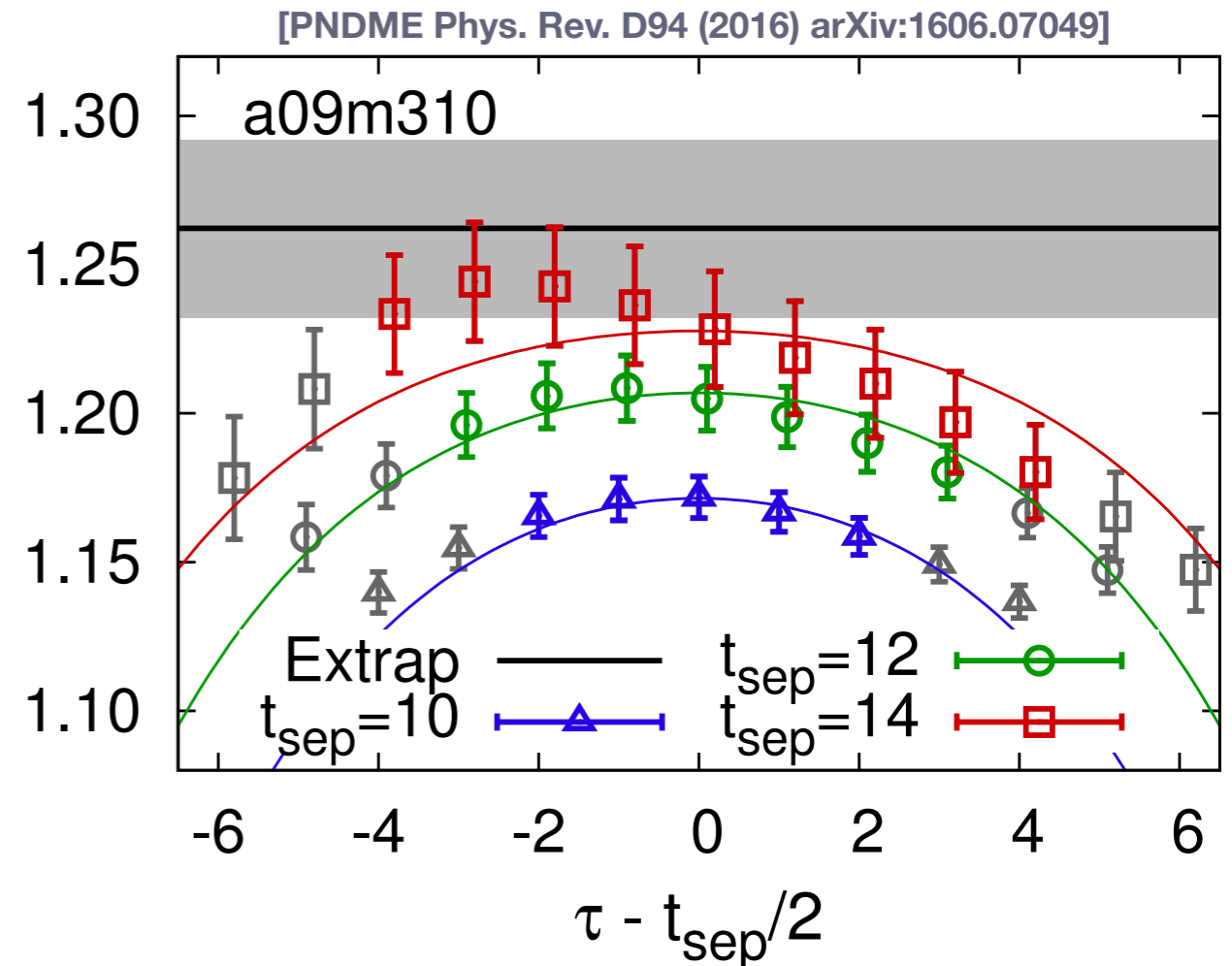
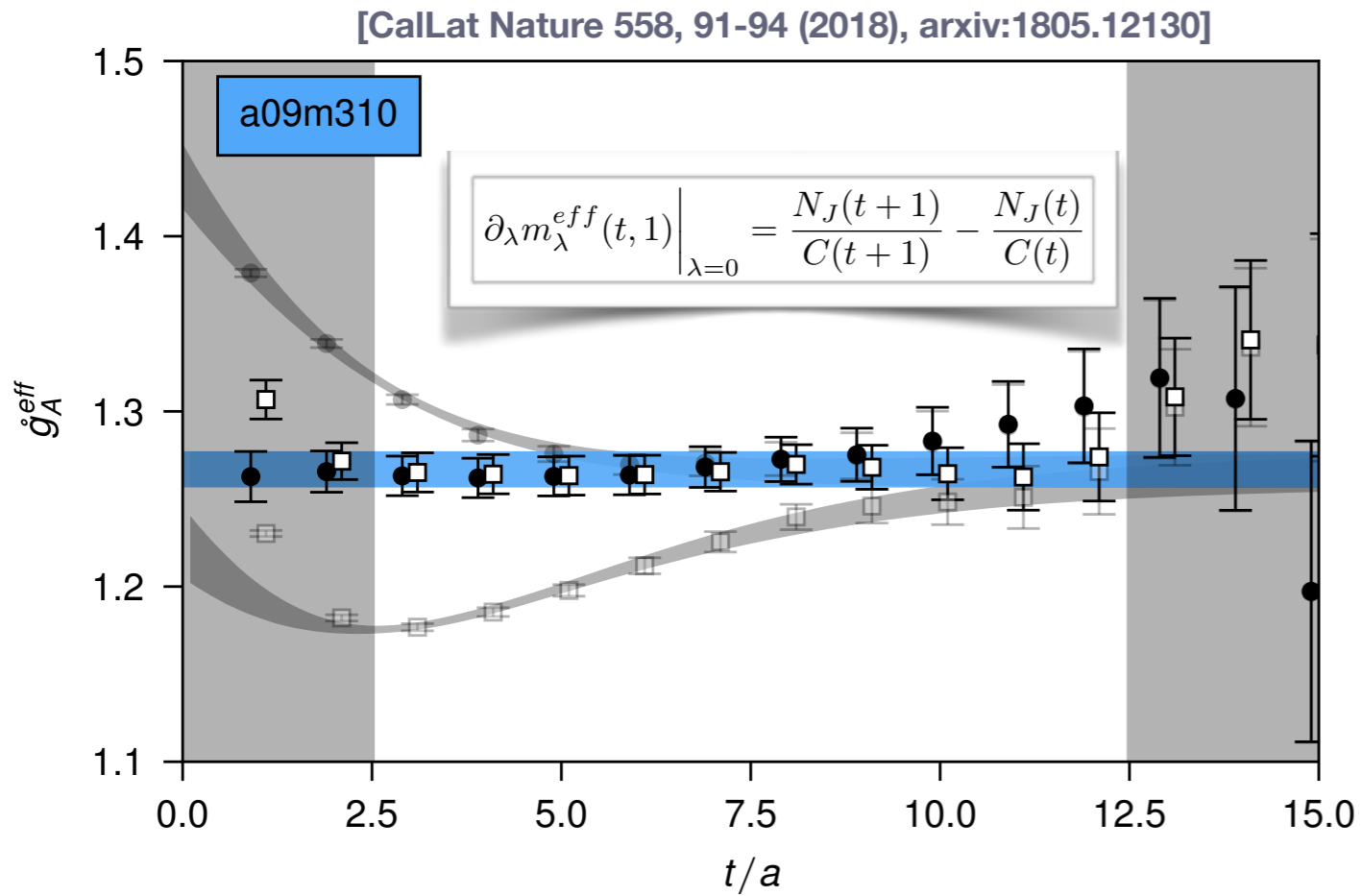
[LHPC arXiv:1703.06703]



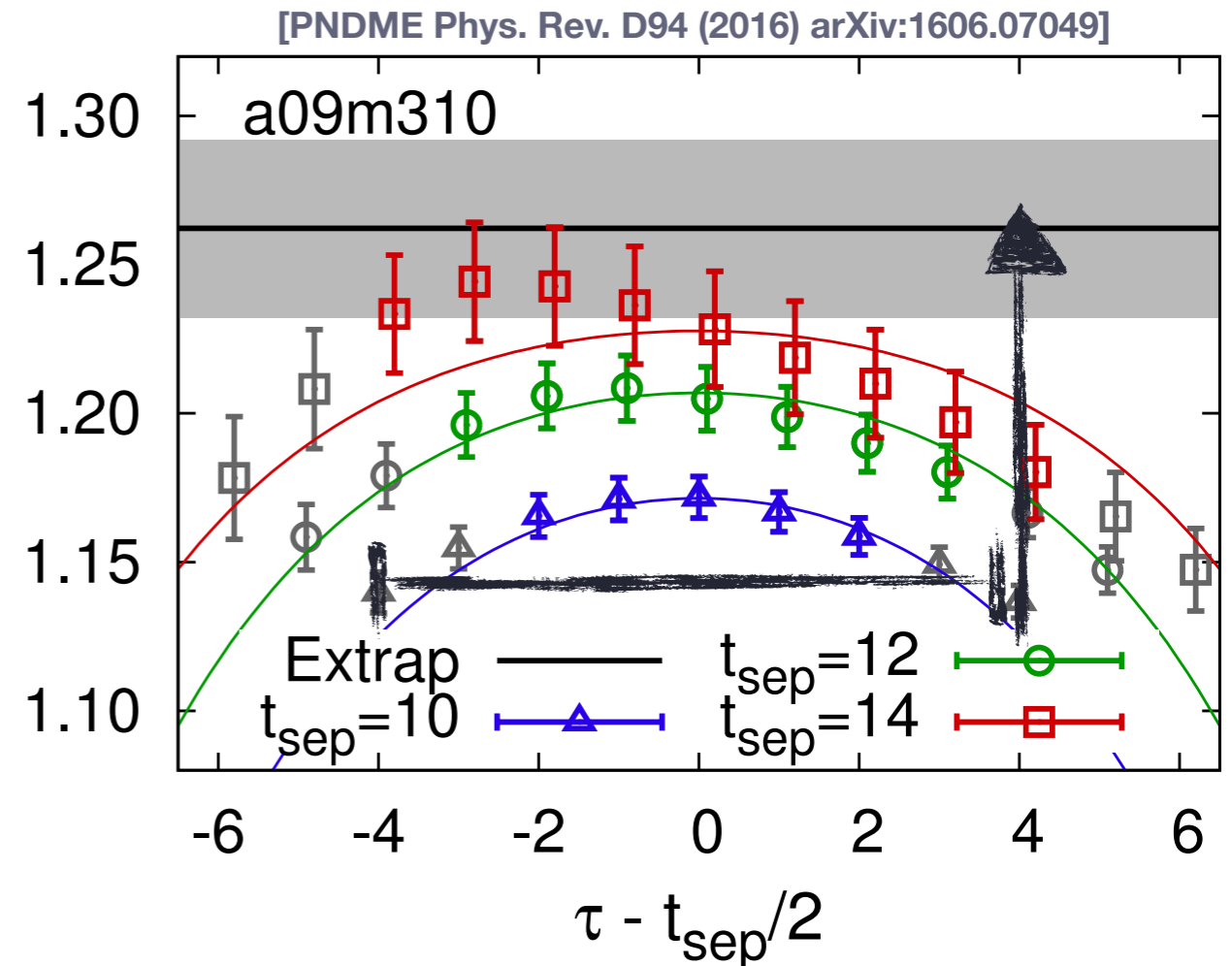
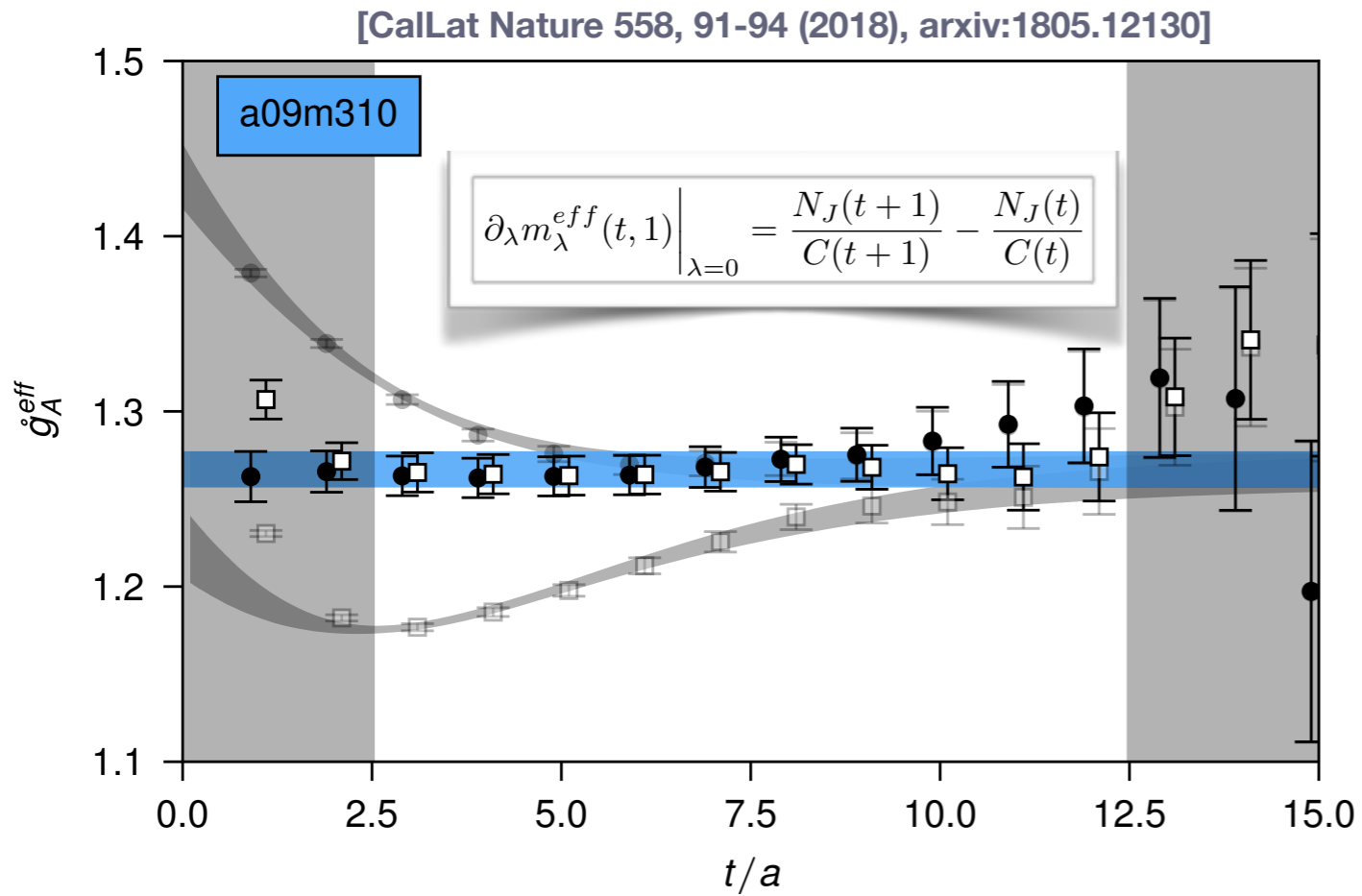
Need to fit in 2 variables:
increased systematics

Should have a plateau at each
 t_{sep} if no exc. states

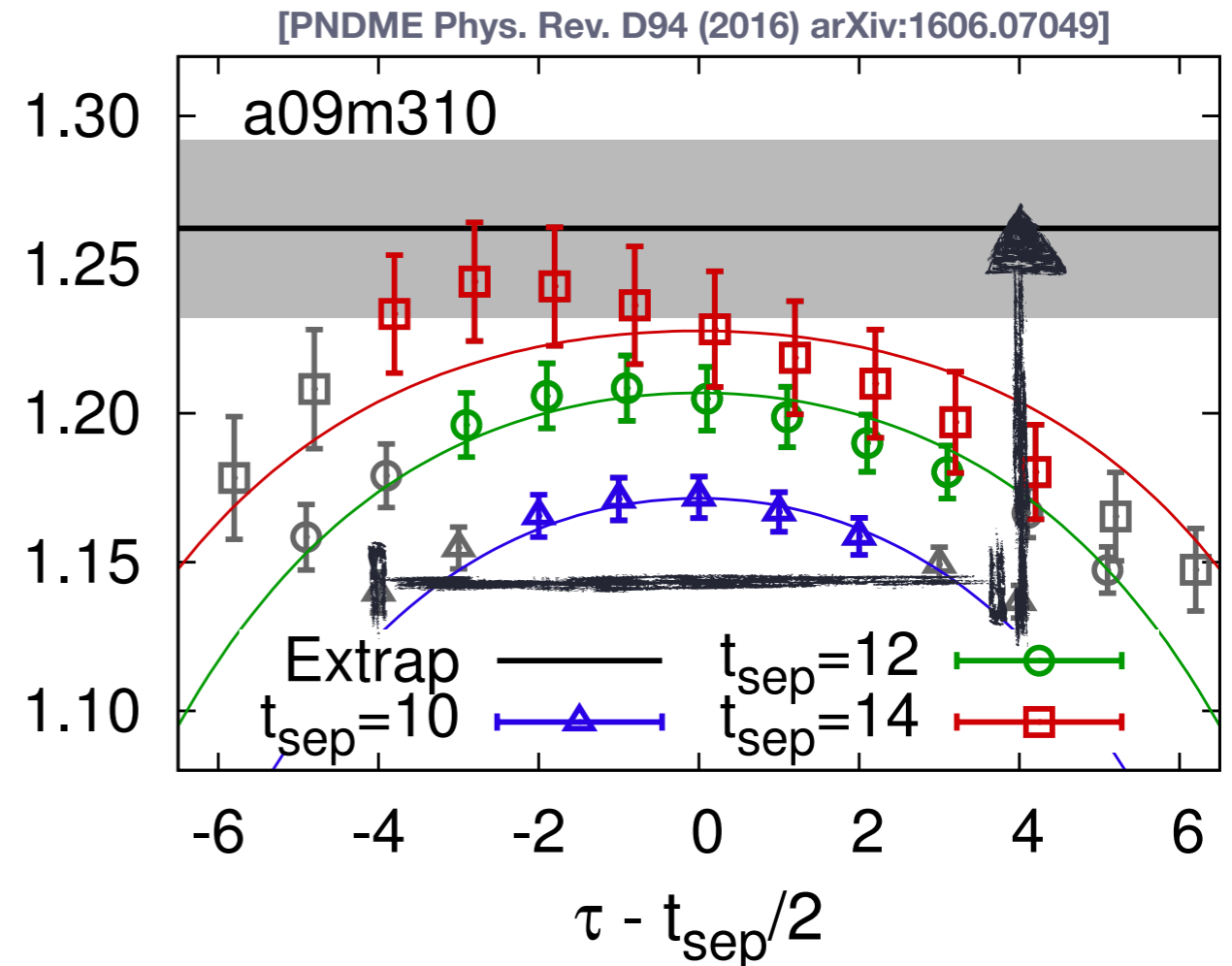
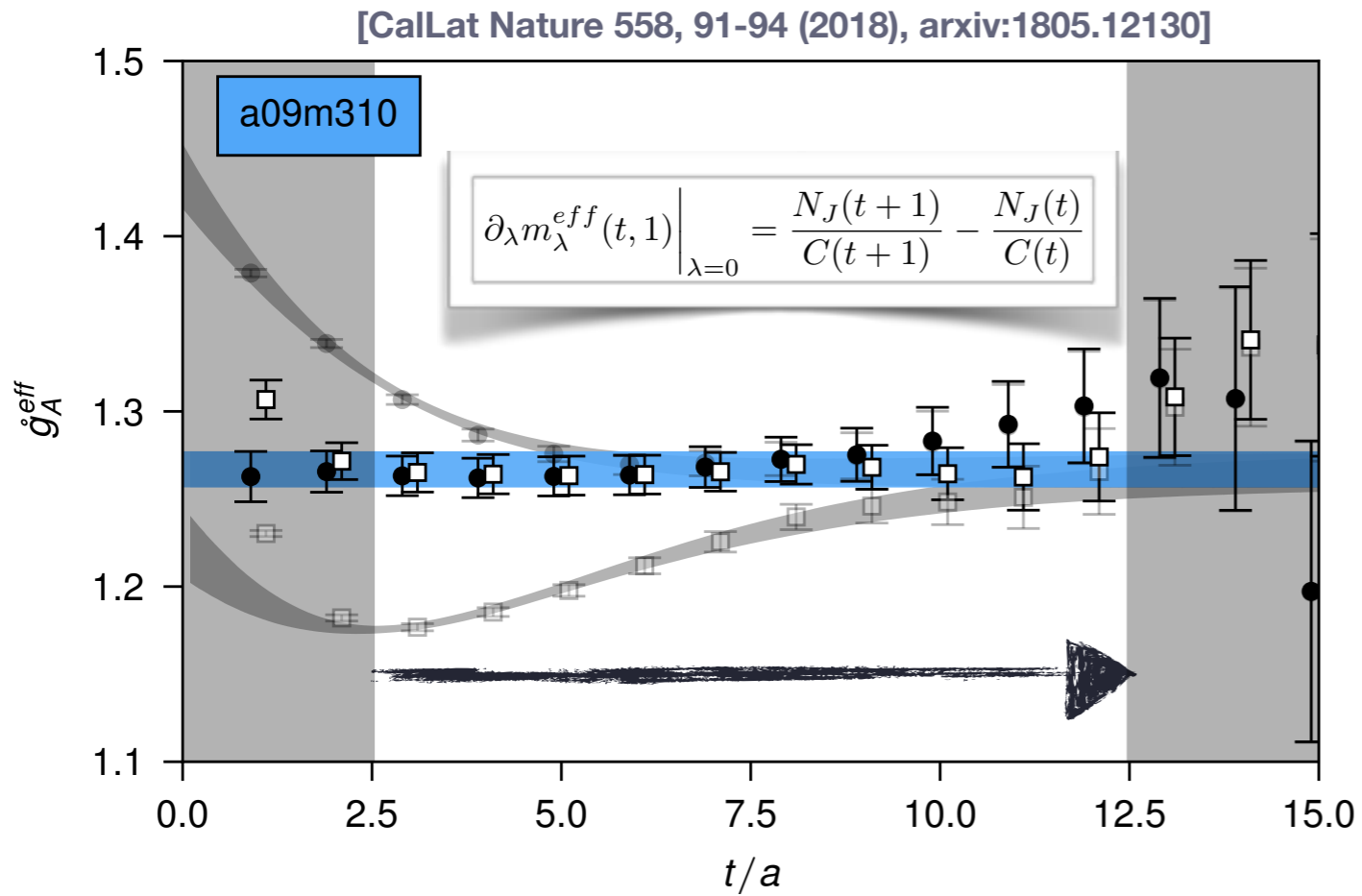
Improved method to reduce excited states



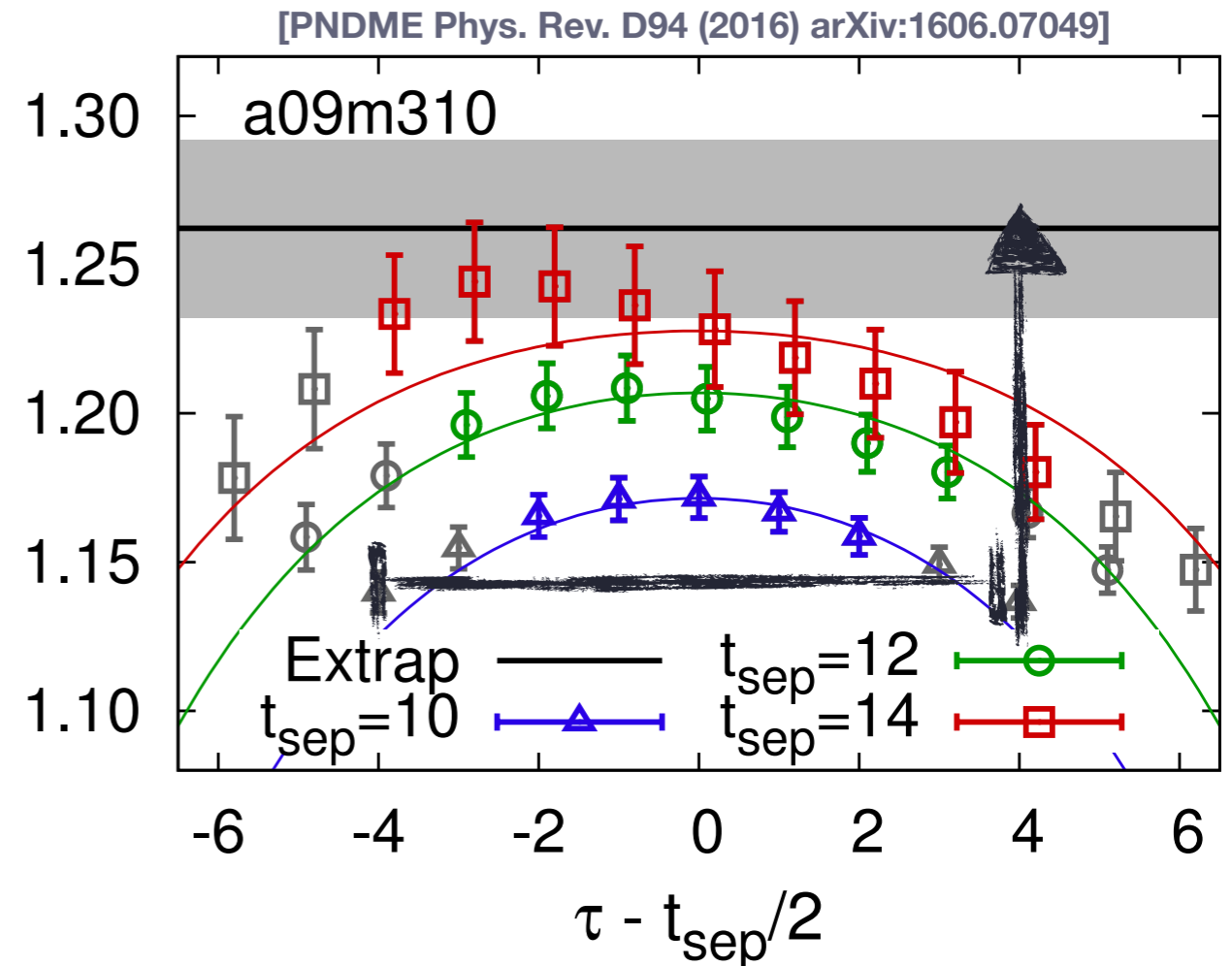
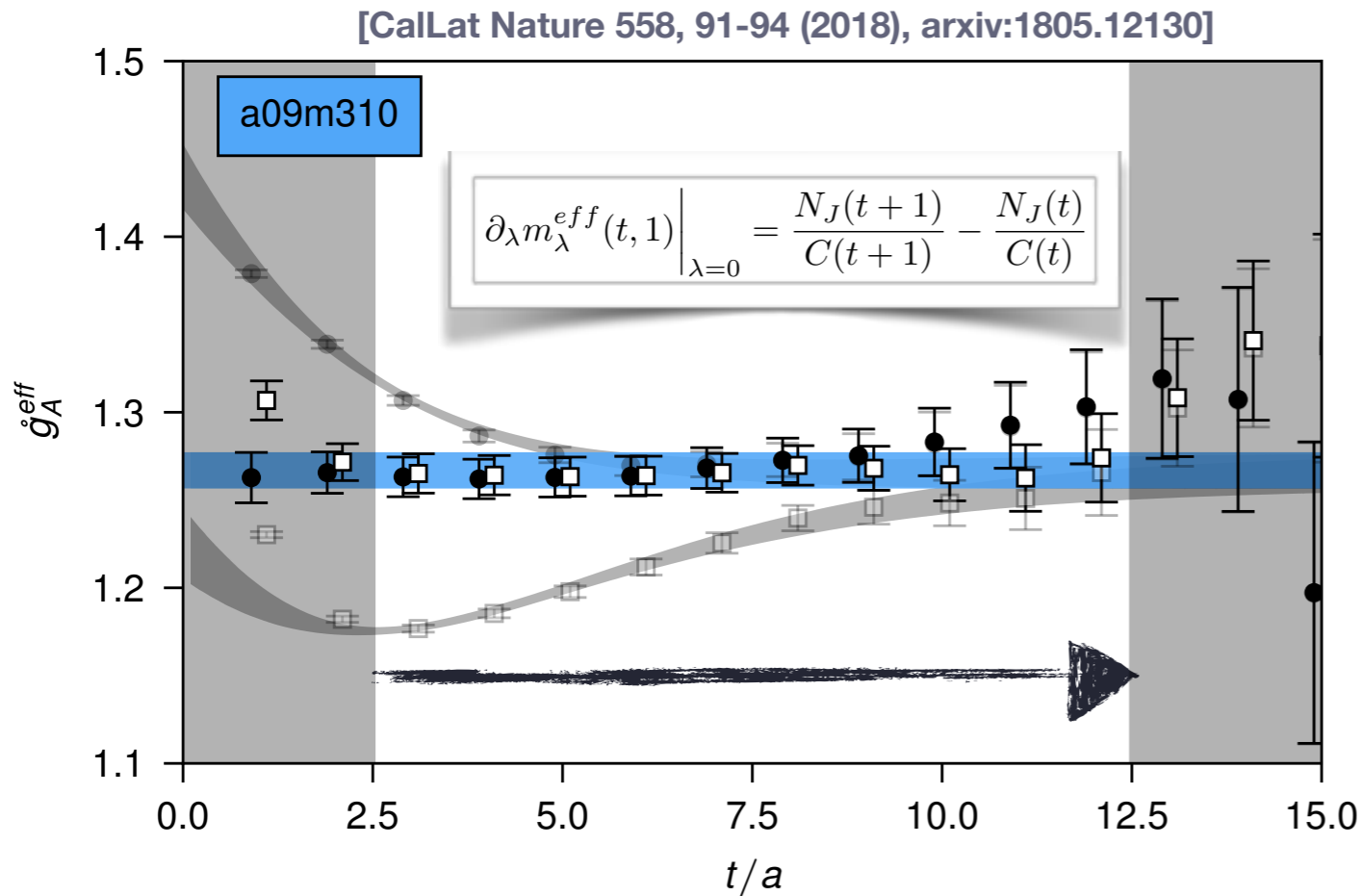
Improved method to reduce excited states



Improved method to reduce excited states



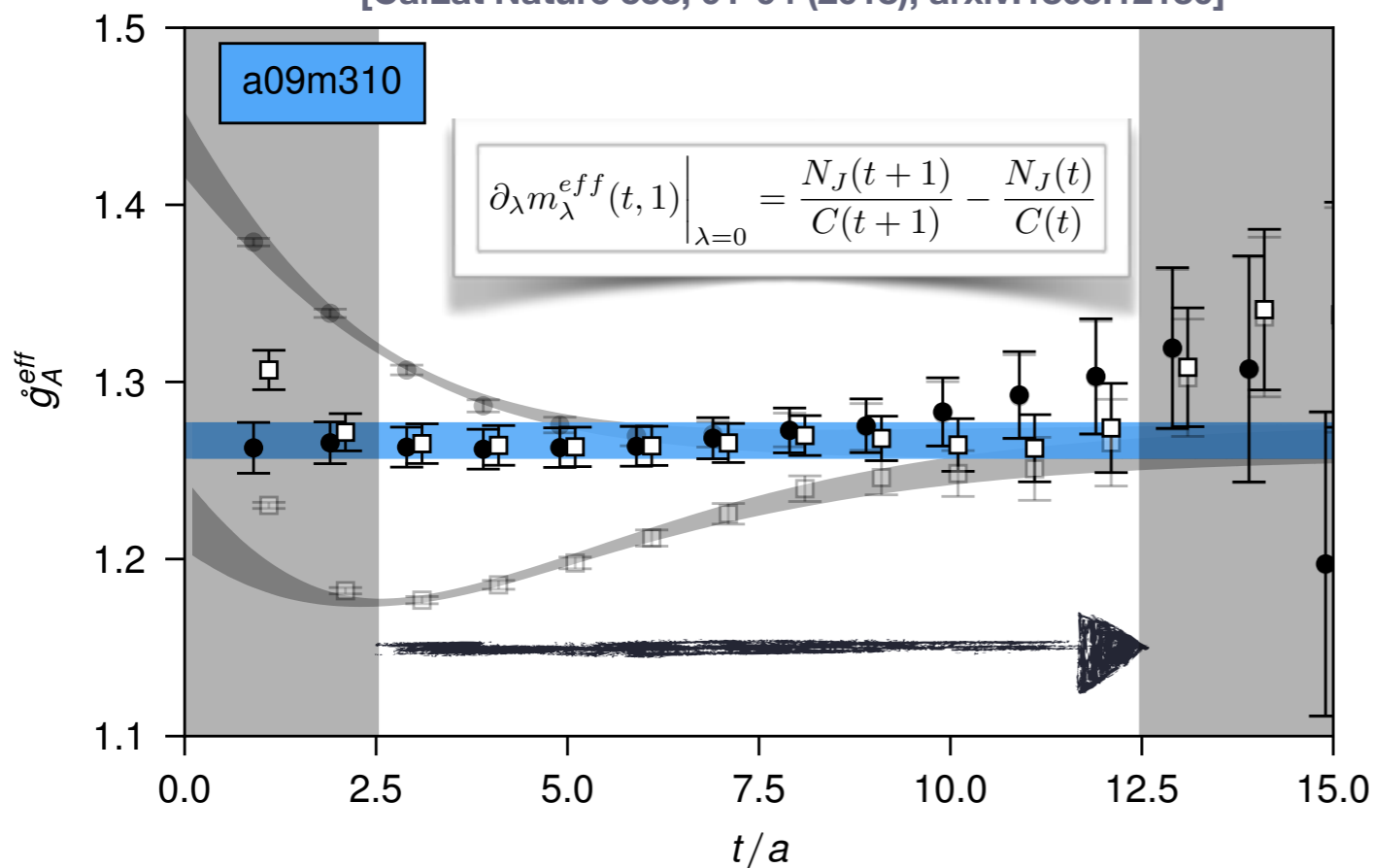
Improved method to reduce excited states



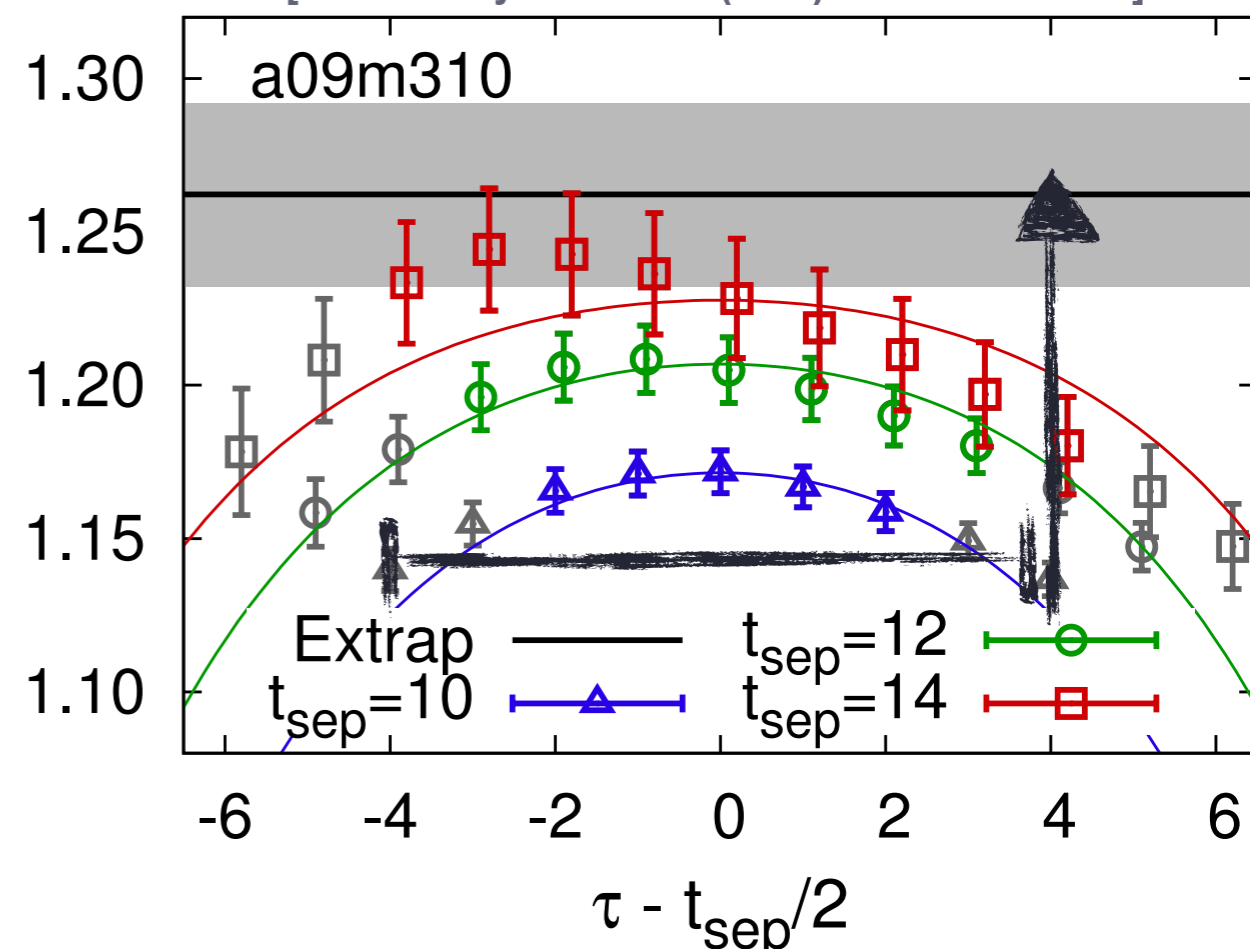
Simple functional form
to isolate ground state at small
times

Improved method to reduce excited states

[Callat Nature 558, 91-94 (2018), arxiv:1805.12130]



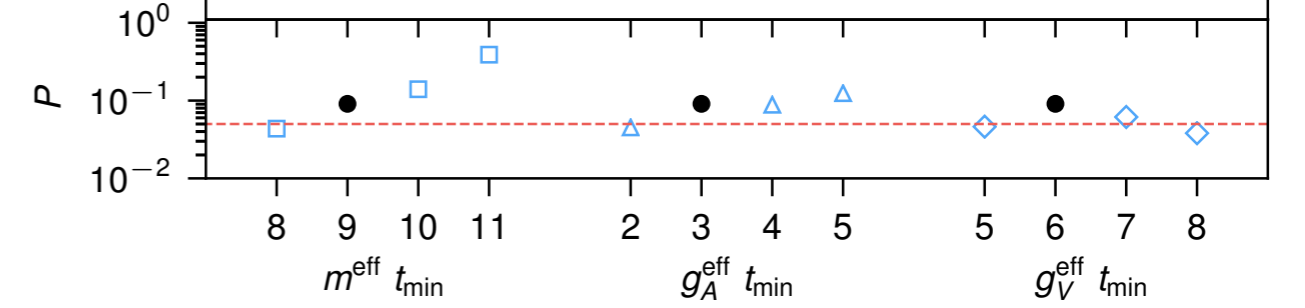
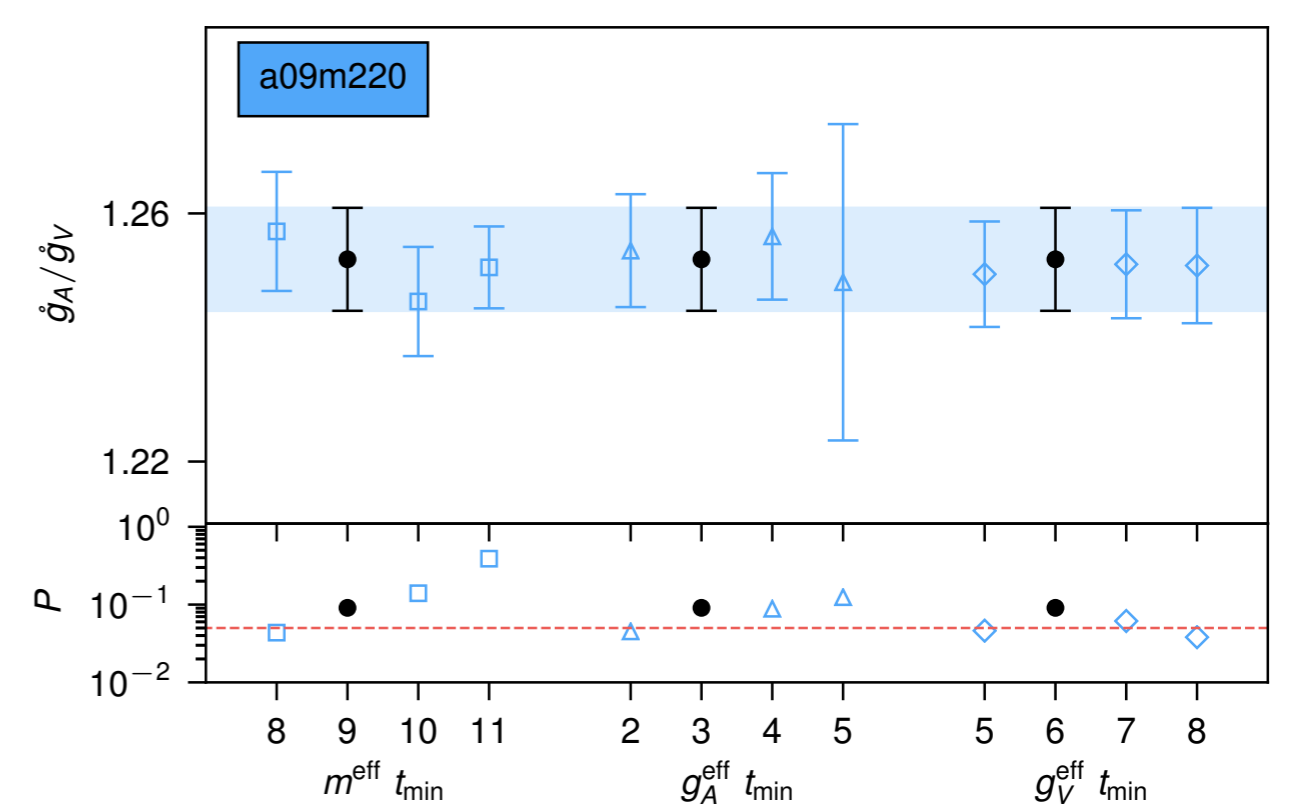
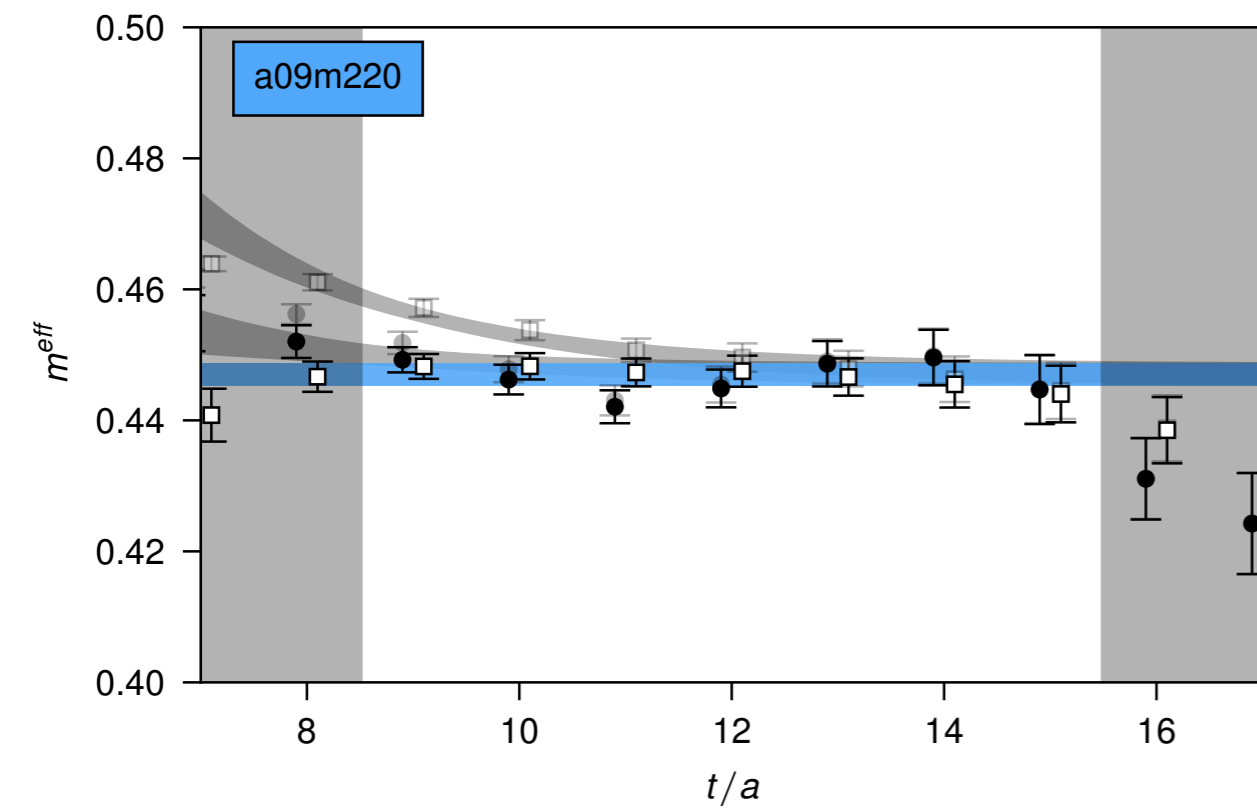
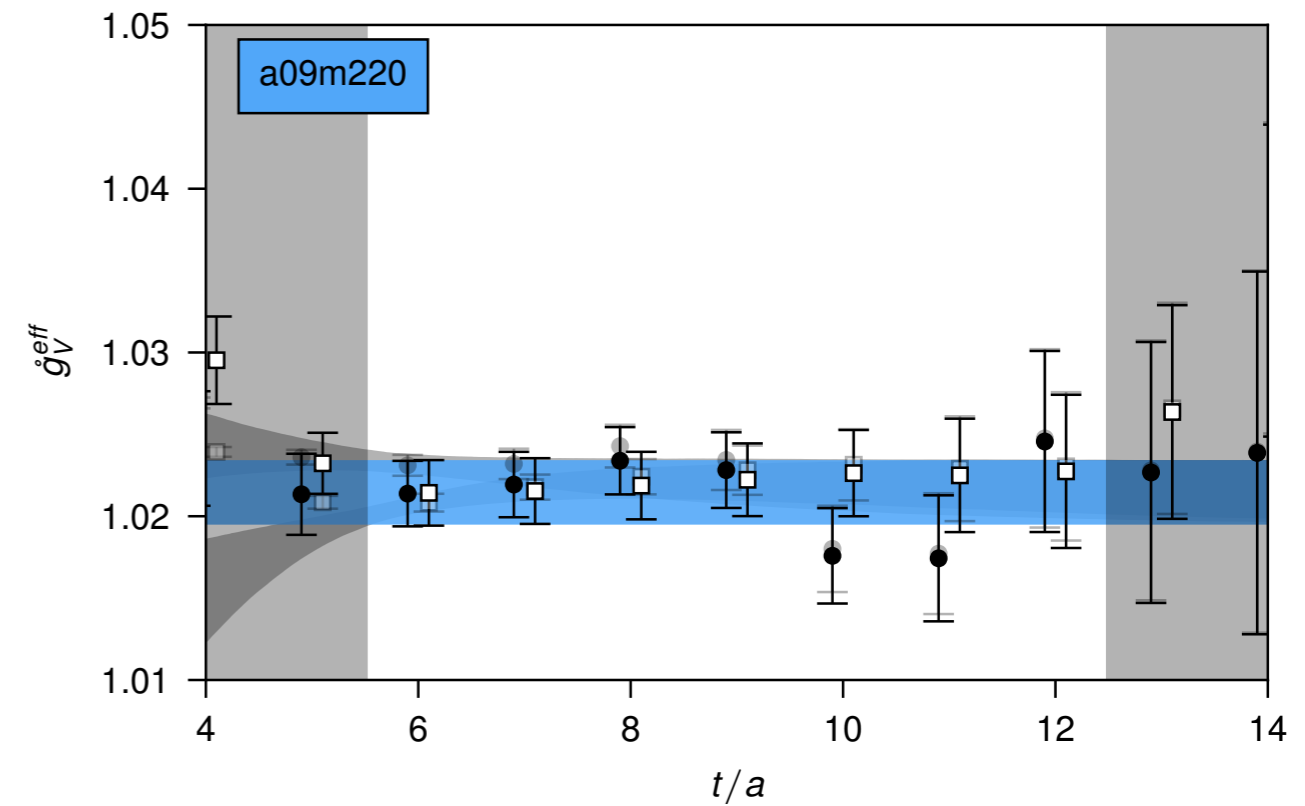
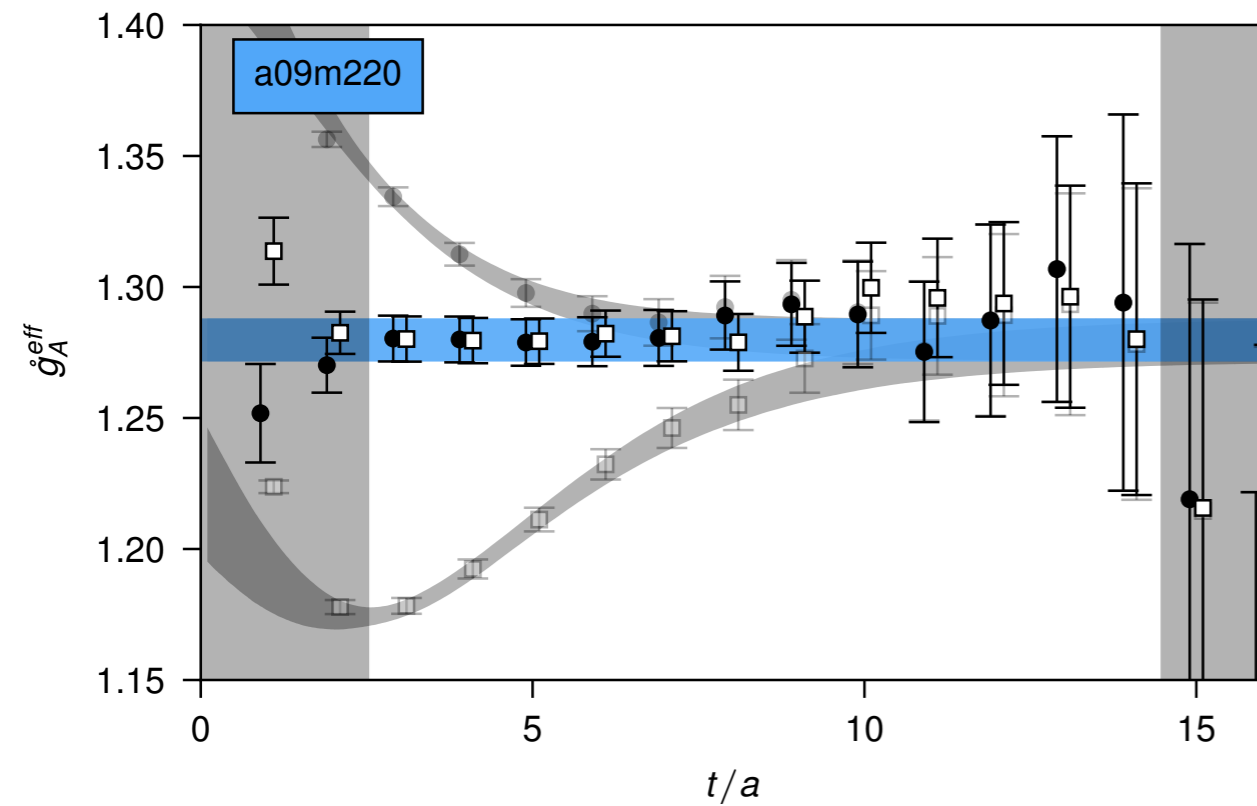
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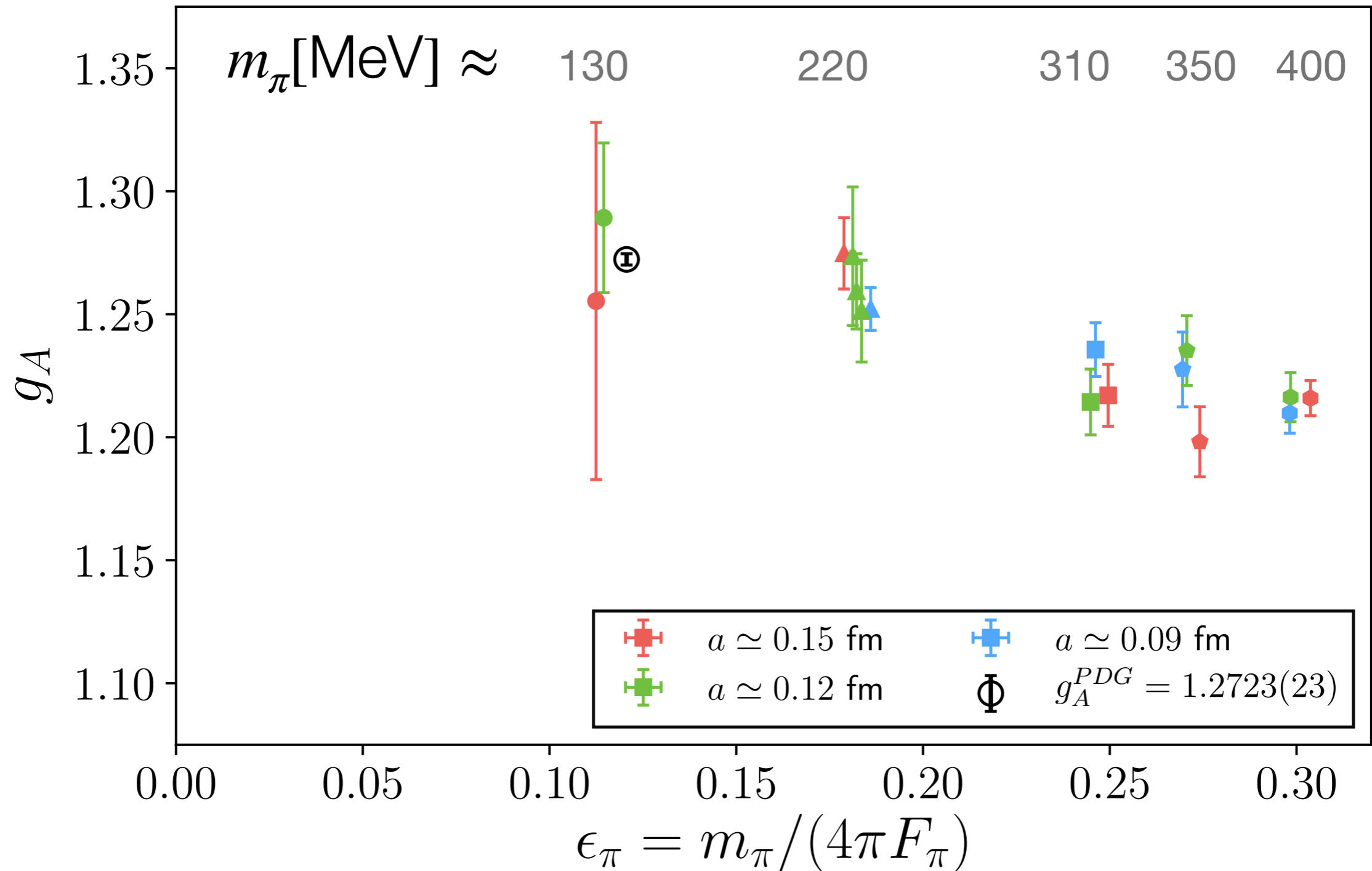
Simple functional form
to isolate ground state at small
times

- ✓ Only one time variable parametrizes exc. states
 - ➔ Reduced fitting systematics
 - ➔ Extract exponentially better signal
- ✓ Improved method contains summation of vertex over all space-time:
 - ➔ Improved statistical sampling

Extracting g_A from LQCD data



Extracting g_A from LQCD data - extrapolations



Chiral and Continuum Extrapolations

SU(2) NNLO baryon χ PT

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \quad \epsilon_a^2 = \frac{1}{4\pi} \frac{a^2}{\omega_0^2}$$

m_π^2 analytic $g_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4$

non-analytic $-\epsilon_\pi^2(g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + g_0c_3\epsilon_\pi^3$

a^2 analytic $a_2\epsilon_a^2 + b_4\epsilon_\pi^2\epsilon_a^2 + a_4\epsilon_a^4$

NLO FV $(8/3)\epsilon_\pi^2[g_0^3F_1(m_\pi L) + g_0F_3(m_\pi L)]$

- ★ Try different chiral, continuum and infinite volume extrapolations, averaged under Bayes framework
- ★ Based on ChPT, MAEFT, and a Taylor expansion around the physical point
- ★ Fits with parameters that can not be constrained are neglected
- ★ Study stability of fits, including variations of Bayes priors, additional discretization effects and cutting data

Chiral and Continuum Extrapolations

SU(2) NNLO baryon χ PT

$$m_\pi^2 \text{ analytic} \quad g_0 + c_2 \epsilon_\pi^2 + c_4 \epsilon_\pi^4$$

$$\text{non-analytic} \quad -\epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3$$

$$a^2 \text{ analytic} \quad a_2 \epsilon_a^2 + b_4 \epsilon_\pi^2 \epsilon_a^2 + a_4 \epsilon_a^4$$

$$\text{NLO FV} \quad (8/3) \epsilon_\pi^2 [g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)]$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$\epsilon_a^2 = \frac{1}{4\pi} \frac{a^2}{\omega_0^2}$$

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$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$\epsilon_a^2 = \frac{1}{4\pi} \frac{a^2}{\omega_0^2}$$

e.g. Taylor exp.
in quark mass

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- ★ Based on ChPT, MAEFT, and a Taylor expansion around the physical point
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Chiral and Continuum Extrapolations

SU(2) NNLO baryon χ PT

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$$\text{non-analytic} \quad -\epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3$$

$$a^2 \text{ analytic} \quad a_2 \epsilon_a^2 + b_4 \epsilon_\pi^2 \epsilon_a^2 + a_4 \epsilon_a^4$$

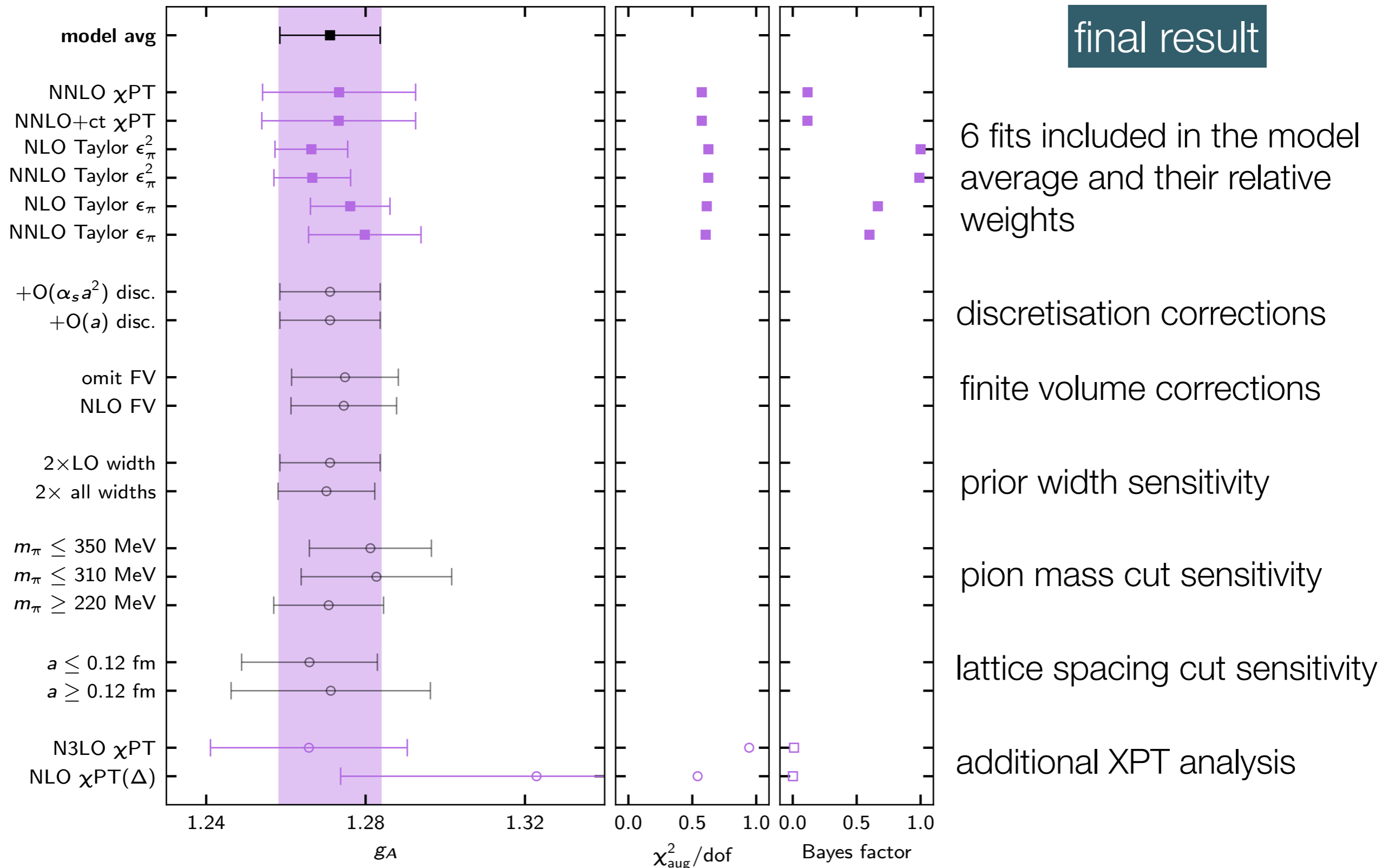
$$\text{NLO FV} \quad (8/3) \epsilon_\pi^2 [g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)]$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \quad \epsilon_a^2 = \frac{1}{4\pi} \frac{a^2}{\omega_0^2}$$

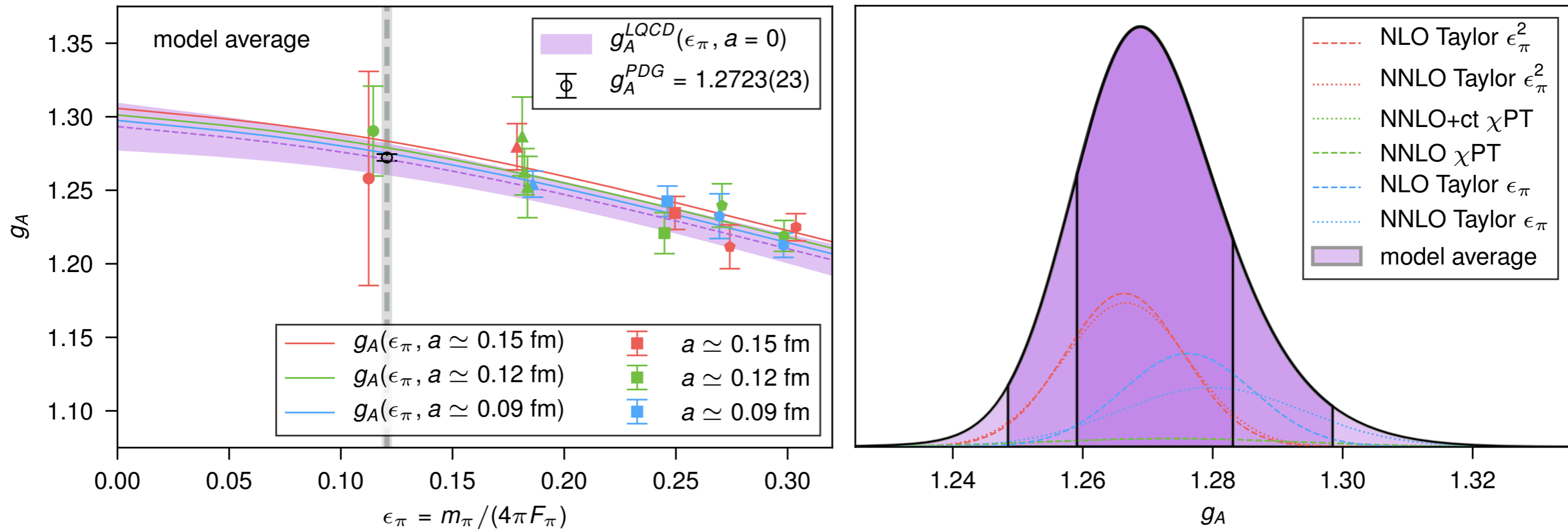
e.g. Taylor exp.
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- ★ Try different chiral, continuum and infinite volume extrapolations, averaged under Bayes framework
- ★ Based on ChPT, MAEFT, and a Taylor expansion around the physical point
- ★ Fits with parameters that can not be constrained are neglected
- ★ Study stability of fits, including variations of Bayes priors, additional discretization effects and cutting data

Extrapolation stability

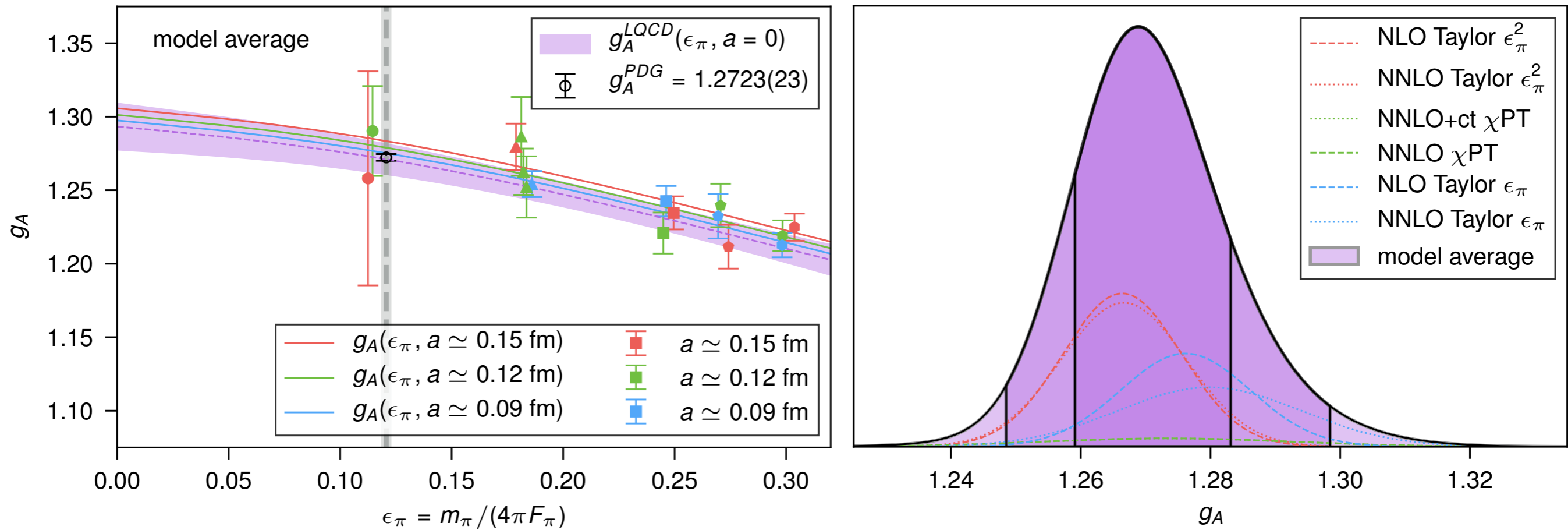


$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^{\chi}(15)^a(19)^V(04)^I(55)^M$$



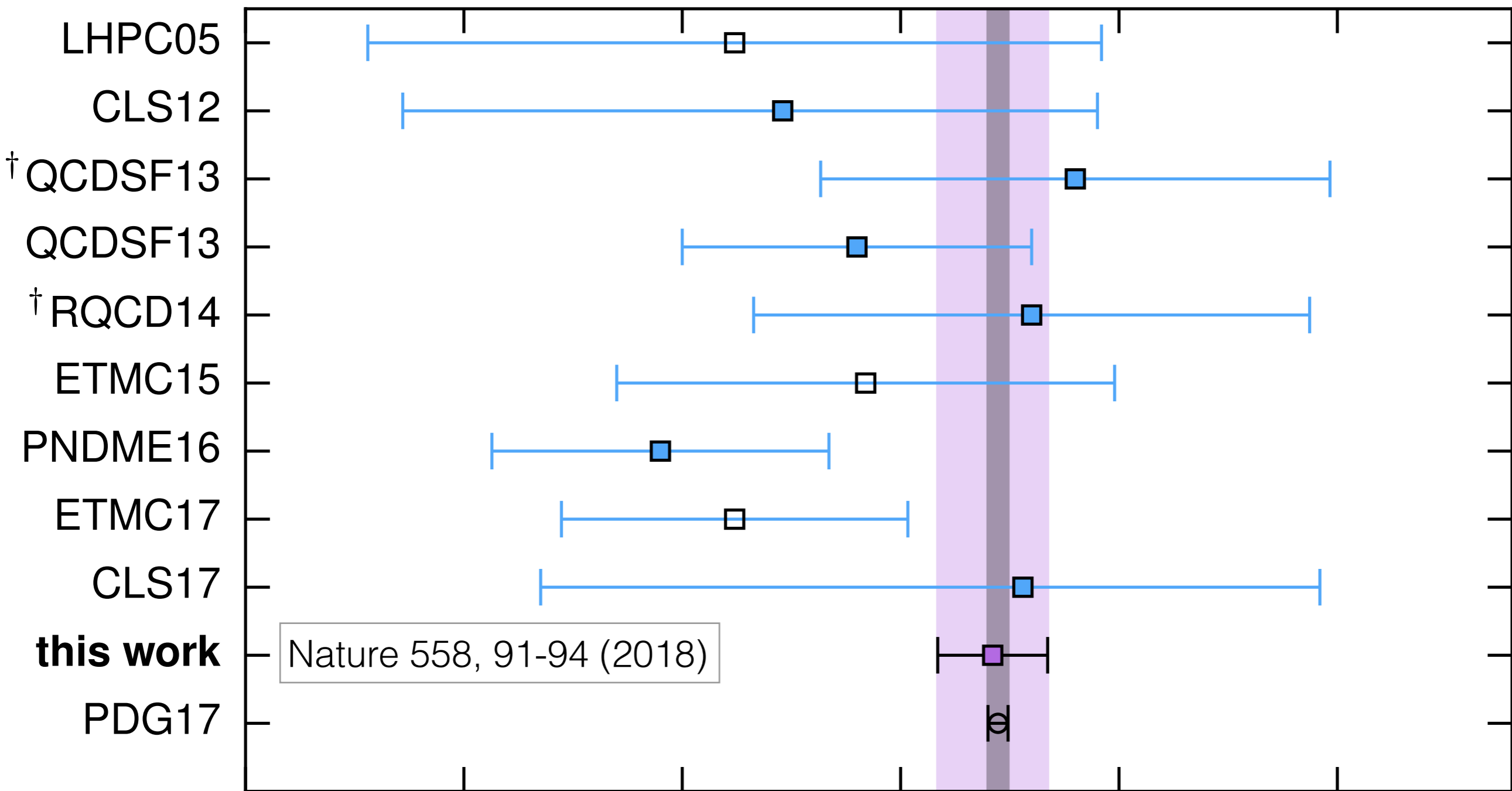
statistical	0.81%
chiral extrapolation	0.31%
$a \rightarrow 0$	0.12%
$L \rightarrow \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%

$$g_A^{\text{QCD}} = 1.2711(103)^s (39)^{\chi} (15)^a (19)^V (04)^I (55)^M$$



dominant sources
of uncertainties

statistical	0.81%
chiral extrapolation	0.31%
$a \rightarrow 0$	0.12%
$L \rightarrow \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



Nature 558, 91-94 (2018)

$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

First percent-level determination of g_A from LQCD

- * result is limited by statistics
- * new supercomputers help!
- * all data is publicly available
https://github.com/callat-qcd/project_gA

LQCD neutron lifetime

- Use **LQCD** values of the **axial coupling** and the **light quark mixing matrix element**

$$\tau_n = \frac{4908.6(1.9)s}{|V_{ud}|^2 (1 + 3g_A^2)}$$

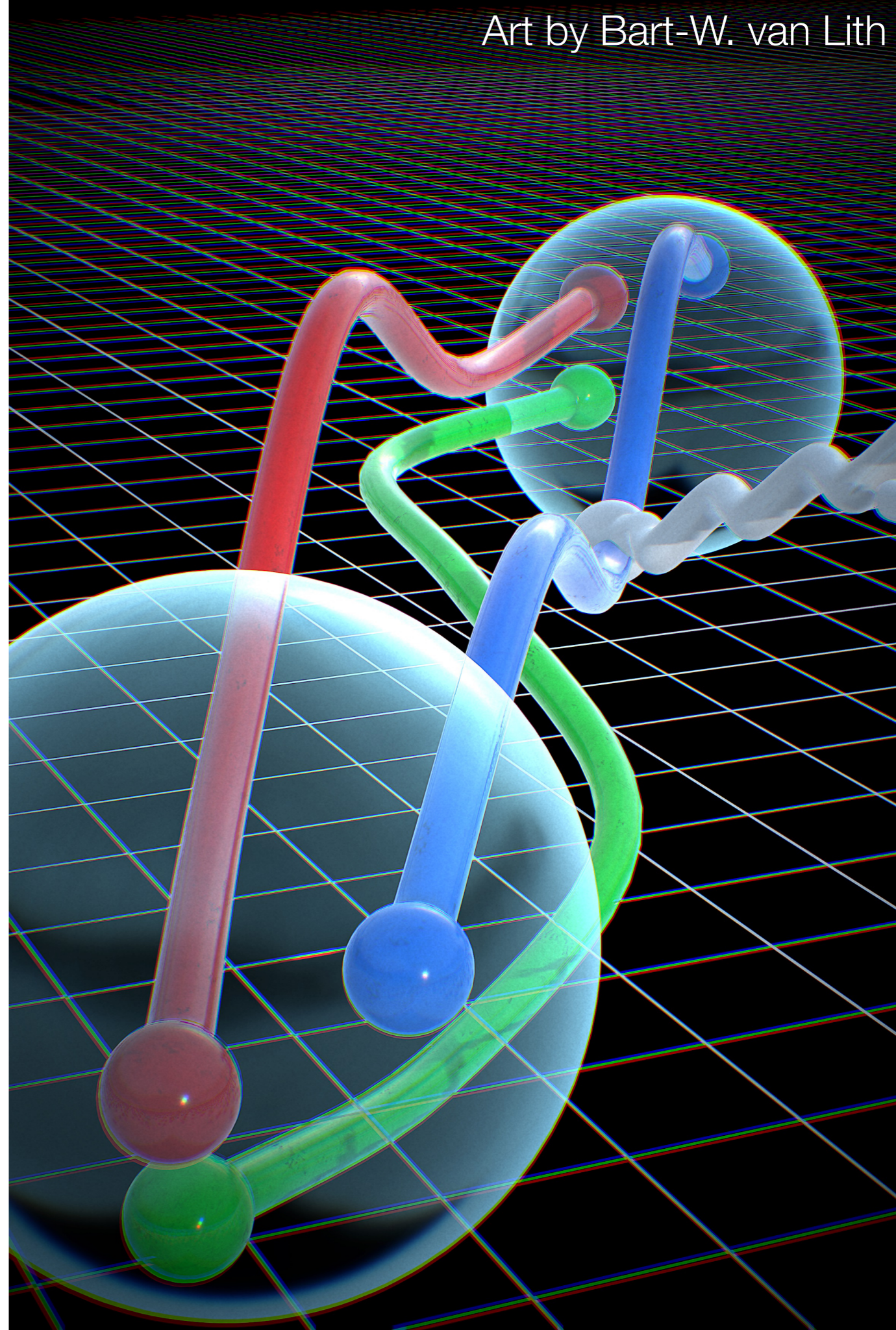
[Czarnecki, Marciano and Sirlin, Phys. Rev. Lett. 120, 202002 (2018)]

$$|V_{ud}| = 0.97438(12)$$

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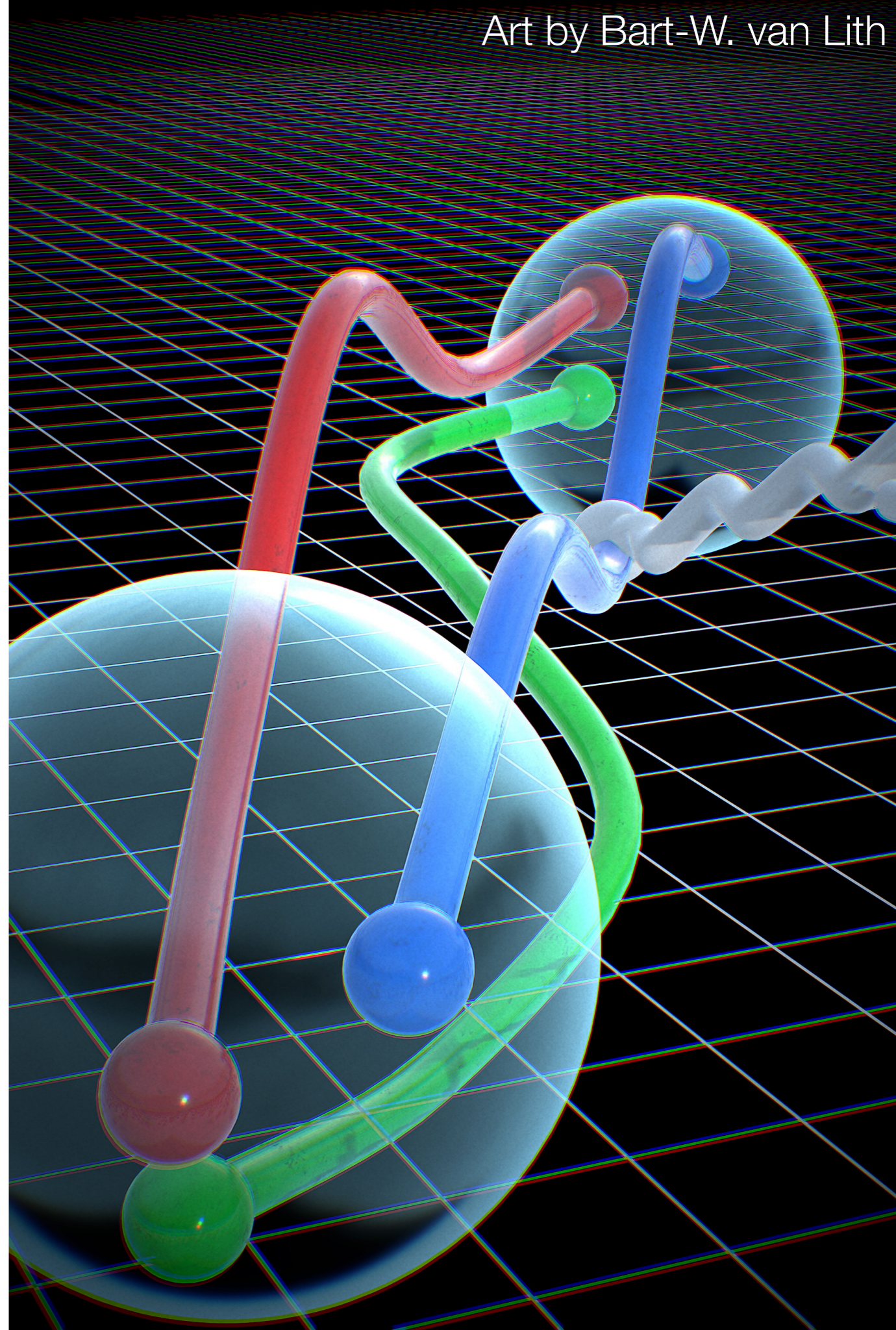
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$$\tau_n = 884(15)s$$



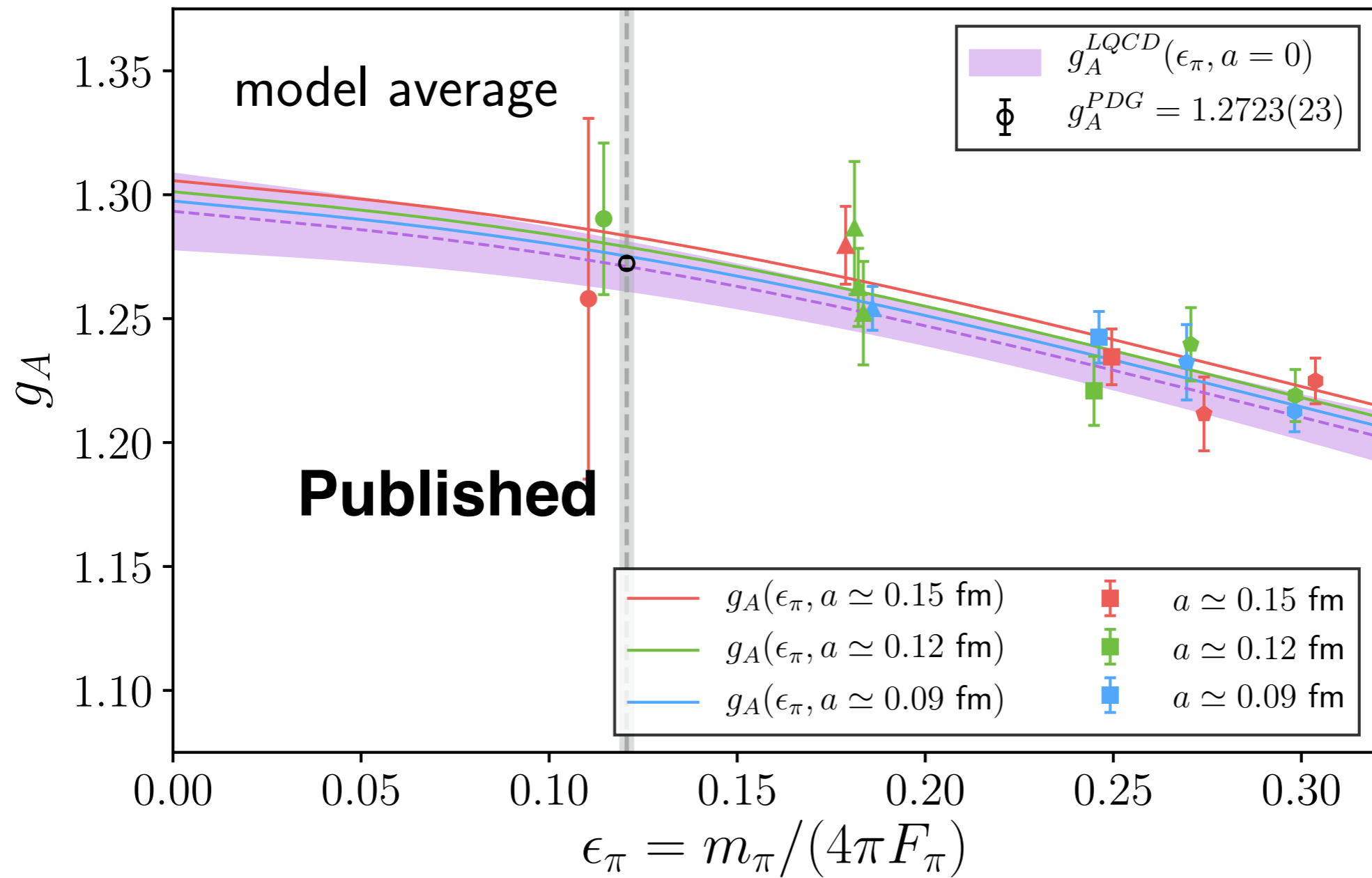


2018 Gordon Bell Finalist
[sc18.supercomputing.org/
presentation/?
id=gb101&sess=sess467](https://sc18.supercomputing.org/presentation/?id=gb101&sess=sess467)

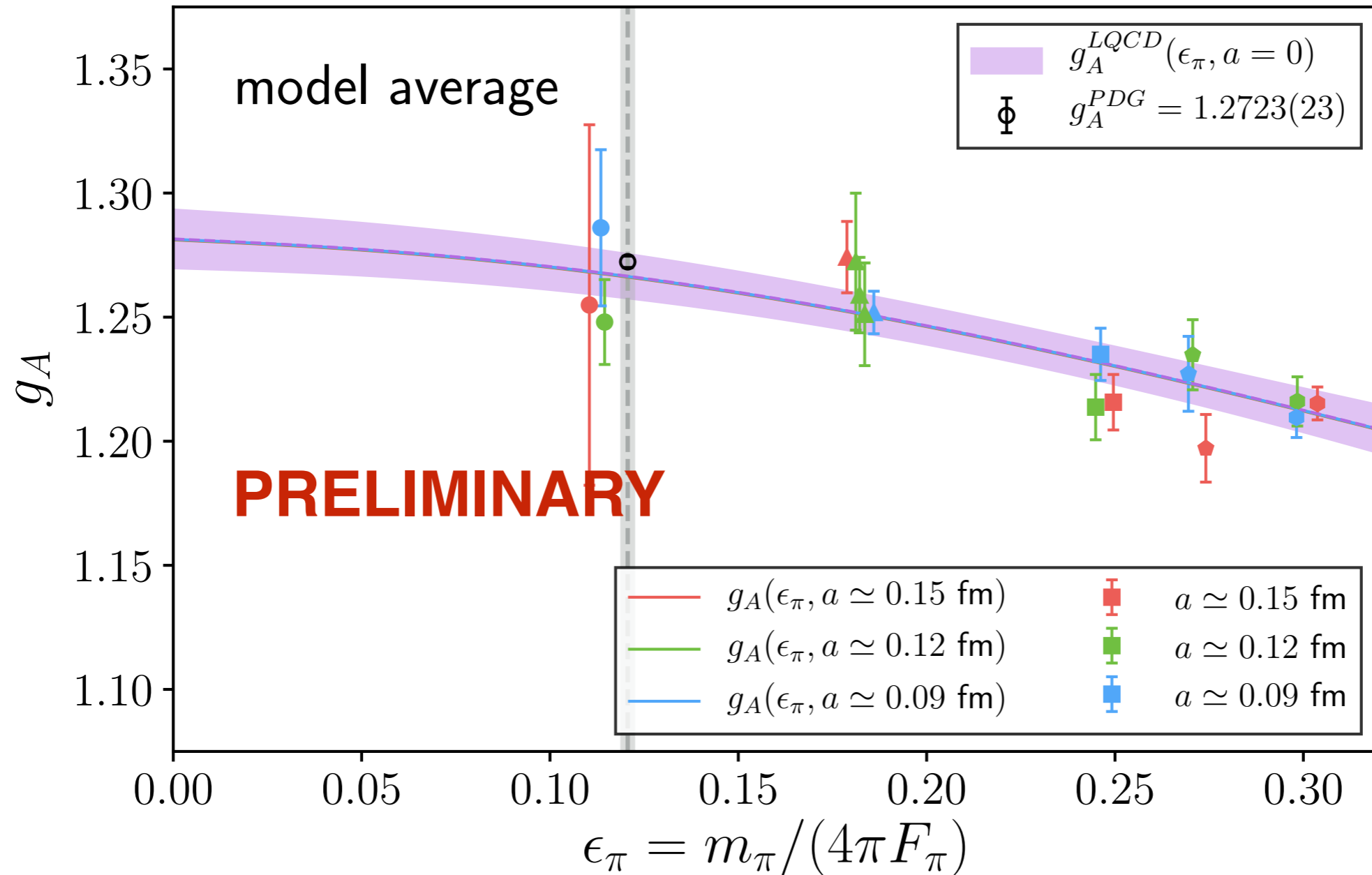


Simulating the weak death of the neutron in a femtoscale universe with near-Exascale computing

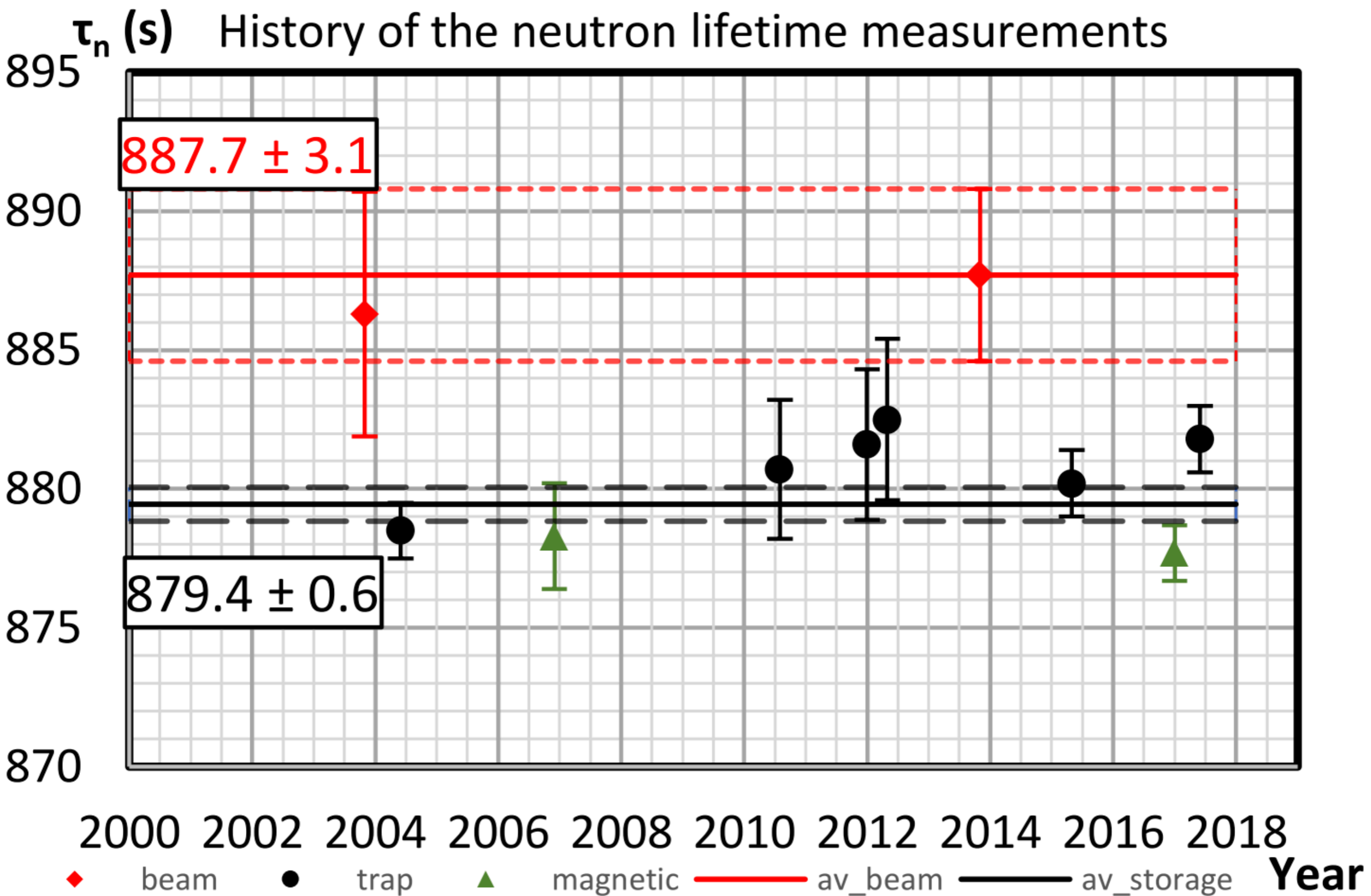
[Evan Berkowitz](#), [M.A. Clark](#), [Arjun Gambhir](#), [Ken McElvain](#), [Amy Nicholson](#), [Enrico Rinaldi](#), [Pavlos Vranas](#), [André Walker-Loud](#), [Chia Cheng Chang](#),
[Bálint Joó](#), [Thorsten Kurth](#), [Kostas Orginos](#)



Code development from Gordon Bell + initial Sierra Early Science time result.
Increase 0.12 fm physical mass statistics by $\sim 5x$, 50% reduction in uncertainty.
New 0.09 fm physical mass 322 configs x 4 sources.
 Preliminary update for $g_A = 1.2670(97)$
 23% reduction in uncertainty \rightarrow 0.77% relative error

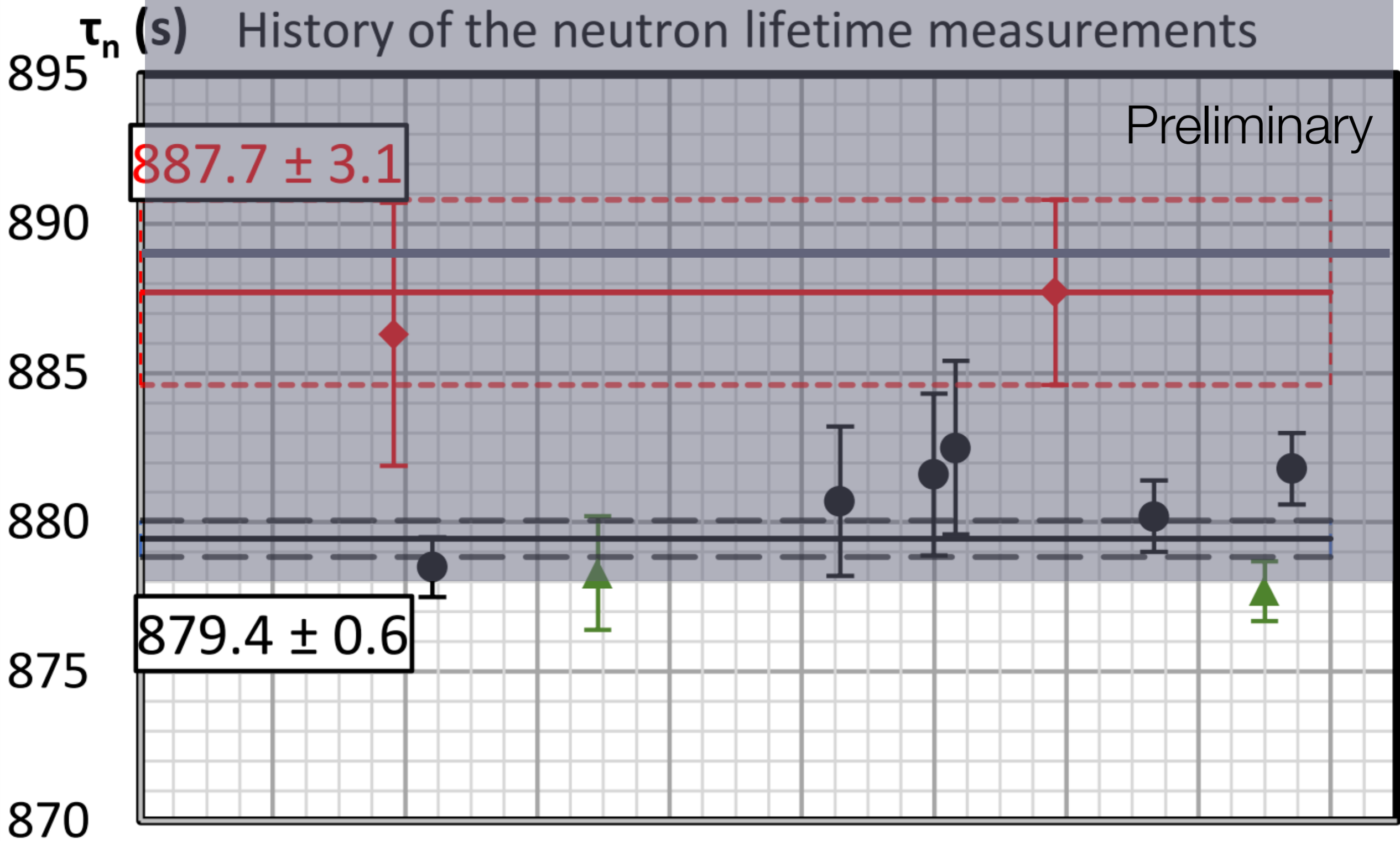


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History of the neutron lifetime measurements

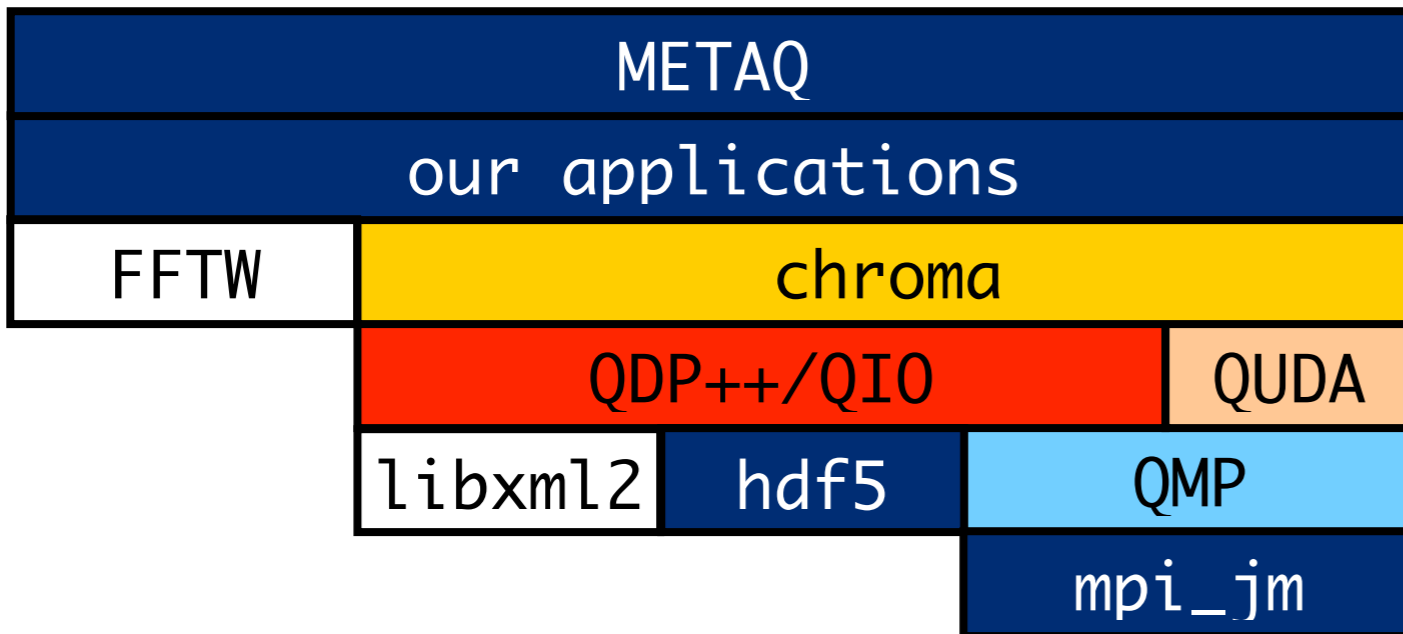
Preliminary



◆ beam ● trap ▲ magnetic — av_beam — av_storage

Summary

- ✓ The neutron lifetime is showing a discrepancy of $\sim 4\sigma$ between different experimental methods
- ✓ The Standard Model predicts a precise relation which allows us to obtain a theoretical value of the neutron lifetime using Lattice QCD non-perturbative calculations
- ✓ The first percent-level calculations of the nucleon axial coupling has been obtained this year, ahead of expectations <https://www.nature.com/articles/s41586-018-0161-8>
 - ✓ Statistical uncertainties $\sim 0.8\%$ can be reduced with the next generation of supercomputers (we “only” used the no. 7 and 33 of the June 2018 *top500* list of supercomputers: <https://www.top500.org/lists/2018/06/>)
 - ✓ A more accurate calculation at the physical point using the no. 1 and 3 *top500* has been accepted as one of the six finalists in the Gordon Bell competition, recognizing outstanding achievement in high-performance computing (<https://awards.acm.org/bell>)



Software

References

METAQ

Berkowitz arXiv:1702.06122 github.com/evanberkowitz/metaq
 Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018)

chroma
 QDP++

Edwards and Joo (SciDAC, LHPC and UKQCD Collaborations) Nucl. Phys. Proc. Suppl 140, 832 (2005)

QUDA

Clark et al. Comput. Phys. Commun. 181 1517 (2010)
 Babich et al. Supercomputing 11, 70

hdf5 in QDP++

Kurth et al PoS LATTICE2014 045 (2015)

qmp

Chen, Edwards, and Watson et al.
<https://github.com/usqcd-software/qmp>

mpi_jm

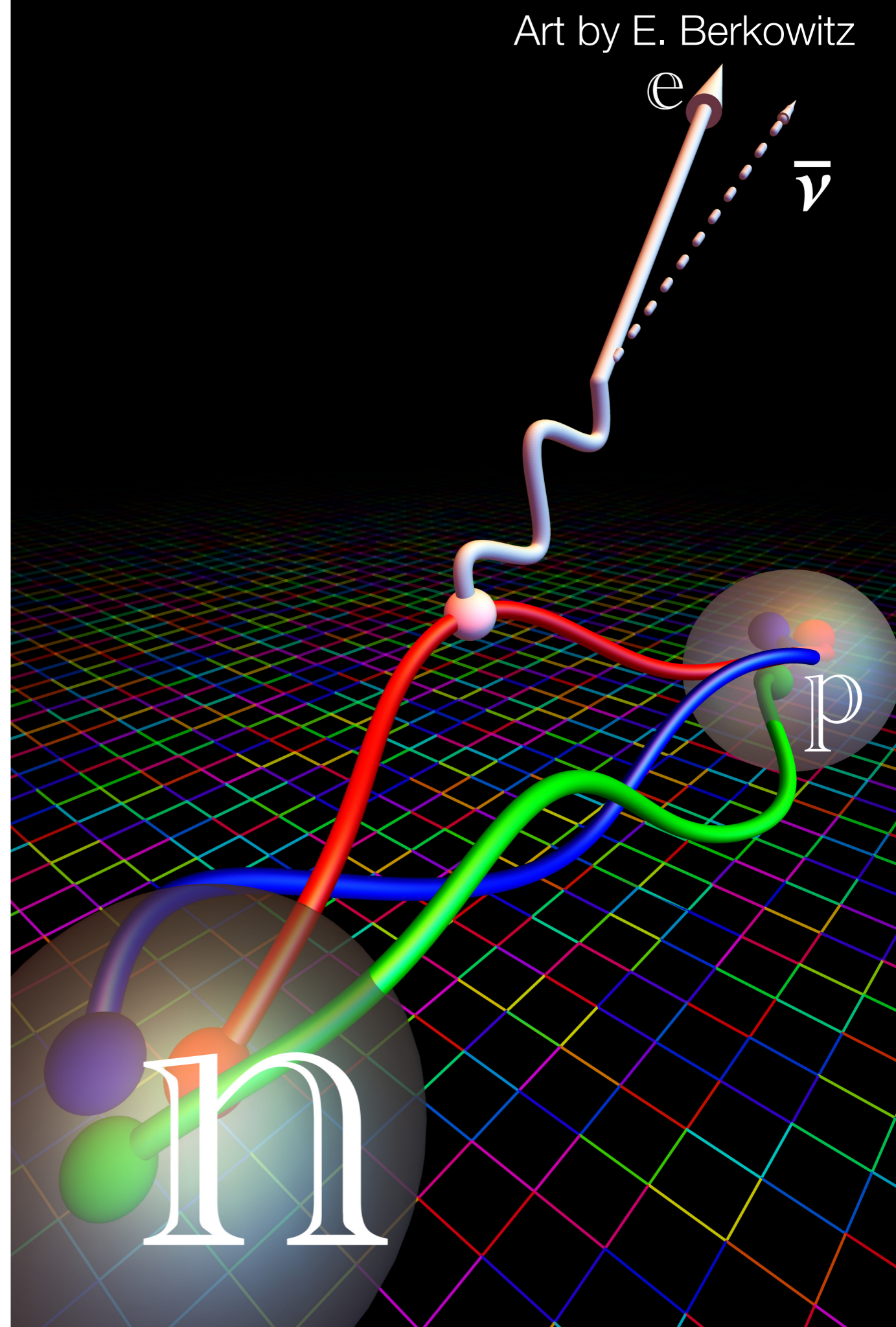
Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018)
 McElvain et al. https://github.com/kenmcelvain/mpi_jm/

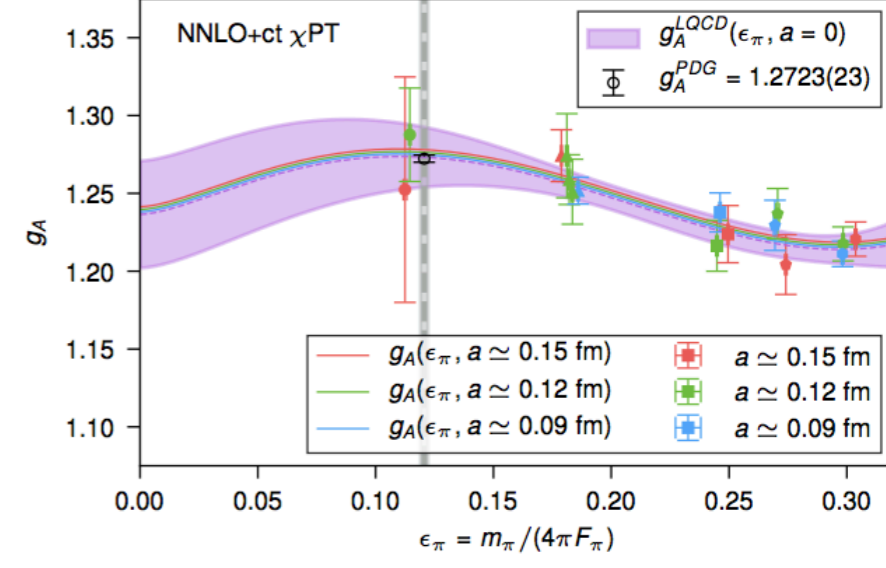
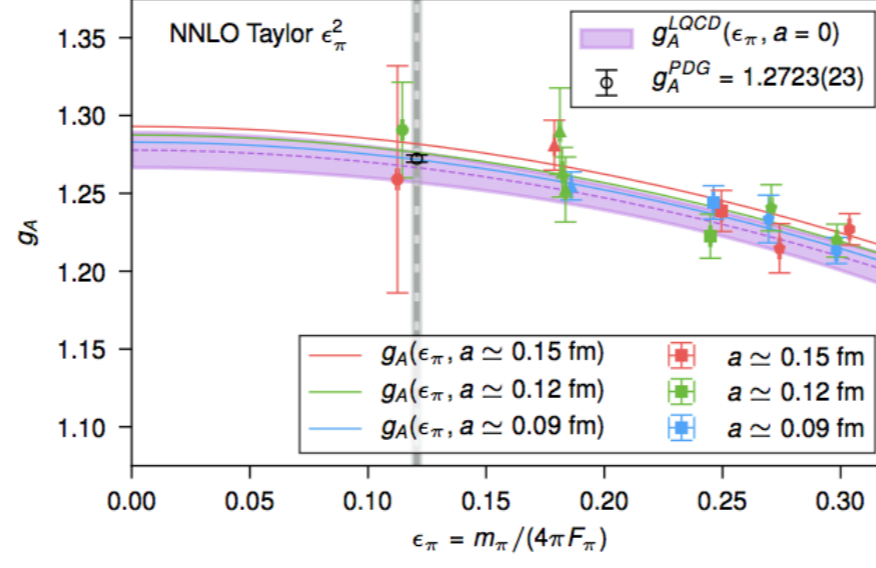
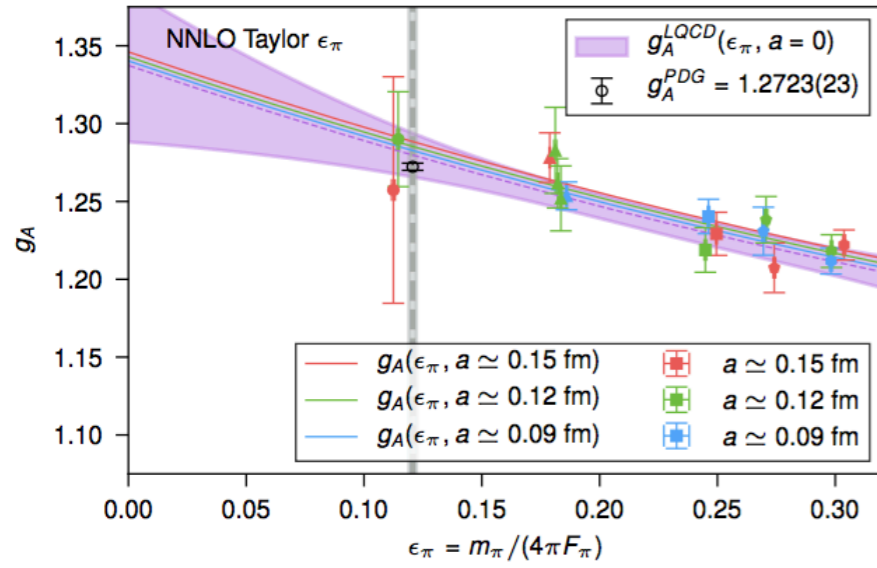
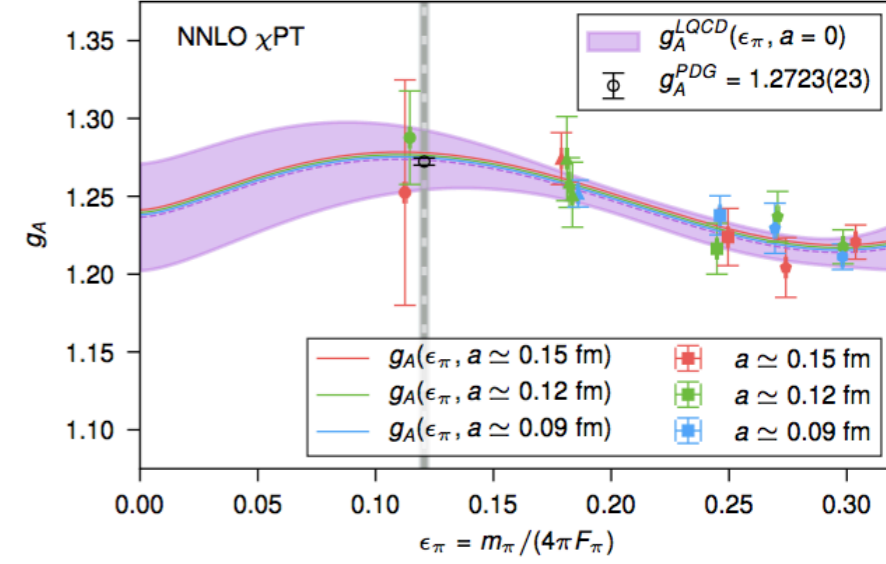
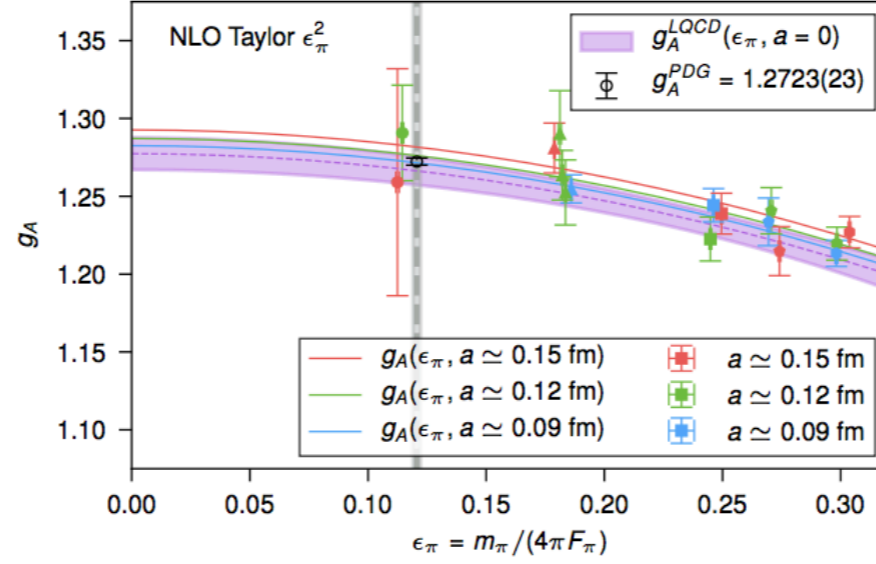
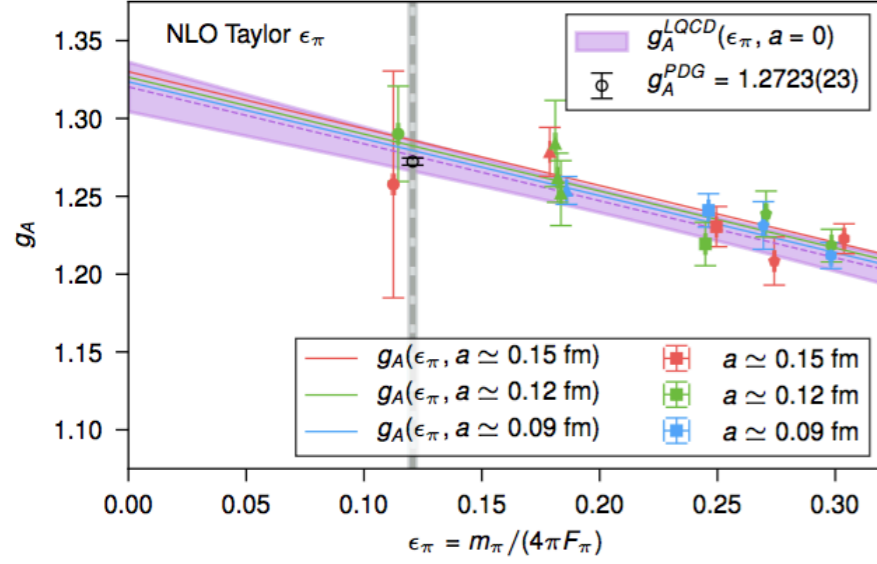
thank you



extra slides

background and more plots





Taylor in m_π

Taylor in $(m_\pi)^2$

χ PT

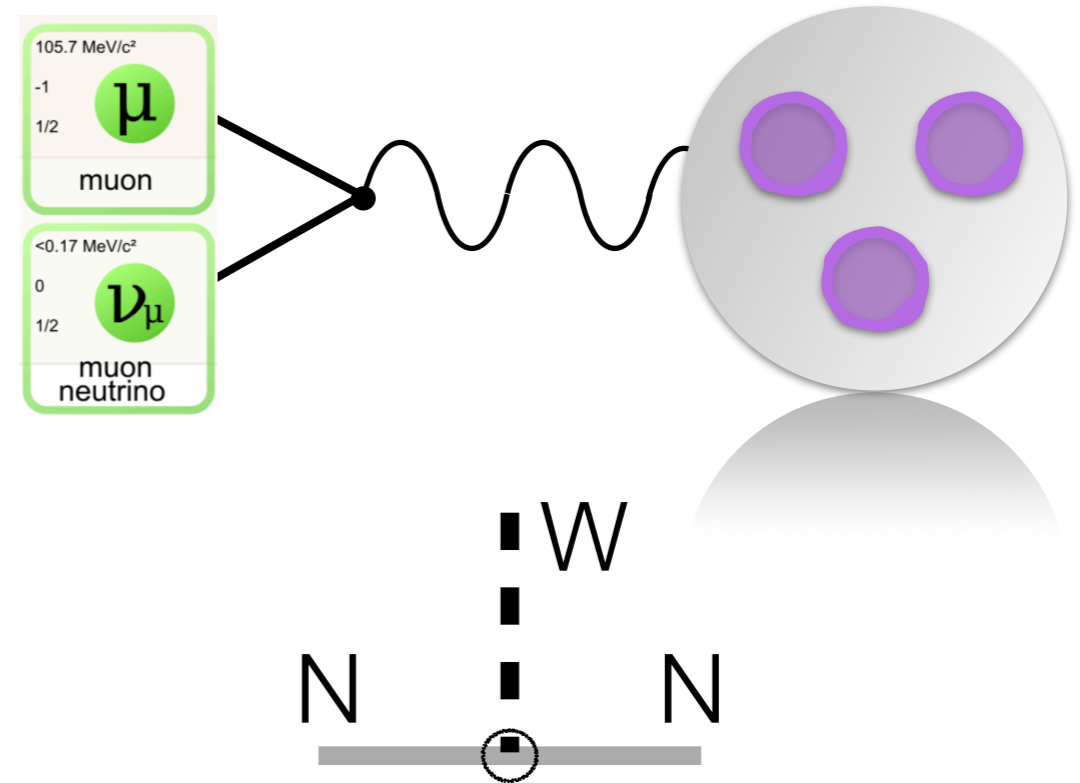
Different models for
extrapolation

	Fit χ^2/dof	$\mathcal{L}(D M_k)$	$P(M_k D)$	$P(g_A M_k)$
NNLO χ PT	0.727	22.734	0.033	1.273(19)
NNLO+ct χ PT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

Axial coupling, g_A

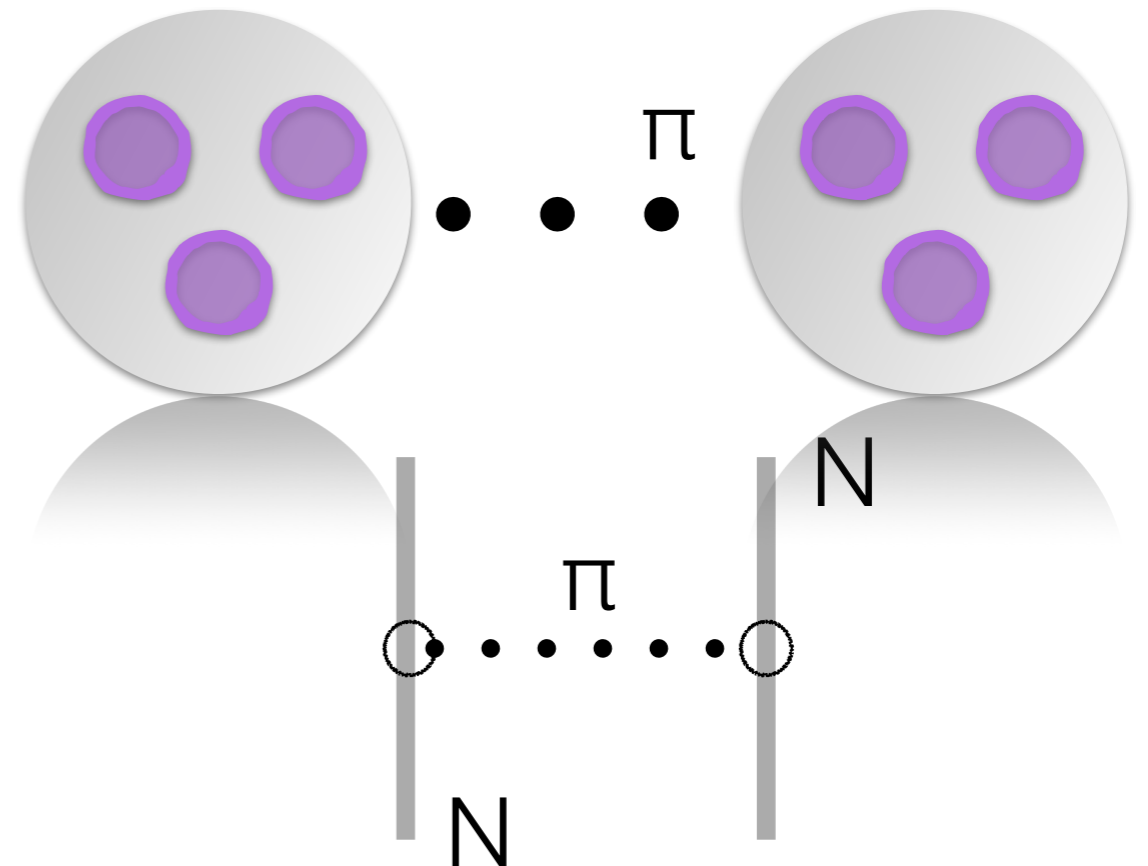
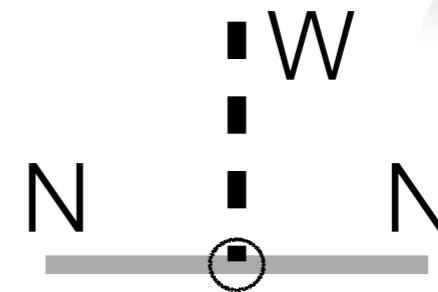
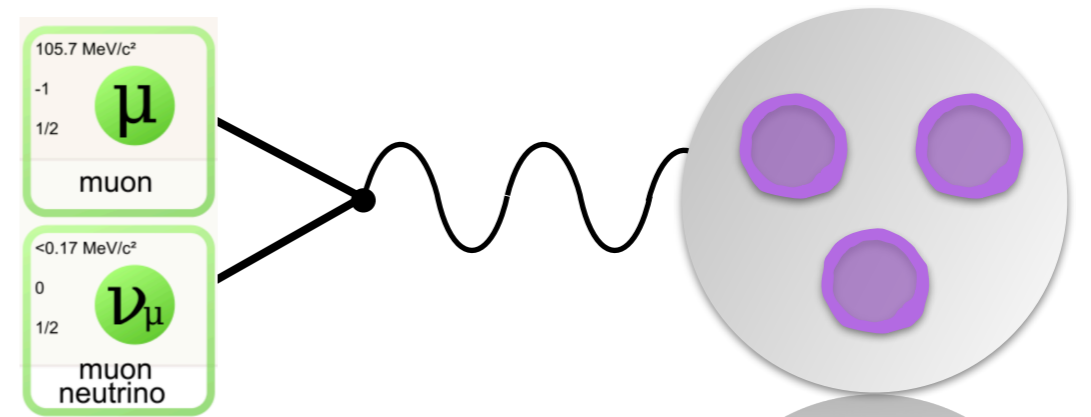
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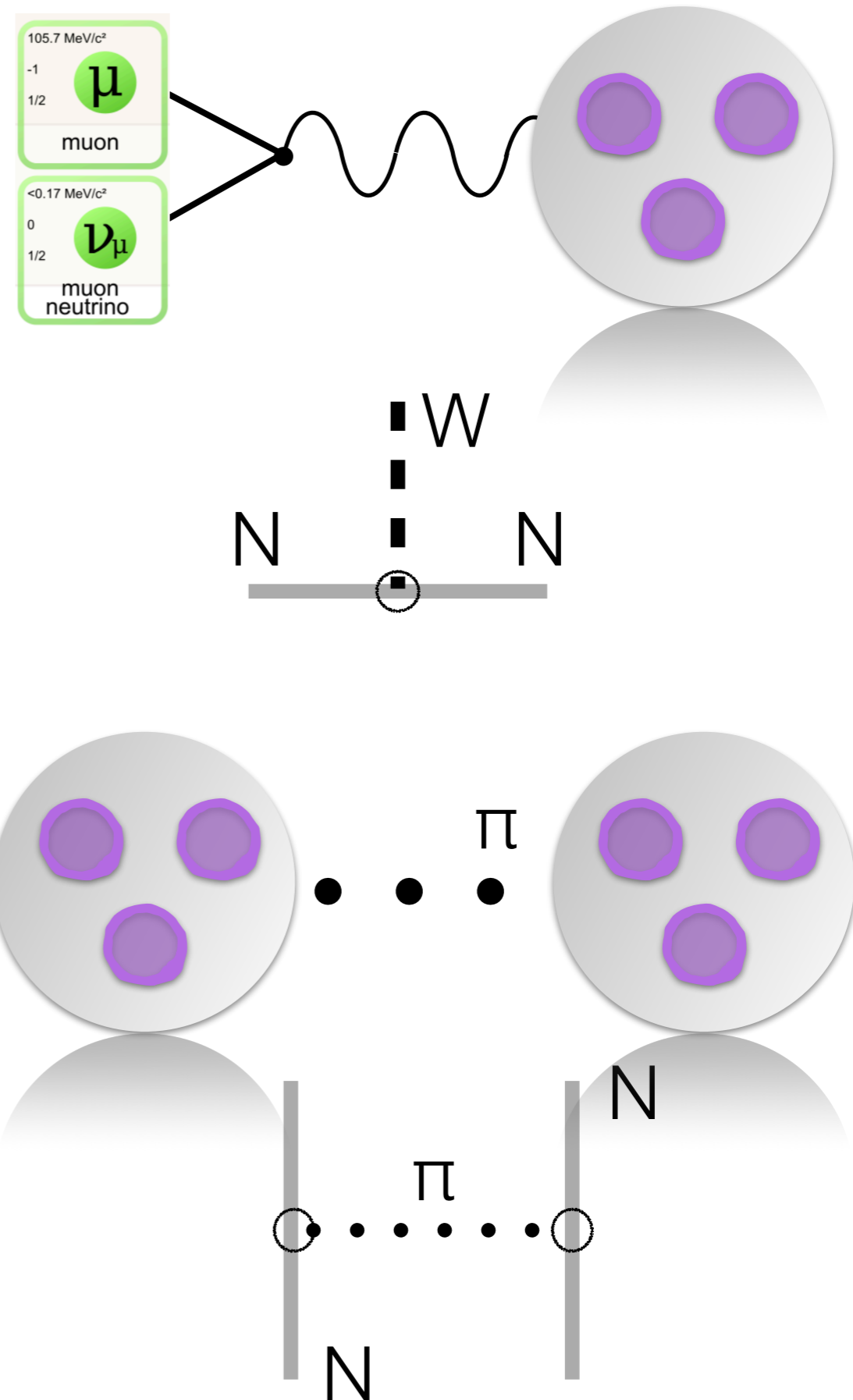
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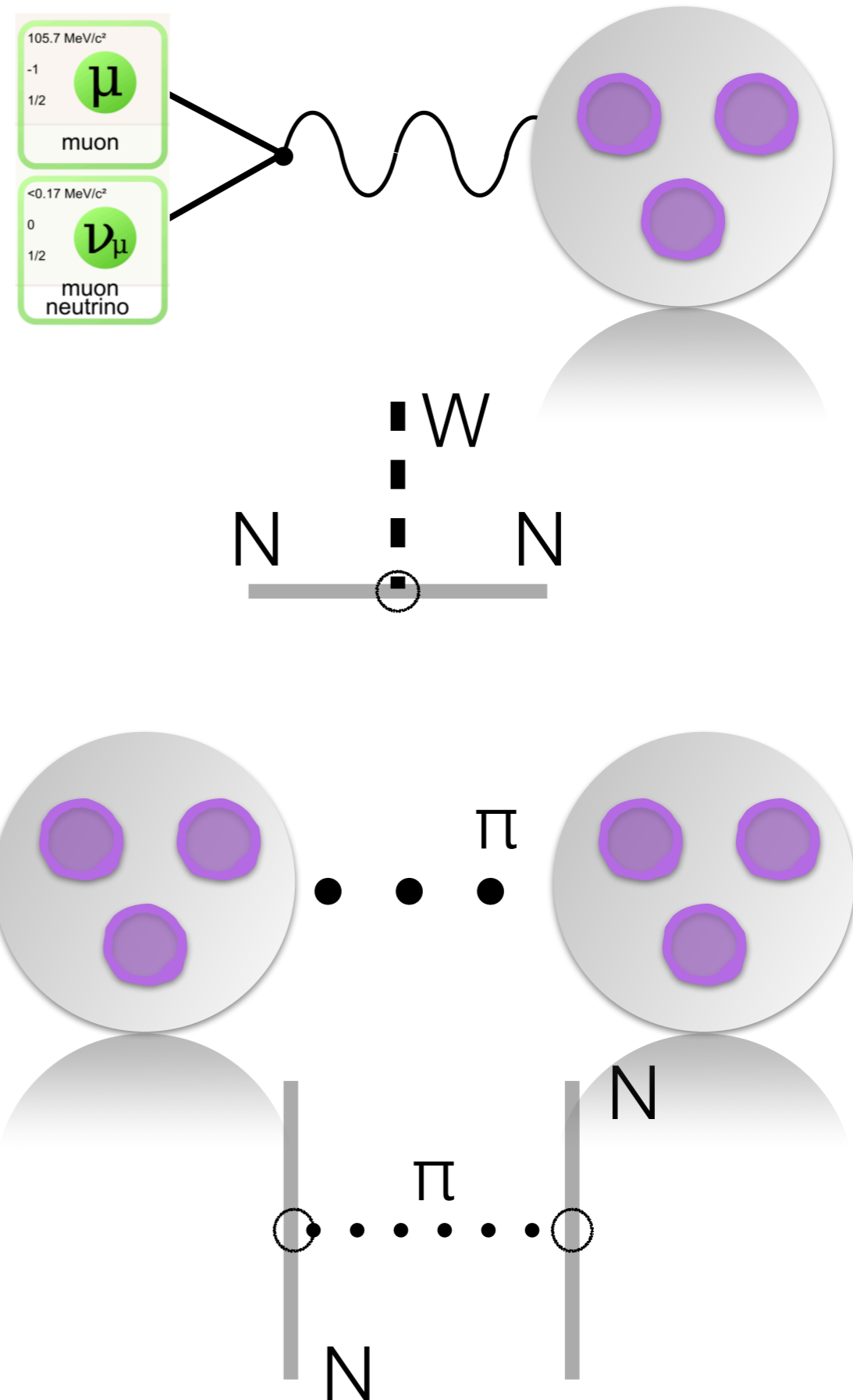
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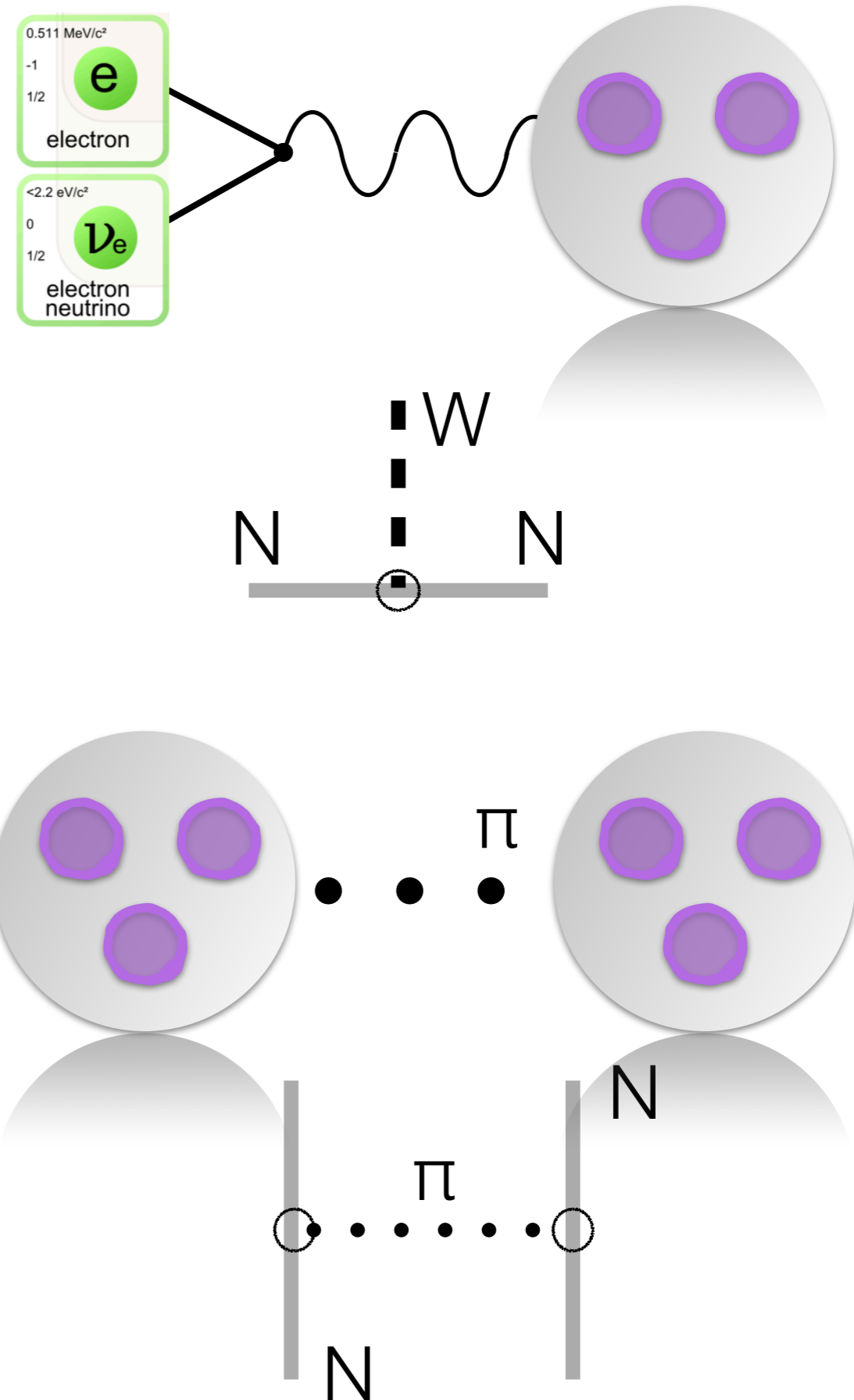
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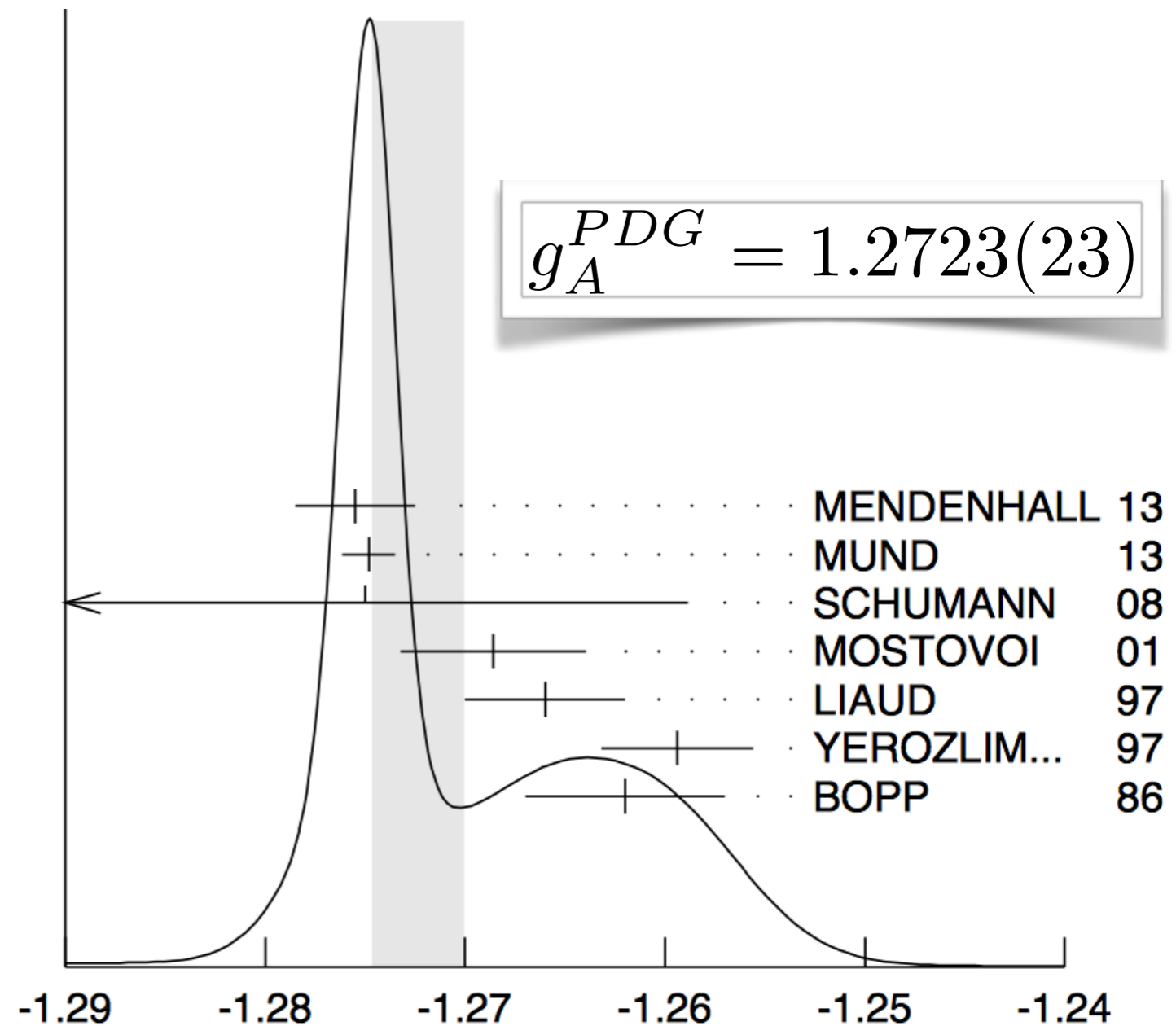
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- ❖ Very well determined experimentally $\sim 0.2\%$ (from angular correlations in cold neutron decays)



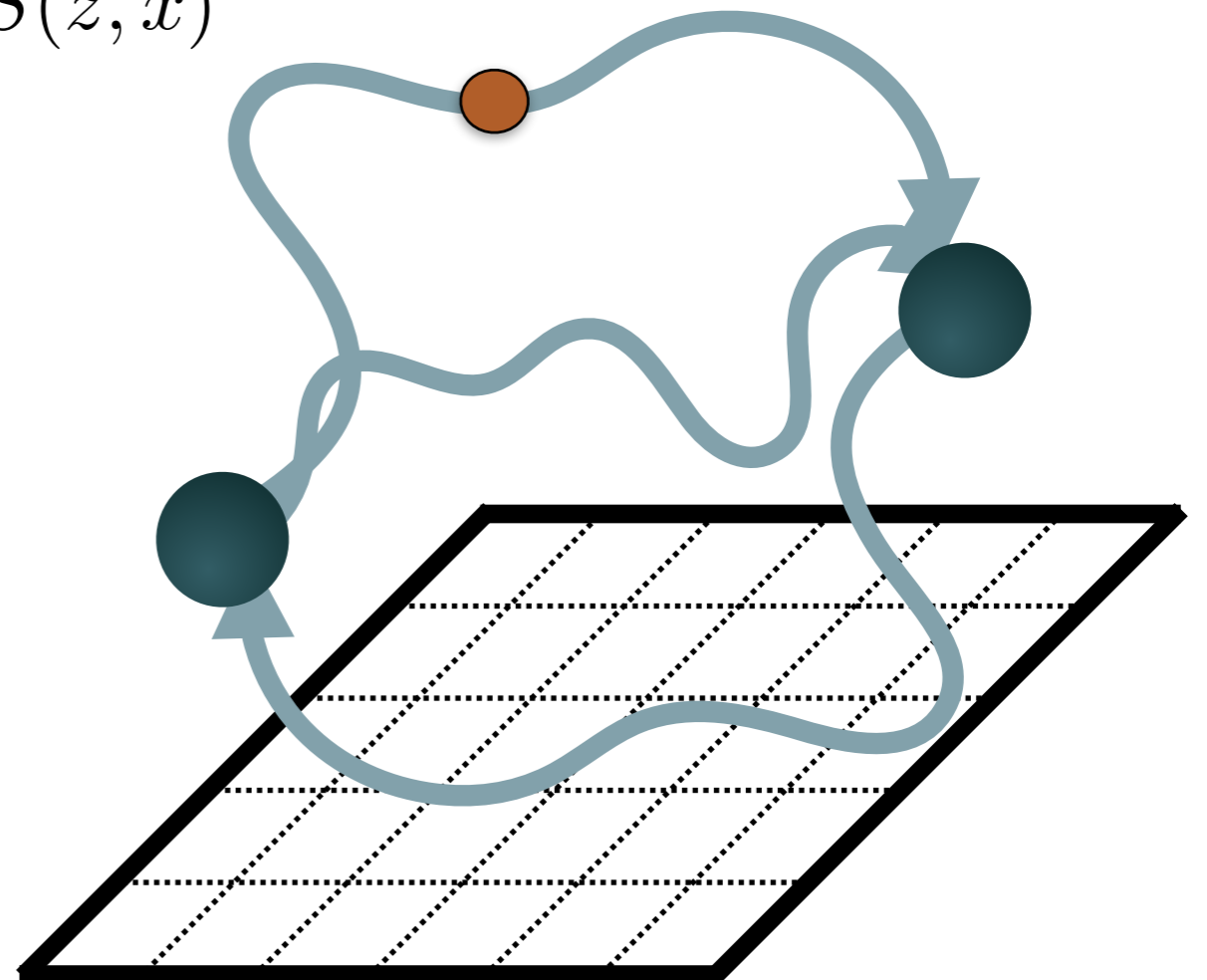
Practical implementation

[Bouchard, Chang, Kurt, Orginos, Walker-Loud, PRD96(014504) - arxiv:1612.06963]

$$\left. \frac{\partial m_\lambda^{eff}(t, \tau)}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\tau} \left[\frac{-\partial_\lambda C_\lambda(t + \tau)}{C(t + \tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)} \right]$$

Feynman-Hellmann propagator

$$\text{---} \circ \text{---} = S_{FH}(y, x) = \sum_z S(y, z) \Gamma(z) S(z, x)$$



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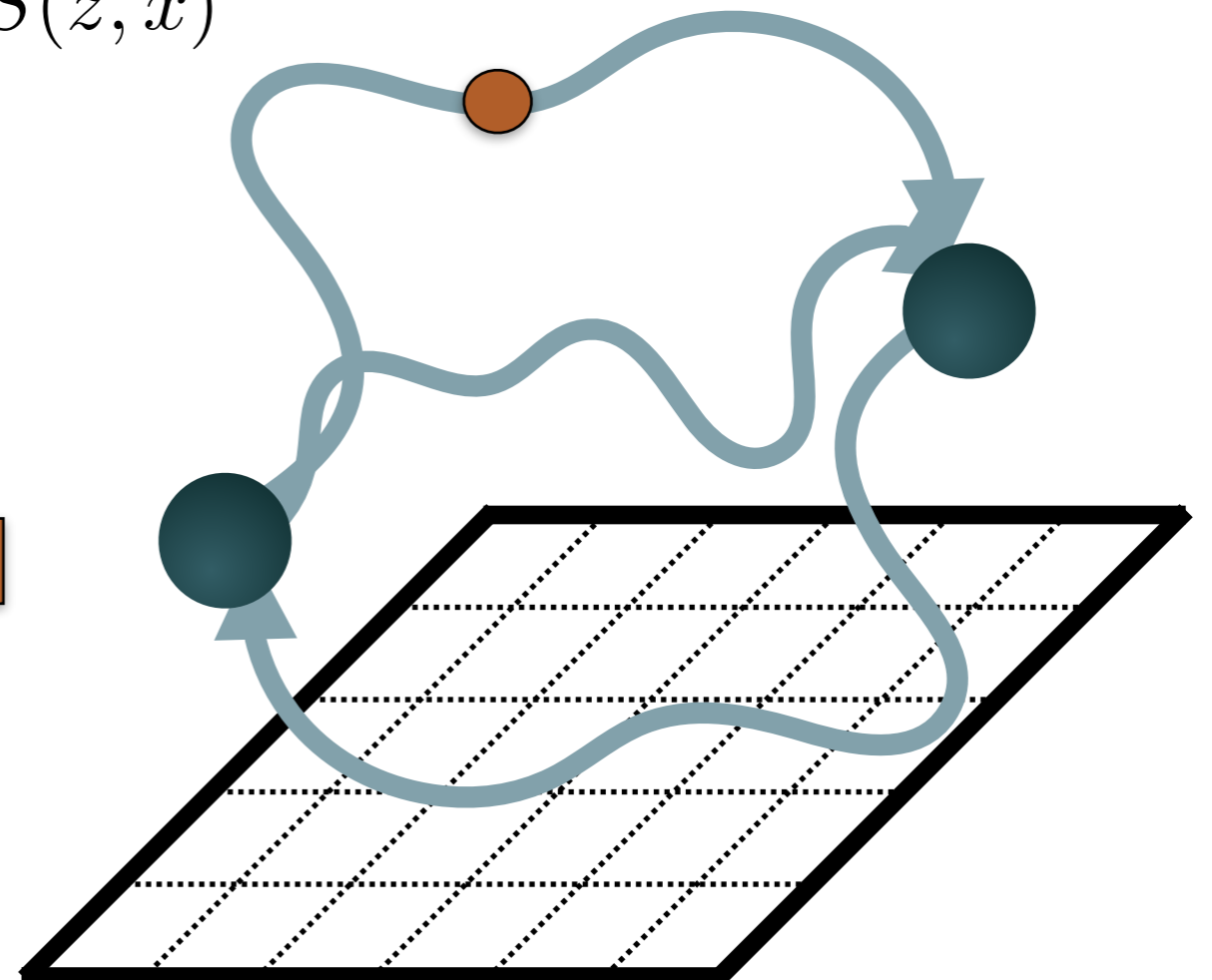
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3-pt function becomes a 2-pt function with FH-prop



Smearred Möbius Domain Wall fermions - I

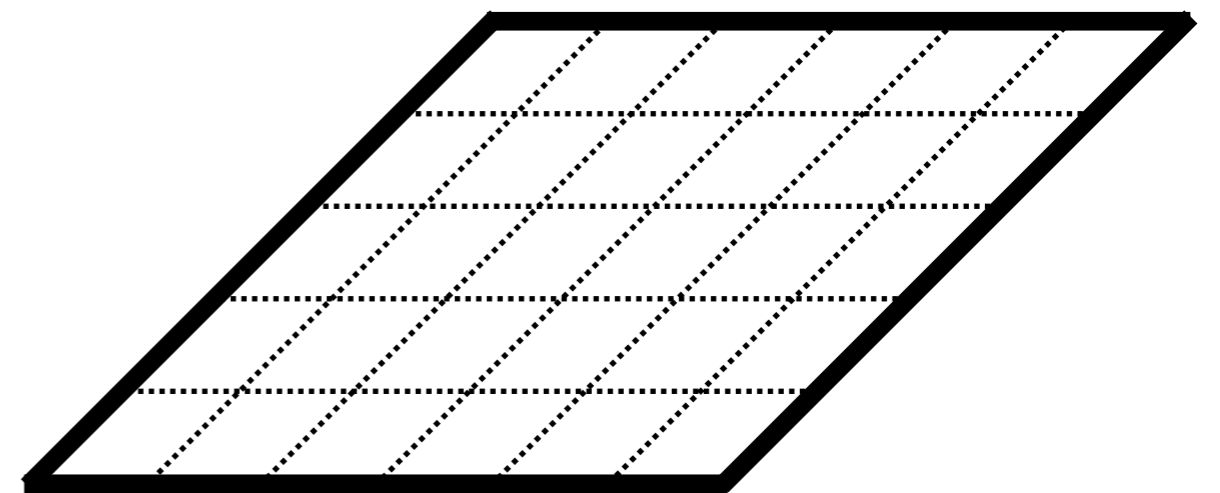
✓ Mixed Action (MA) Lattice QCD

→ **tradeoff** between
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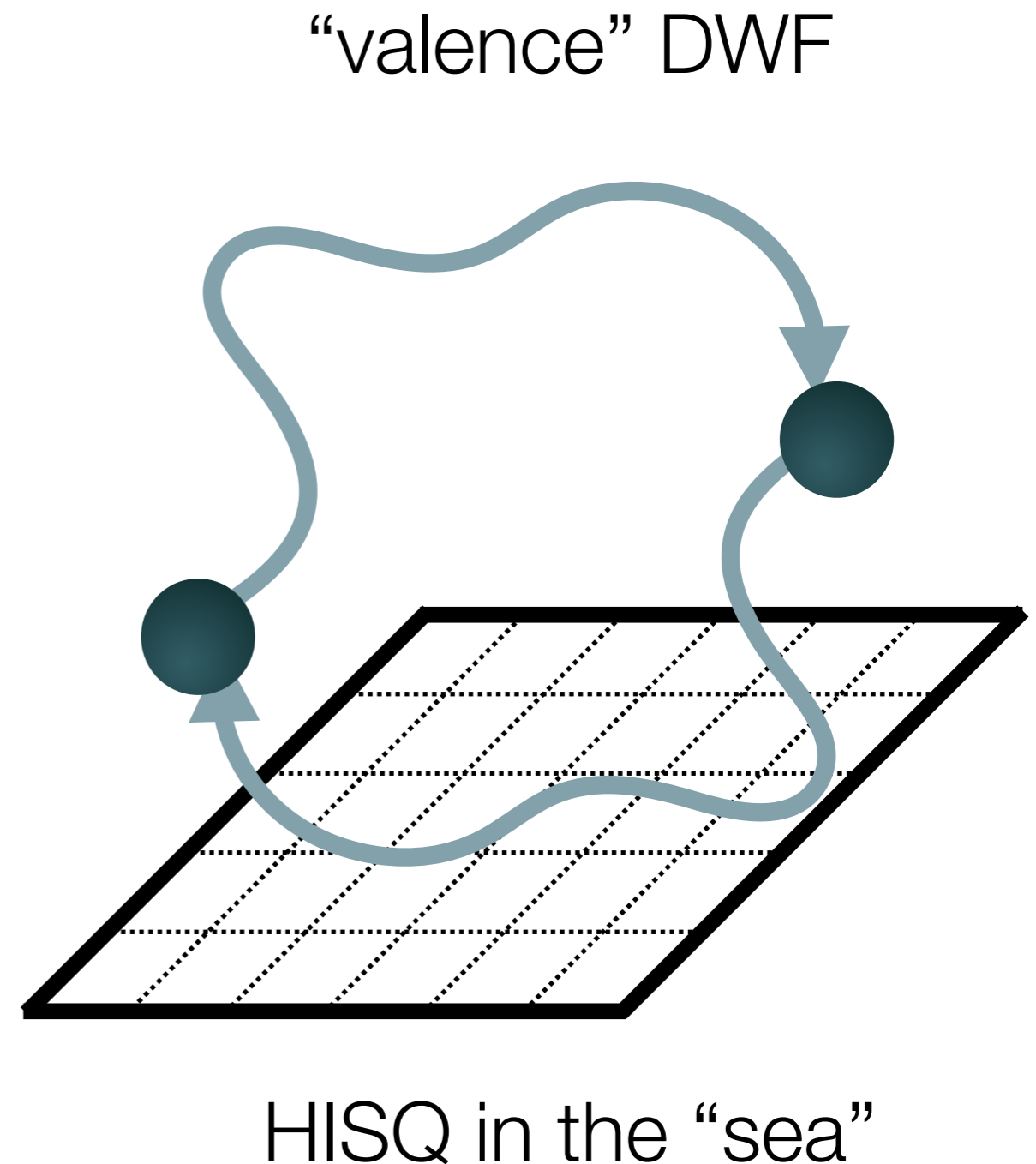


HISQ in the “sea”

Smeared Möbius Domain Wall fermions - I

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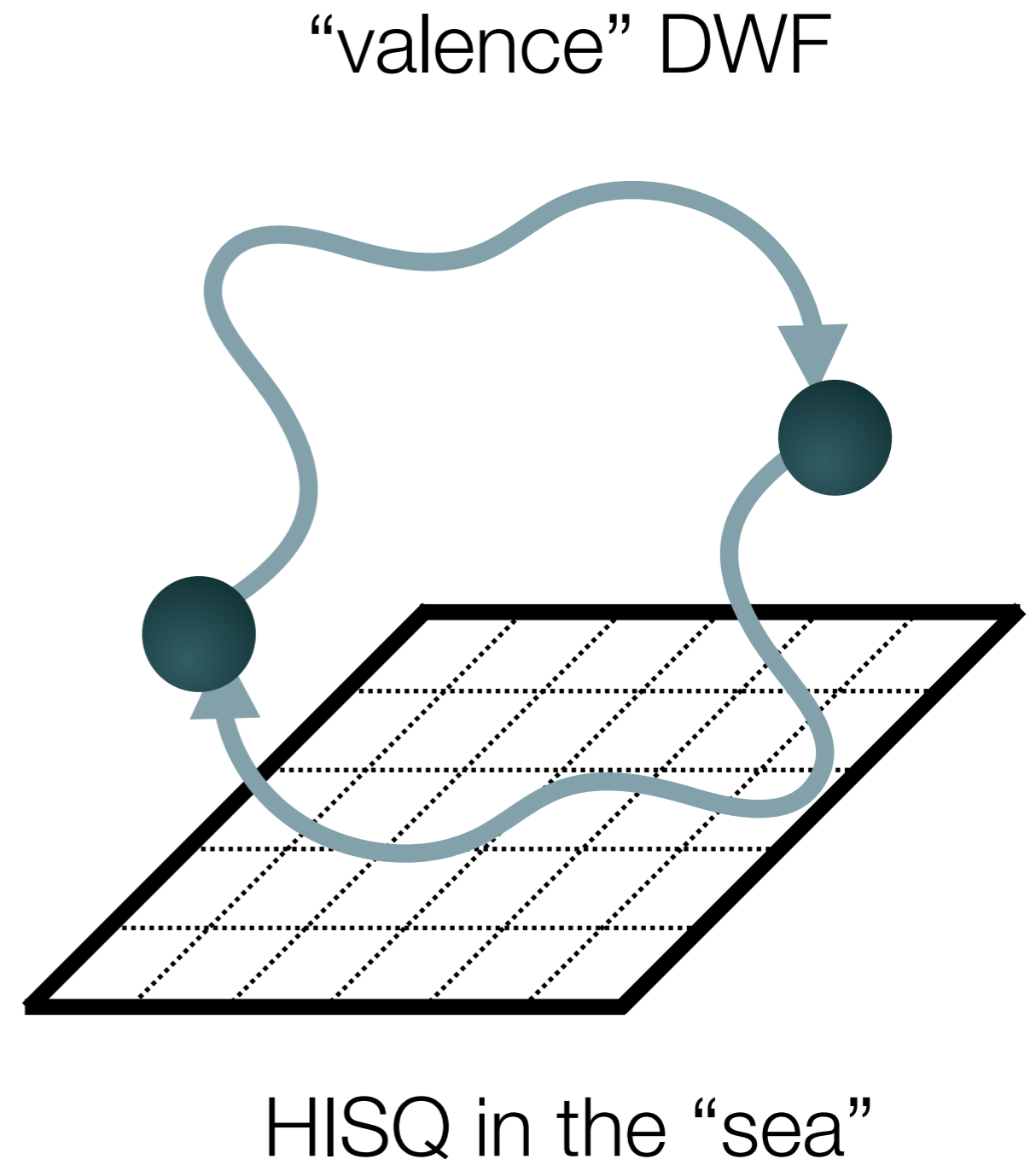
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→ **simplify** treatment of the EFT
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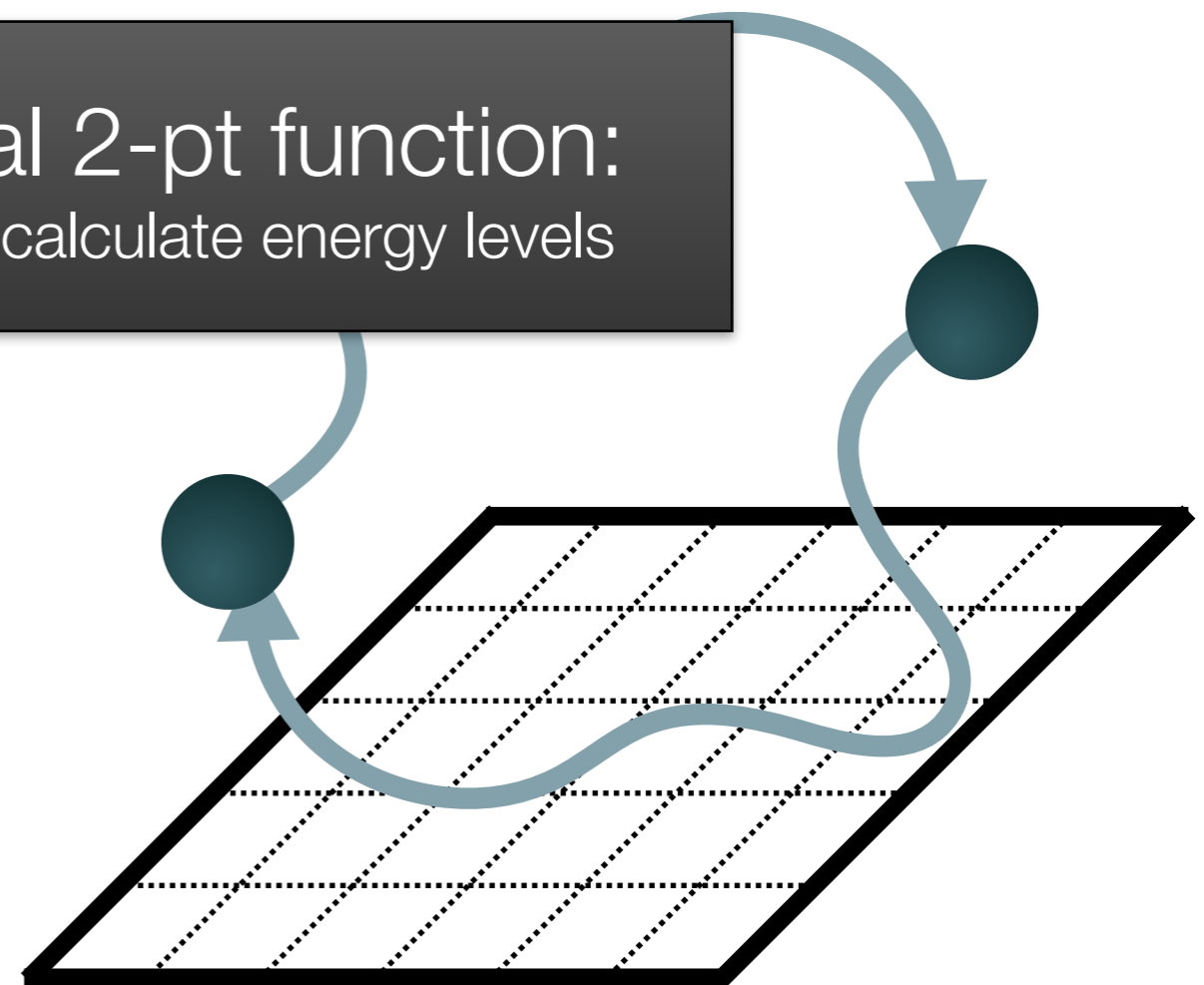
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“valence” DWF

Typical 2-pt function:
used to calculate energy levels



HISQ in the “sea”

Smearred Möbius Domain Wall fermions - II

✓ Gradient flow smeared gauge links

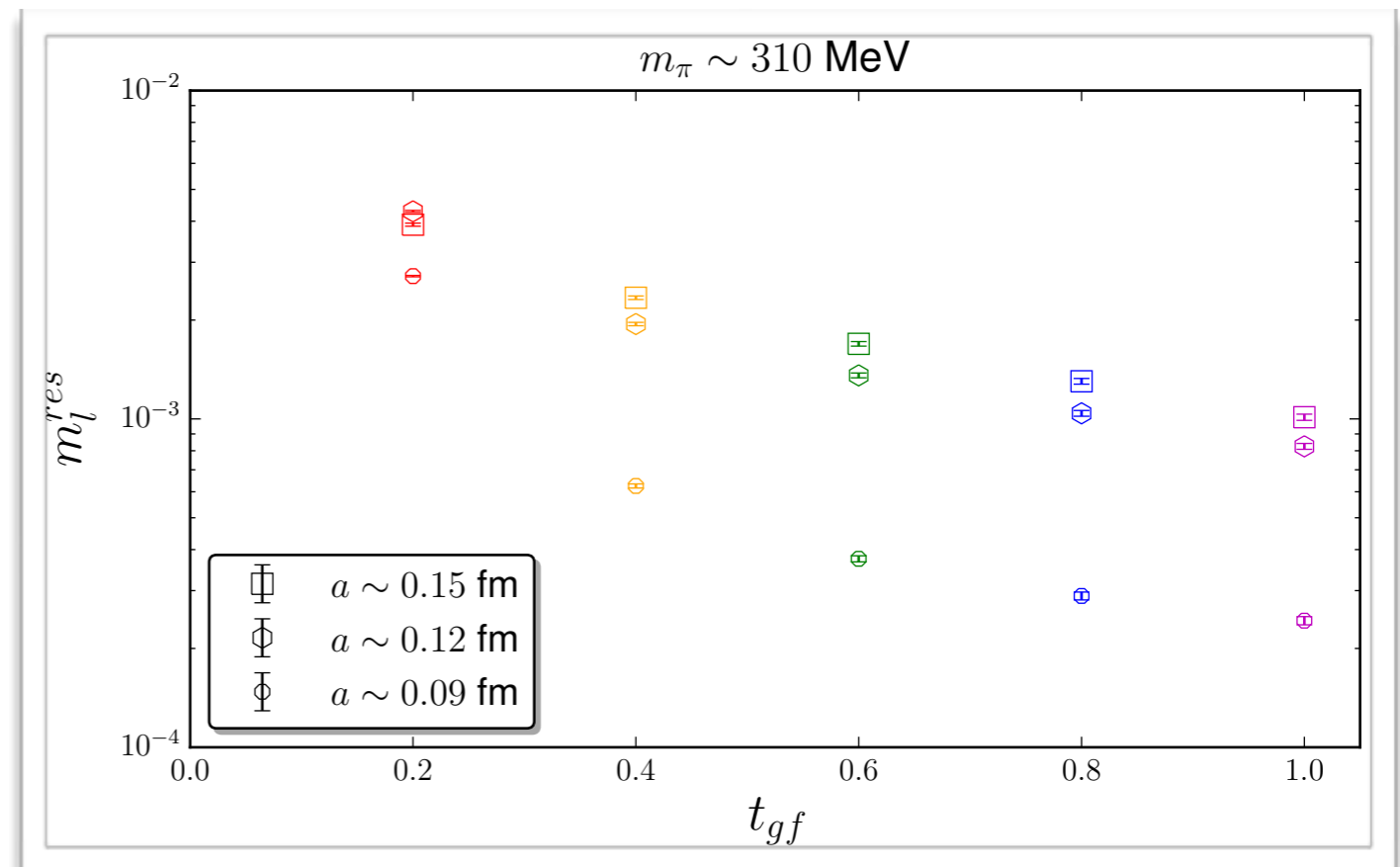
→ parametrized by t_{gf}

✓ Reduces sources of residual chiral symmetry breaking

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→ Z_A has suppressed lattice spacing dependence and is close to unity

✓ Dependence on t_{gf} is removed when performing the continuum limit for physical observables



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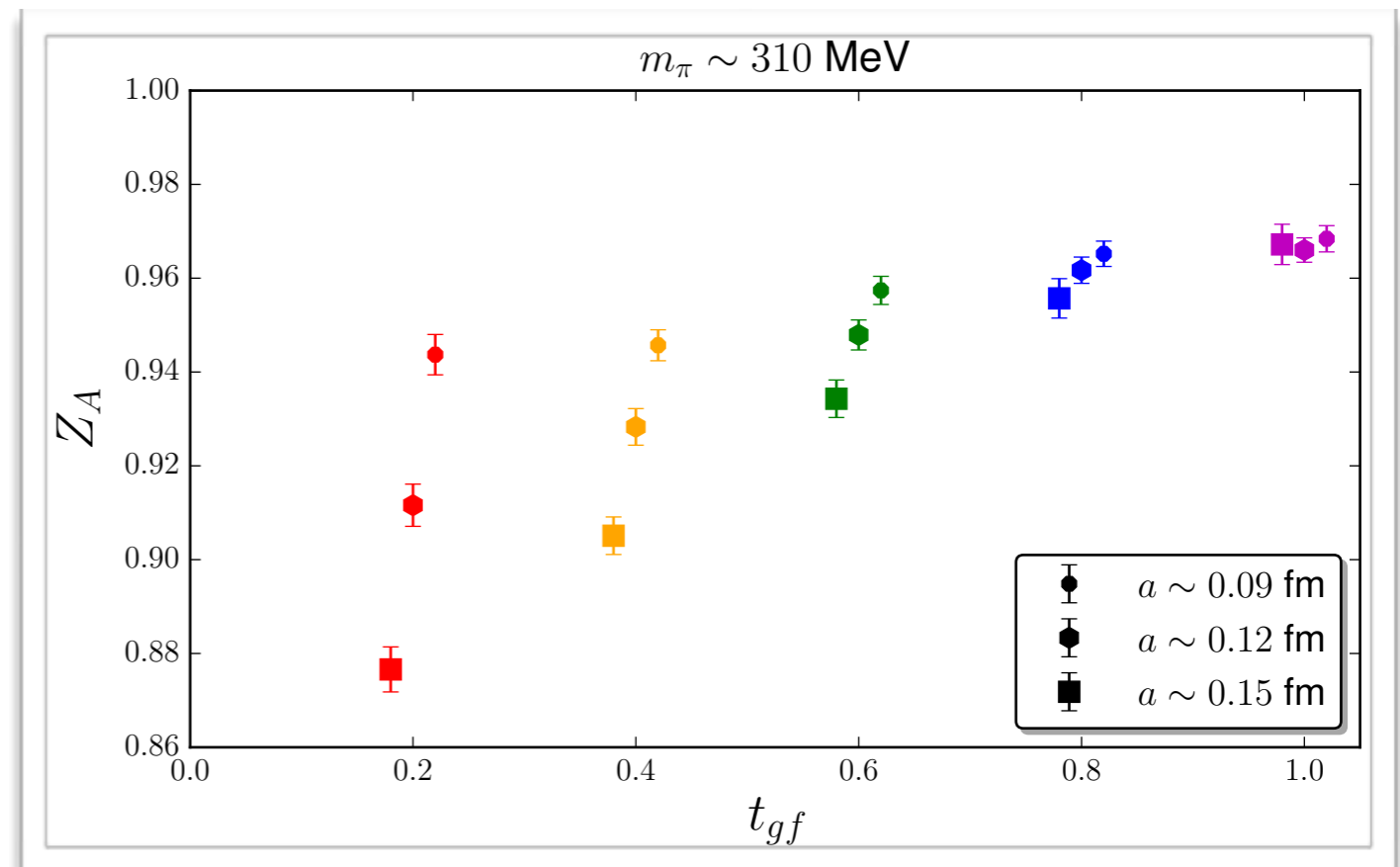
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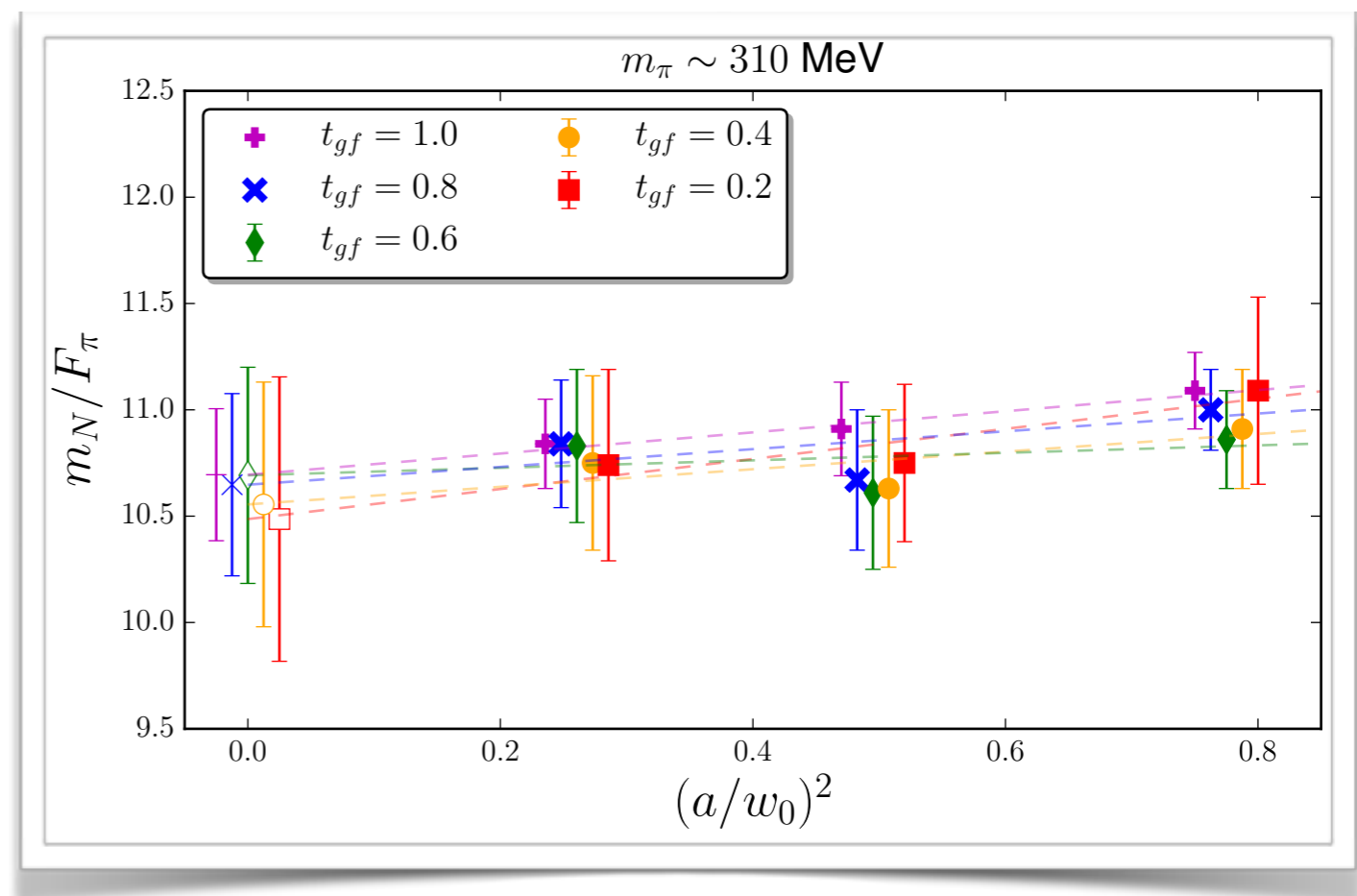
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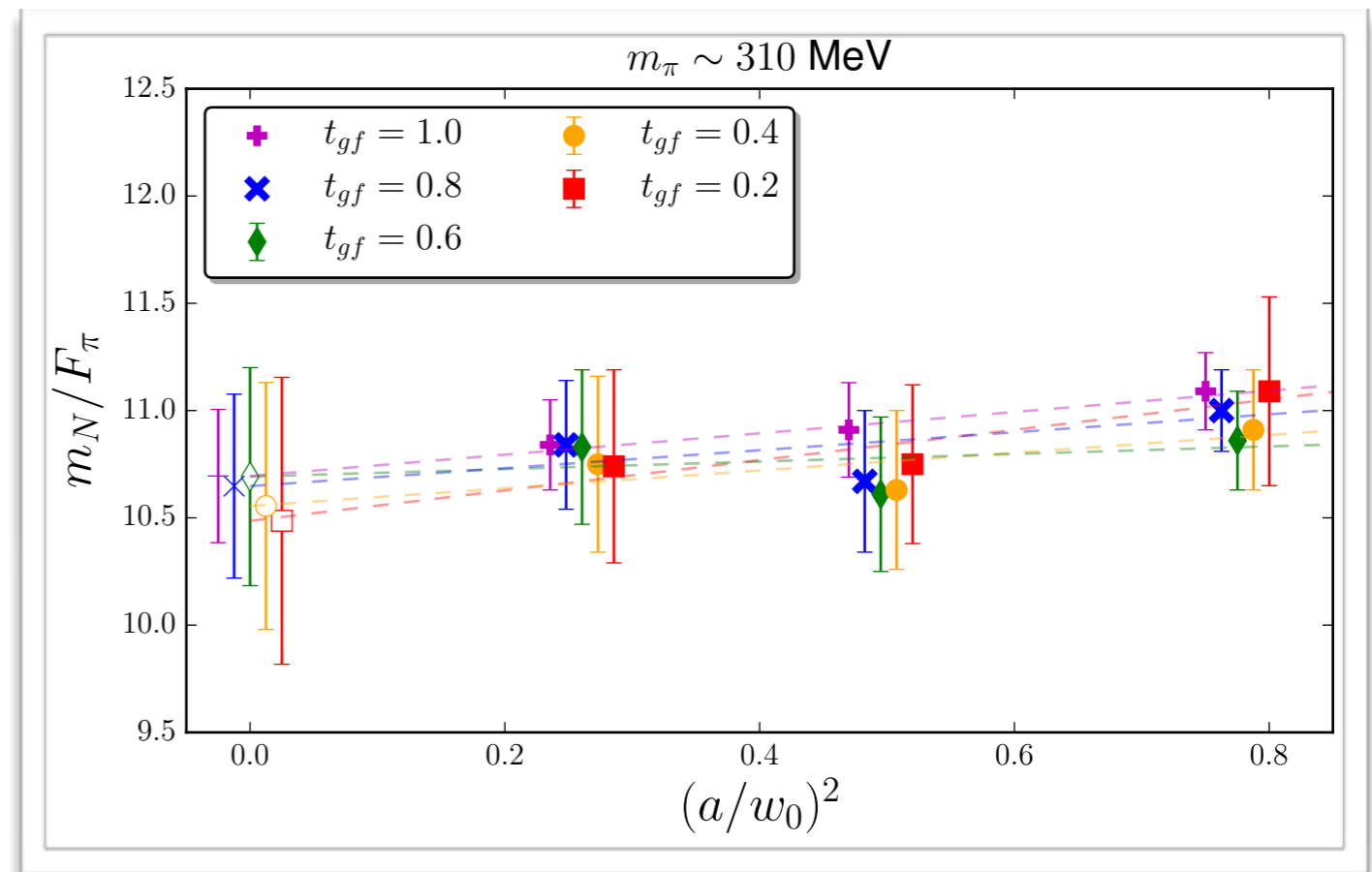
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Improvement of statistical and extrapolation uncertainties

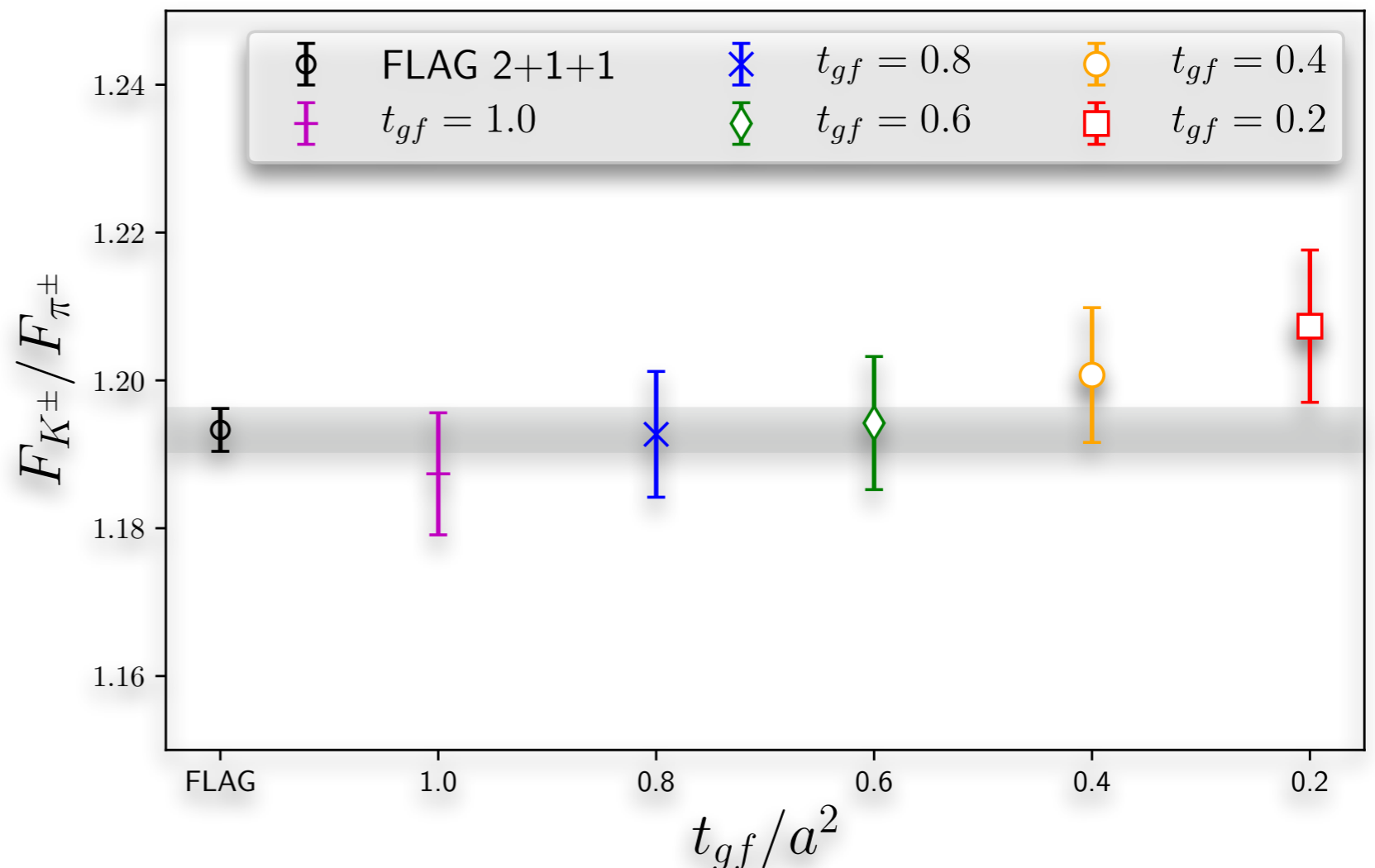
Smeared Möbius Domain Wall fermions - III

✓ Chiral and continuum extrapolation at various t_{gf} values:

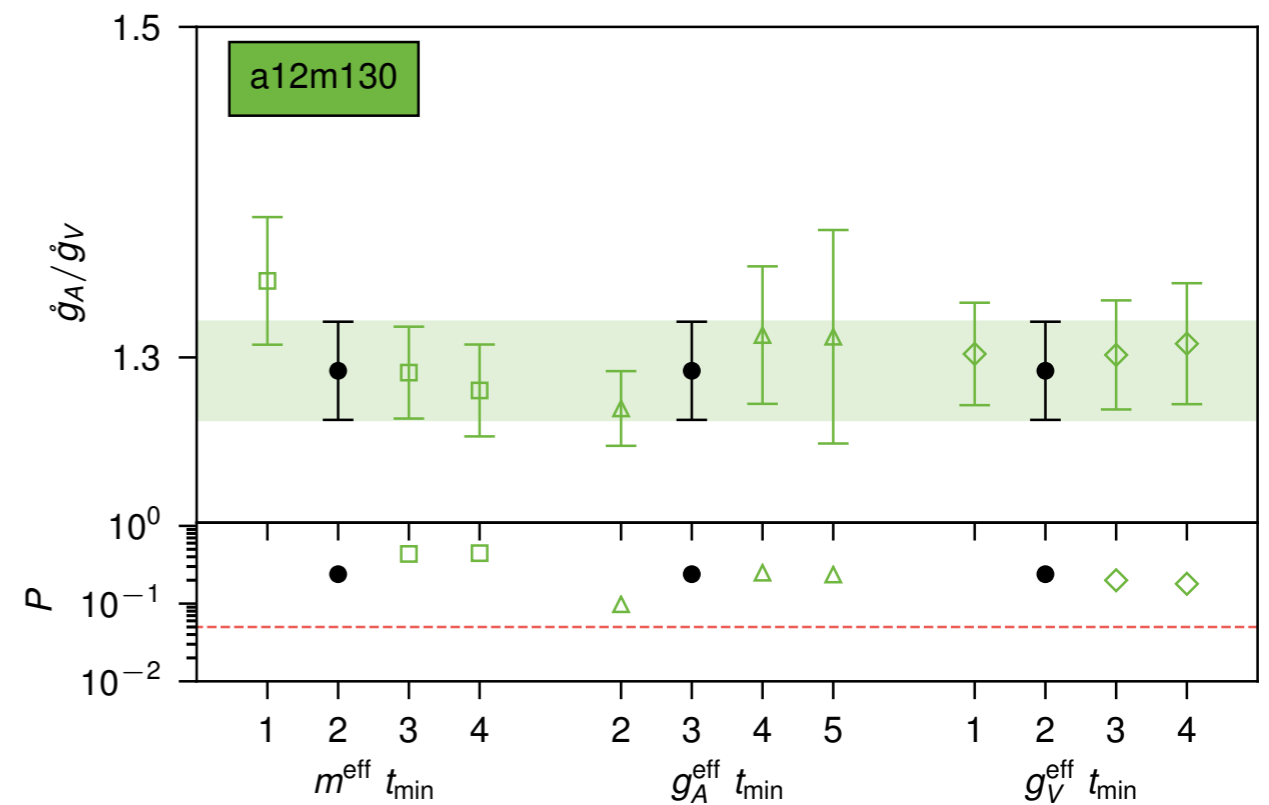
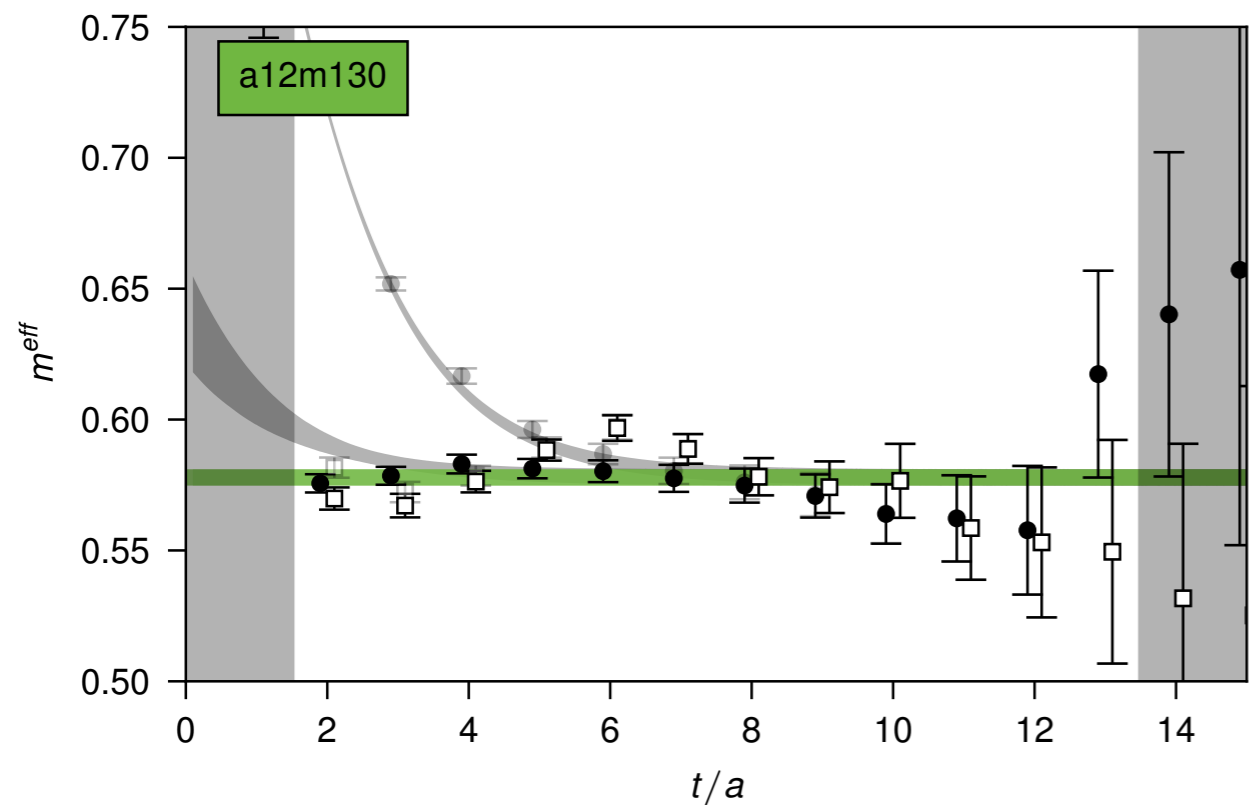
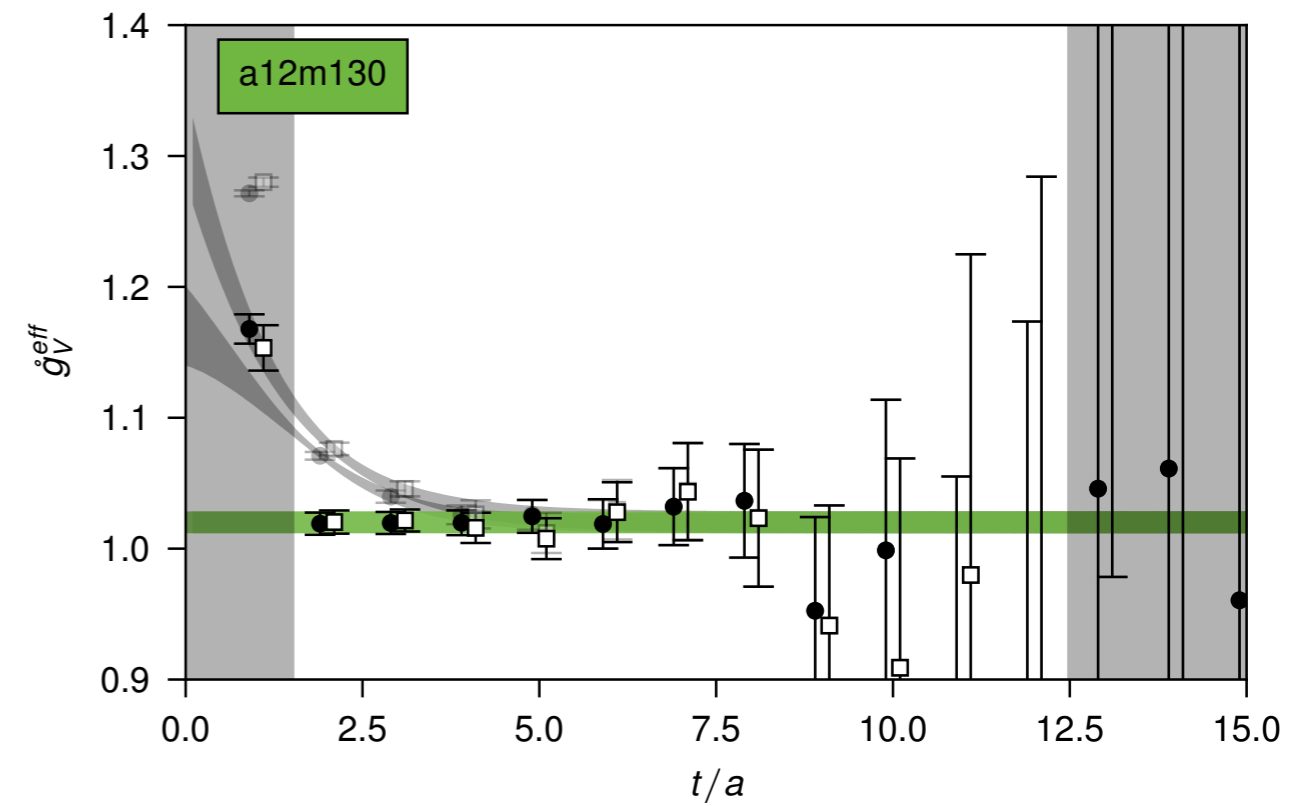
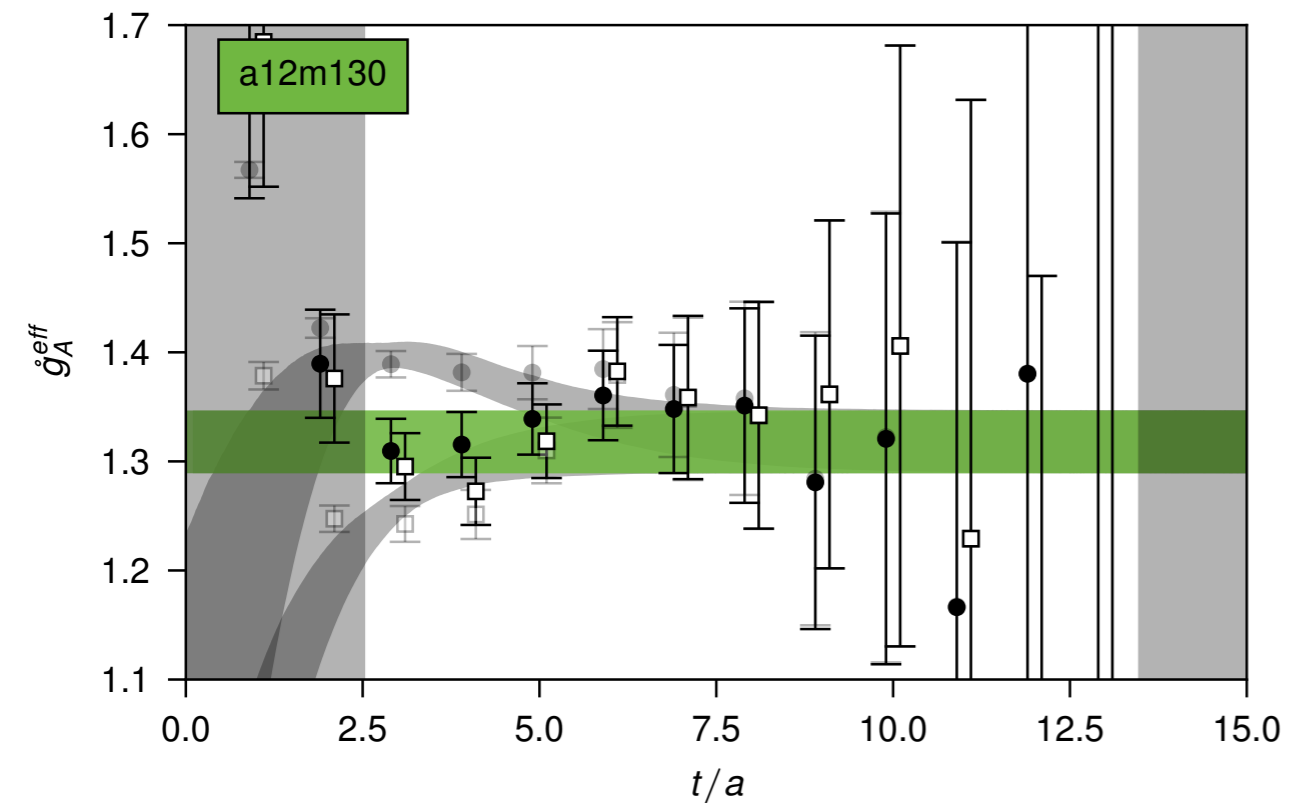
- ➔ 3 lattice spacings, 2 pion masses
- ➔ include a^2 effects and NLO ChiralPT terms
- ➔ negligible finite volume effects

✓ No dependence on t_{gf} and results are consistent with “world average” from FLAG

Benchmark calculation of meson decay constants



Extracting g_A from LQCD data



New method for matrix elements

[Bouchard, Chang, Kurt, Orginos, Walker-Loud, PRD96(014504) - arxiv:1612.06963]

- ❖ Handle the next major source of systematic effects: excited states contamination.
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New method for matrix elements

[Bouchard, Chang, Kurt, Orginos, Walker-Loud, PRD96(014504) - arxiv:1612.06963]

❖ Handle the next major source of systematic effects: excited states contamination.

❖ Based on the Feynman-Hellmann theorem

$$\frac{\partial E_n}{\partial \lambda} = \langle n | H_\lambda | n \rangle$$

$$H = H_0 + \lambda H_\lambda$$

❖ relates matrix elements to linear variations in the energy spectrum with respect to external source

➔ rewording: relates a 3-point correlation function to a change in a 2-point function induced by an external source

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- ❖ related to the background field method (but no need for multiple field values) [NPLQCD arxiv:1610.04545]

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References related to the new method

Similar methods (other FH / GEVP):

J. Bulava *et. al.* JHEP 01,140 (2012)

F. Bernardoni *et. al.* Phys. Lett. B740, 278-284 (2015)

A.J. Chambers *et. al.* Phys. Rev. D 90, 014510

A.J. Chambers *et. al.* Phys. Rev. D 92, 114517

M.J. Savage *et. al.* Phys. Rev. Lett. 119, 062002

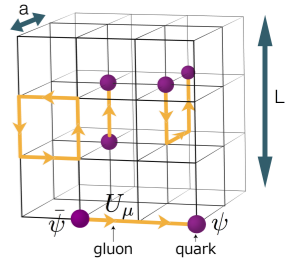
Similar fit function:

S. Capitani *et. al.* Phys. Rev. D 86, 074502

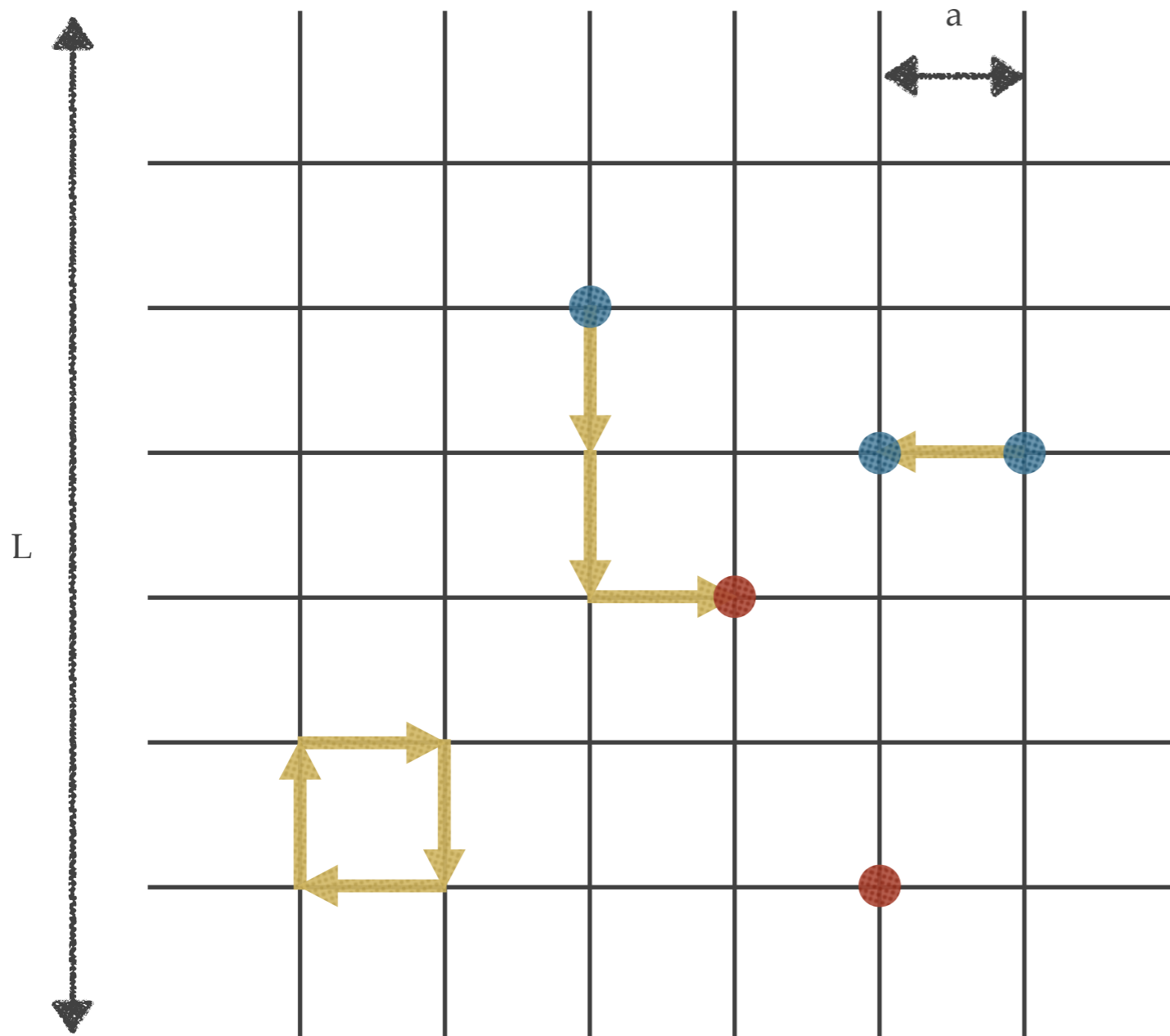
Similar propagator construction:

L. Maiani *et. al.* Nucl. Phys. B293 (1987)

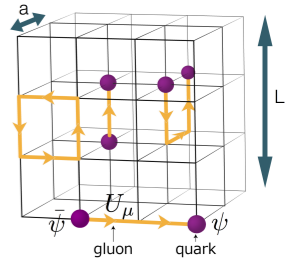
G.M. de Divitiis *et. al.* Phys. Lett. B718 (2012)



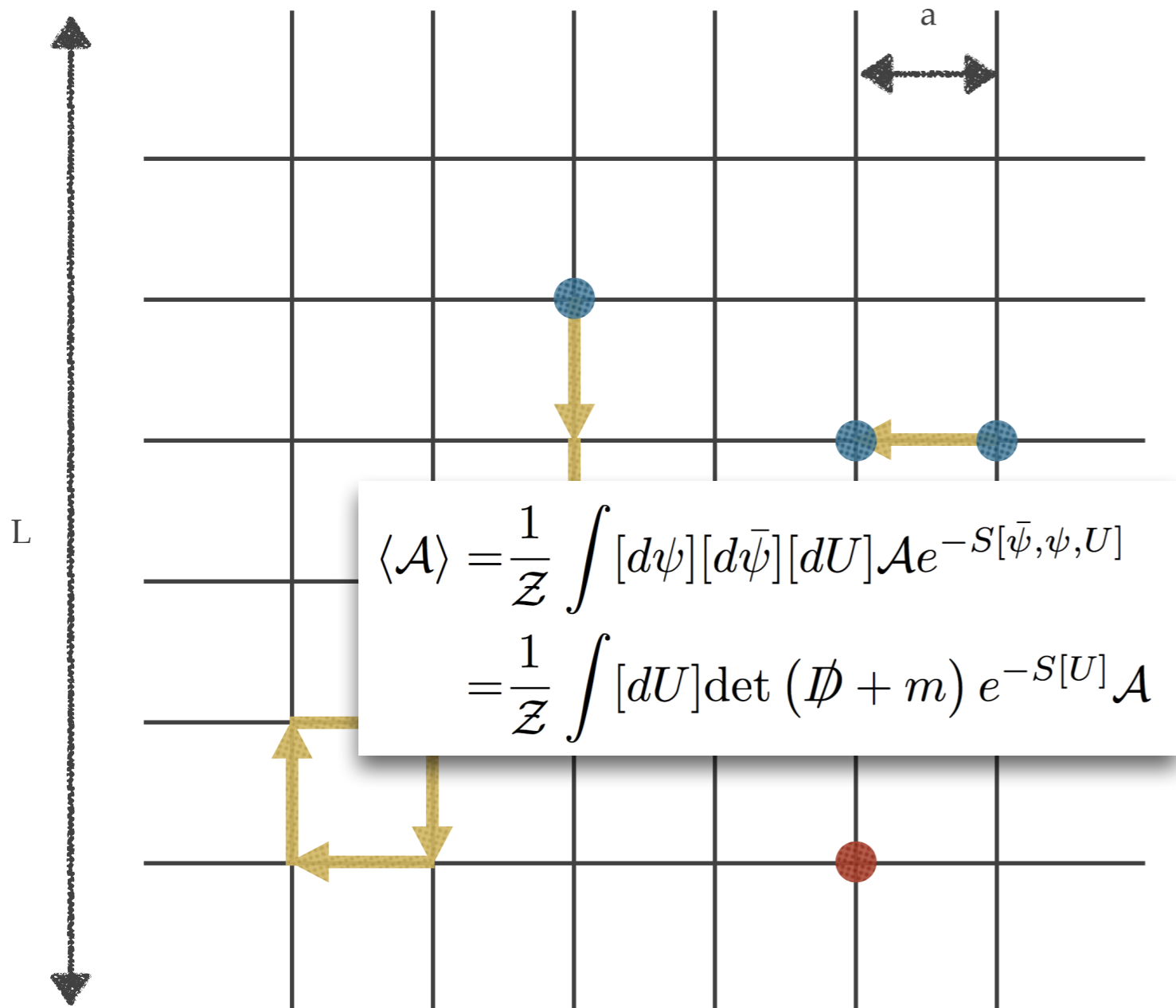
Lattice QCD - basics



- Discretize space and time
 - lattice spacing “a”
 - lattice size “L”
- Keep all d.o.f. of the theory
 - not a model!
 - no simplifications
- Amenable to numerical methods
 - Monte Carlo sampling
 - use supercomputers
- Precisely quantifiable and improvable errors
 - Systematic
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