

# EDMs, CP-odd Nucleon Correlators & QCD Sum Rules

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Based on (older) work with M. Pospelov, see e.g. the review  
M. Pospelov & AR, Ann. Phys. 318, 119 (2005) [hep-ph/0504231]  
(plus some updates)

# Experimental EDM Limits

$$H = -d\vec{E} \cdot \frac{\vec{S}}{S}$$

- EDMs are powerful (amplitude-level) probes for new (T,P) violating sources, motivated e.g. by baryogenesis.
- Best current limits from neutrons, para- and dia-magnetic atoms and molecules

Neutron EDM

$|d_n| < 3 \times 10^{-26} \text{ e cm}$  [Baker et al. '06]

Diamagnetic EDMs

$|d_{\text{Hg}}| < 3 \times 10^{-29} \text{ e cm}$  [Griffith et al '09]

$|d_{\text{Xe}}| < 4 \times 10^{-27} \text{ e cm}$  [Rosenberry & Chupp '01]

Paramagnetic EDMs

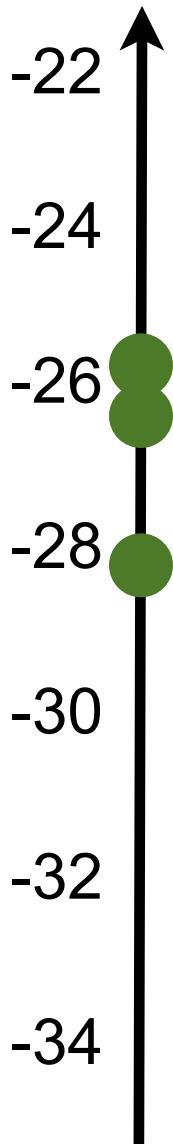
$\Delta E_{\text{ThO}}/\varepsilon_{\text{ext}} < 3 \times 10^{-22} \text{ e cm}$  [Baron et al. '13]

$\Delta E_{\text{YbF}}/\varepsilon_{\text{ext}} < 1.4 \times 10^{-21} \text{ e cm}$  [Hudson et al. '11]

**Negligible SM (CKM) background** - contribution is (at least) 4-5 orders of magnitude below the current neutron sensitivity, and lower for the atomic EDMs

# Summary of the bounds

$\log(d [e \text{ cm}])$



Real sensitivity to underlying sources of CP violation depends on significant enhancement and suppression factors

$d_q$  and  $\tilde{d}_q$  from the neutron

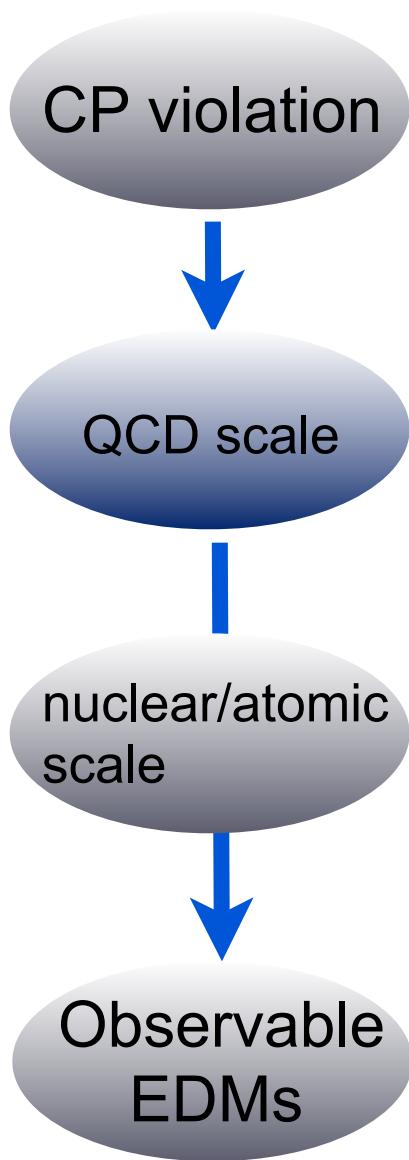
$\tilde{d}_q$  from Hg

$d_e$  from ThO

impact of recent order of magnitude improvement in paramagnetic EDM sensitivity

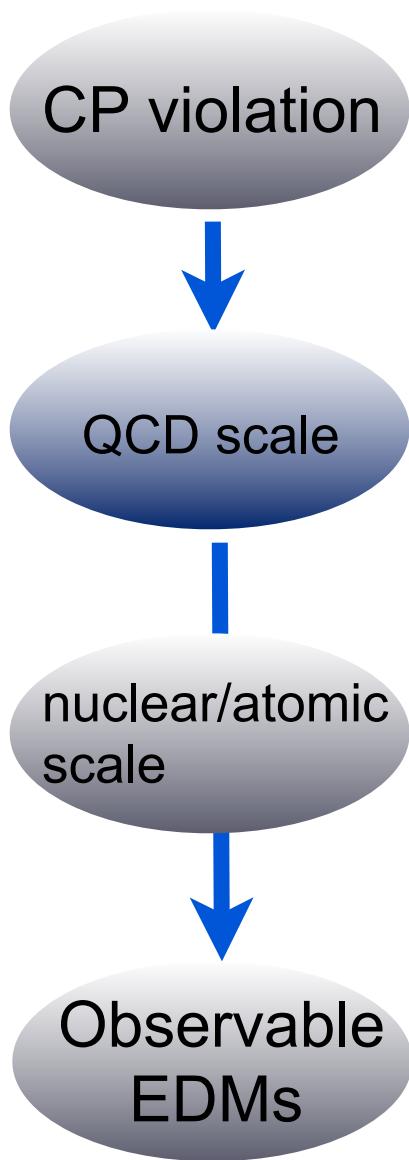
The generic sensitivity to new physics follows from taking  $d_f \propto m_f$

# Multi-scale calculational scheme



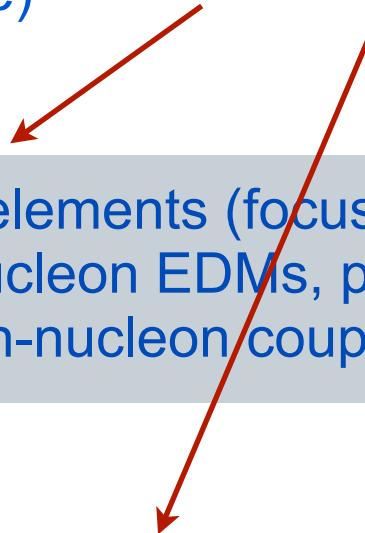
- Model-dependent (e.g. perturbative)
- Nucleon matrix elements (focus of this meeting), nucleon EDMs, pion-nucleon, nucleon-nucleon couplings
- Nuclear scale, e.g. Schiff moment, magnetic quadrupole
- Atomic/Molecular EDM

# Multi-scale calculational scheme

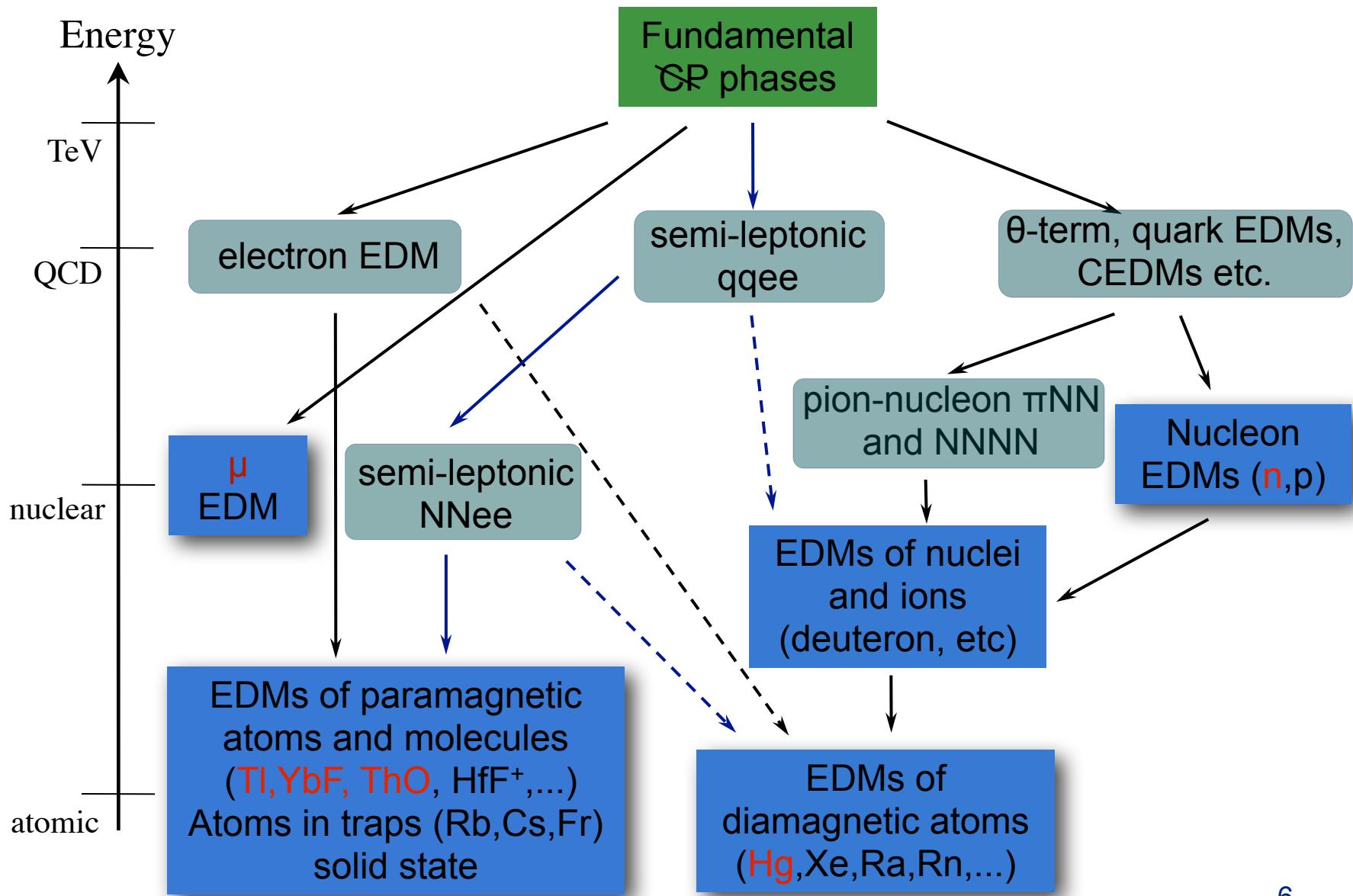


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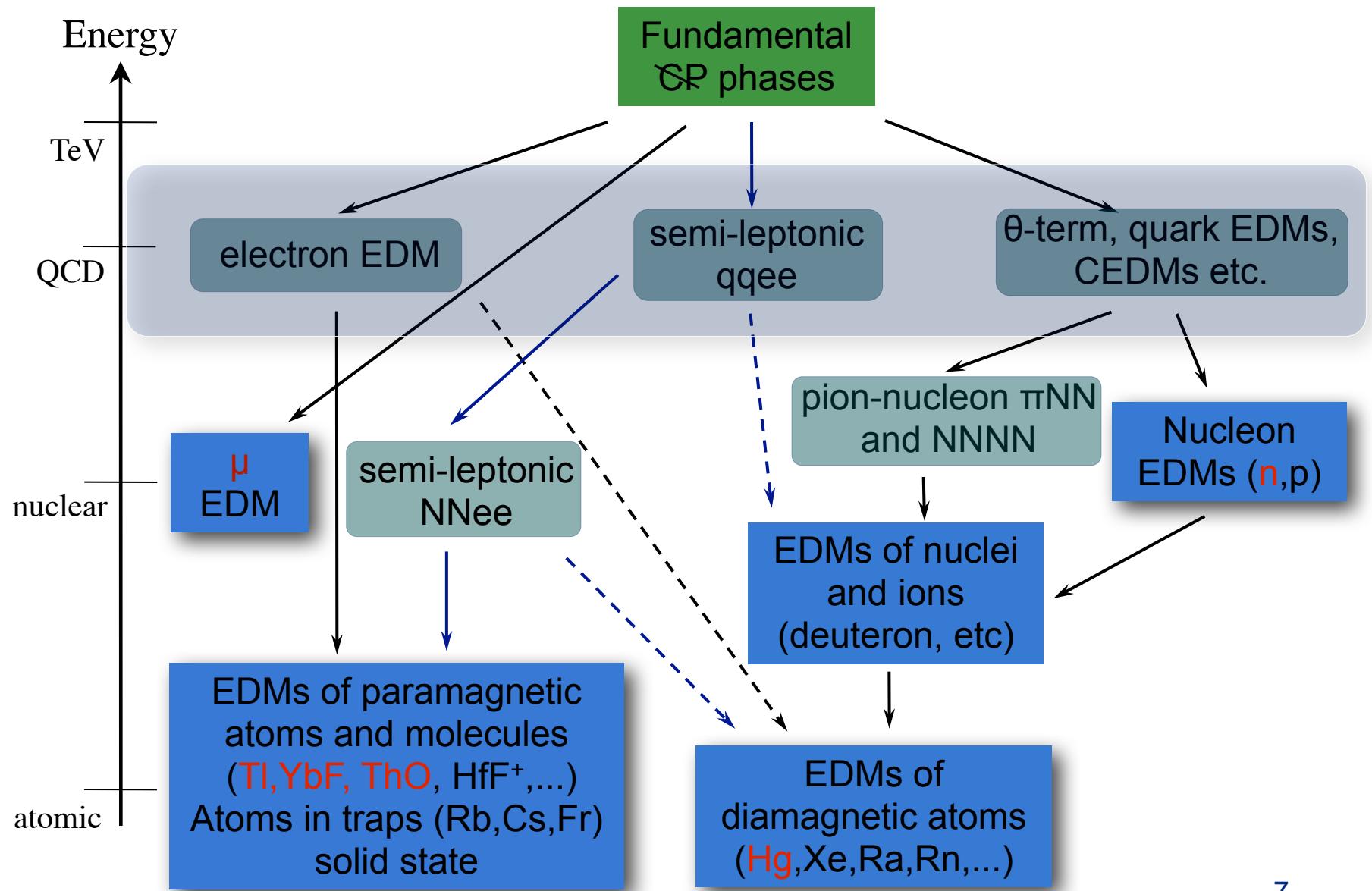
Significant uncertainties  
for nucleon, nuclear and  
diagmagnetic EDMs



# EDM Sensitivity to (short distance) CP-violation



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# CP-odd operator expansion (at $\sim 1\text{GeV}$ )

(Flavor-diagonal) CP-violating operators at  $\sim 1\text{GeV}$

$$\mathcal{L}_{\text{eff}} = \sum_n \frac{c_n}{\Lambda^{d-4}} \mathcal{O}_d^{(n)}$$

[ hadronic sector  
discussed in detail in  
Jordy's talk]

$$\mathcal{L}_{\text{dim } 4} \supset \bar{\theta} \alpha_s G \tilde{G}$$

$$\bar{\theta} = \theta_0 - \text{ArgDet}(M_u M_d) \equiv \theta_0 - \theta_q$$

- NB: (i) Basis at 1 GeV is simpler than at EW scale, after integrating out W,Z,h etc.  
(ii) Use of QCD dofs assumes that the new physics scale is above 1 GeV)

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$$d_i \sim c Y_i \frac{v}{\Lambda^2}$$

$$\mathcal{L}^{\text{"dim 6"}} \supset \sum_{q=u,d,s} \left( d_q \bar{q} F \sigma \gamma_5 q + \tilde{d}_q \bar{q} G \sigma \gamma_5 q \right) + \sum_{l=e,\mu} d_l \bar{l} F \sigma \gamma_5 l$$

$$\mathcal{L}_{\text{dim } 6} \supset w g_s^3 G G \tilde{G}$$

# CP-odd operator expansion (at $\sim 1\text{GeV}$ )

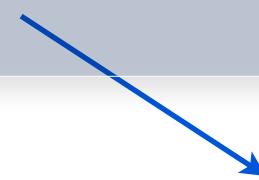
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Schematic form of a few special 4-fermion operators,  
requiring no Higgs insertion - suppressed without new  
UV sources of LR mixing

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$$C_{ij} \sim c Y_i Y_j \frac{v^2}{\Lambda^4}$$

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NB: Relative importance of different operators is very model-dependent, and the expansion can be misleading. E.g. for the SM (and SUSY and 2HDM regimes at large tanbeta), these 4-fermion sources are dominant

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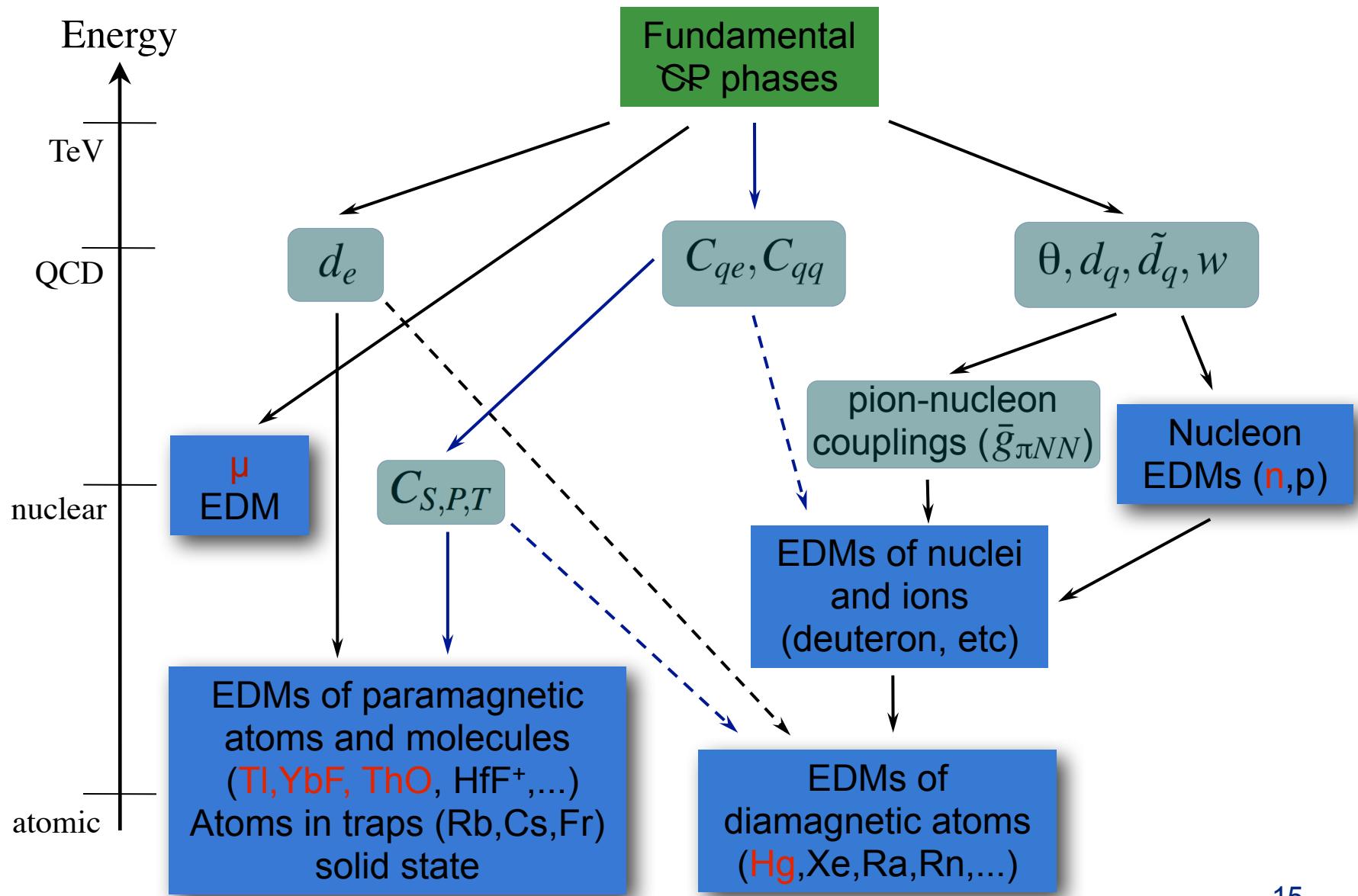
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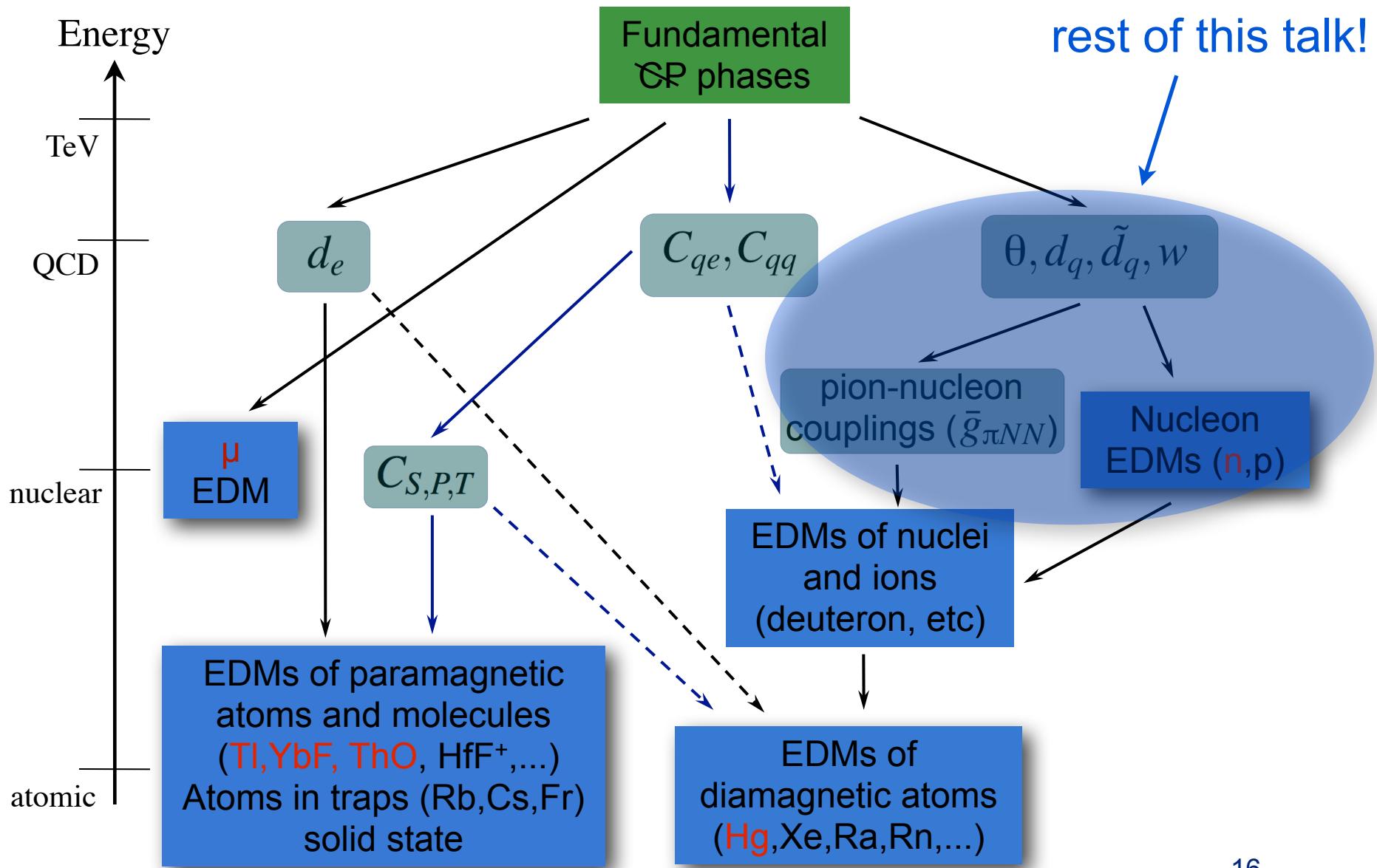
nucleon/nuclear scales

$$d_{(n,p)} \bar{N} F \sigma \gamma_5 N + \bar{g}_{\pi NN}^{(1)} \bar{N} \pi^0 N + \bar{g}_{\pi NN}^{(0)} \bar{N} \sigma \cdot \pi N + (4 - \text{nucleon}) + \dots$$
$$d_e \bar{e} F \sigma \gamma_5 e + C_S^{(0)} \bar{N} N \bar{e} i \gamma_5 e + \dots$$

# EFT hierarchy



# EFT hierarchy



# The QCD scale

- Chiral EFT (chiral constraints) [➡ Emanuele's talk]

$$\mathcal{L} = \mathcal{L}(\pi, (K), N, \dots)$$

$$= -\frac{i}{2} \bar{N} (d_n \tau^- + d_p \tau^+) F \sigma \gamma_5 N - \bar{N} (\bar{g}_{\pi NN}^{(0)} \tau^a \pi^a + \bar{g}_{\pi NN}^{(1)} \pi^0) N + \dots$$

low energy  
constants  $\left\{ \begin{array}{l} d_N(\bar{\theta}, d_q, \tilde{d}_q, w, C_{ij}, \dots) \\ \bar{g}_{\pi NN}^{(0,1)}(\bar{\theta}, \tilde{d}_q, C_{ij}, \dots) \end{array} \right.$

[Crewther et al '79; Hisano & Shimizu '04;  
Stetcu et al '08, de Vries et al '11, '12; An et al  
'12; Guo & Meissner '12, Bsaisou et al '14 ]

# The QCD scale

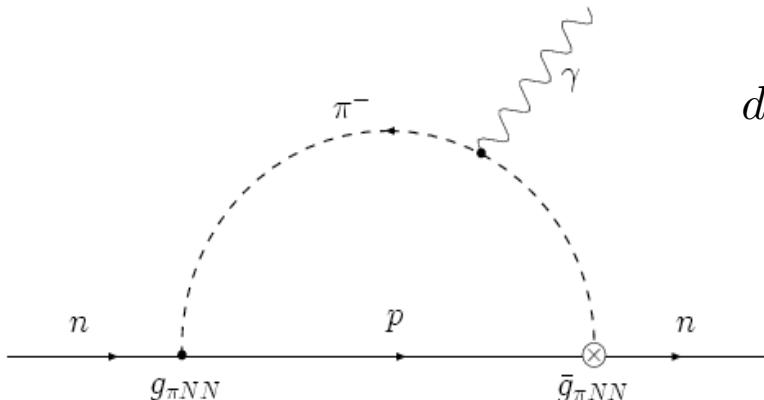
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- LEC's related by IR loops (chiral logs)
  - still need input to fix counterterms



$$d_n = \frac{e}{4\pi^2 m_n} g_{\pi NN} \bar{g}_{\pi NN}^{(0)} \ln \frac{\Lambda}{m_\pi} + C_{ct}$$

need UV threshold corrections

# The QCD scale

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- LEC's related by IR loops (chiral logs)
  - still need input to fix counterterms
- Simplest option is NDA -  $\Lambda_{\text{had}}/f_\pi \sim g_s(\mu) \sim 4\pi$     $m_{q, \text{av}} \sim m_\pi^2/\Lambda_{\text{had}}$

	$\theta_q$	$d_q$	$\tilde{d}_q$
$d_n$	$\frac{em_q}{\Lambda_{\text{had}}^2}$	$\mathcal{O}(1)$	$\frac{eg_s}{4\pi}$
$\bar{g}_{\pi NN}^{(0)}$	$\frac{m_q}{f_\pi}$	$\sim \mathcal{O}(\alpha)$	$\frac{\Lambda_{\text{had}}^2}{f_\pi}$

# Why do better than NDA?

- (Our) pre-historic motivations...
  - back in the 1990's, when SUSY (MSSM) was “just around the corner”, there was a focus on combining multiple EDM contributions to search for cancelations to ameliorate the SUSY CP problem.
  - This requires a systematic procedure to add contributions from multiple CP-odd sources, with relative signs!
  - At the time, there weren't many viable approaches, and we utilized QCD sum rules
- Current (and more generic) motivations...
  - disentangle CP-violating sources, given one (or more) detections
  - indicate possible enhancement/suppression factors (cf. NDA)
  - still require a systematic procedure able to handle multiple CP-odd sources

# (CP-odd) nucleon correlators

- Consider the two-point function of the nucleon interpolating current in the presence of CP-odd sources

$$\int d^4x e^{ip \cdot x} \langle \bar{j}_n(x), j_n(0) \rangle_{QP,F} = \Pi_0(p) + \Pi_1^{\mu\nu}(p) F_{\mu\nu} + \dots$$

- Features:
  - multiple interpolating currents with lowest dimension
  - New - chiral “ambiguity” of the nucleon current

$$j_n = 2\epsilon_{abc}(d_a^T C \gamma_5 u_b)d_c + \beta \times 2\epsilon_{abc}(d_a^T C u_b)\gamma_5 d_c$$

unphysical parameter

$$\langle 0 | j_n | n \rangle = (\lambda_1 + \beta \lambda_2) e^{i\alpha\gamma_5/2} v$$

CP-odd sources introduce an unphysical phase in the coupling of the nucleon current and the physical state, which can (unphysically) mix d and  $\bar{u}$

# (CP-odd) nucleon correlators

- Another (related) new feature - the nucleon current can now mix with CP-conjugate currents,  $\tilde{j}_n = CPj_nCP$

- Need to account for mixing by re-diagonalizing at linear order in the source

$$j_n \rightarrow j_n + i\epsilon_{CP}\tilde{j}_n$$

- So the correlator is correspondingly rotated

$$\int d^4x e^{ip \cdot x} \langle \bar{j}_n(x), j_n(0) \rangle_{CP,F} = \Pi_0(p) + \Pi_1^{\mu\nu}(p) F_{\mu\nu} + \dots$$



$$\langle \bar{j}j \rangle_{F,CP} + i\epsilon_{CP} \langle j\bar{j} + \tilde{j}\bar{j} \rangle_F + \dots$$

$$\frac{i}{2} \frac{\langle \tilde{j}_n \bar{j}_n - j_n \bar{\tilde{j}}_n \rangle_{CP}}{\langle j_n \bar{j}_n - \tilde{j}_n \bar{\tilde{j}}_n \rangle}$$

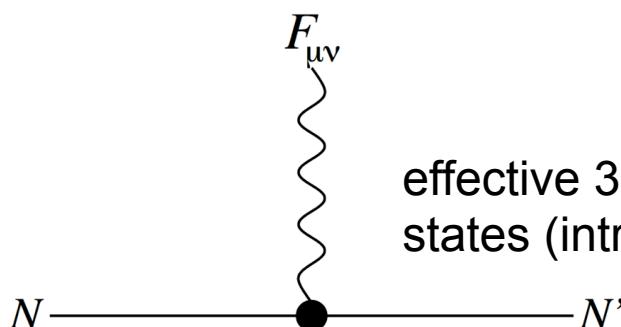
# QCD sum rules

- Work in a general basis of CP sources, as a cross-check
- Account for CP-odd current mixing to linear order (as above)
- Isolate EDM from a chirally-invariant structure in off-shell dipole form-factor (2-pt function, to first order in F)

$$\Pi_1(p) \cdot F \sim \{F\sigma\gamma_5, p\} \left( \frac{d_n \lambda^2 m_n}{(p^2 - m_n^2)^2} + \frac{A}{p^2 - m_n^2} + \dots \right) + \dots$$

Isolate tensor structure independent of unphysical phase  $\alpha$  (avoids mixing of d and  $\mu$  structures)

3-pt mixing with excited states cannot be exponentially suppressed with a Borel transform, due to lack of positivity in dispersive integral for 3-pt correlators. Must include mixing coefficient A explicitly in the fit.



effective 3-pt vertex allows mixing with excited states (introduces new fitting parameter A)

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- Perform a self-consistent fit for the EDM, using other CP-even nucleon sum rules (mass,  $\sigma_N$ , etc) to determine  $\{m_n, \lambda, A\}$

$$\lambda_1 + \beta \lambda_2$$

FAC criterion fixes  $\beta=1$ , i.e. to “optimize convergence” of the OPE

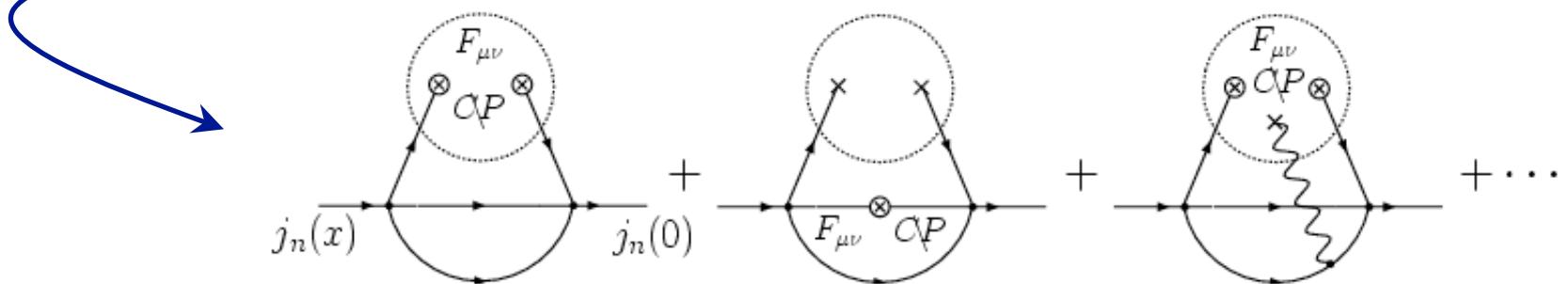
# Neutron EDM

- schematic structure of the OPE

[Pospelov & AR '99-'00]

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- depends on vacuum condensates, e.g.  $\langle \bar{q} \sigma_{\mu\nu} q \rangle_F = \chi e_q F_{\mu\nu} \langle \bar{q} q \rangle$   
 $\langle \bar{q} G \sigma q \rangle = -m_0^2 \langle \bar{q} q \rangle$
- implicit dependence of condensates on the CP-odd sources determined via xPT, and saturation with  $\pi$  and  $\eta$  exchange.  
(vacuum “realignment”)

# Neutron EDM

- Results:

$$d_n(\bar{\theta}) = (1 \pm 0.5) \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3} \bar{\theta} \times 2.5 \times 10^{-16} e \text{ cm}$$

- If the axion relaxes  $\theta$ , the CEDM sources shift the minimum of the axion potential  $V(\theta)$  away from zero [Bigi & Uraltsev]

$$\theta_{ind} = \frac{1}{2} m_0^2 \sum_{q=u,d,s} \frac{\tilde{d}_q}{m_q}$$

$$d_n = (0.4 \pm 0.2) \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3} \left[ 4d_d - d_u + \underbrace{\frac{1}{2} \chi m_0^2 (4e_d \tilde{d}_d - e_u \tilde{d}_u)}_{2.7e(\tilde{d}_d + 0.5\tilde{d}_u)} + \dots \right] + O(d_s, w, C_{qq})$$

[Pospelov & AR '99,'00;  
Hisano et al '12]

Sensitive only to ratios of light quark masses  
(via GMOR relation, given  $d_q \sim m_q$  etc.)

- at this order, s-quark CEDM contribution cancels under axion relaxation (appears accidental)

# Neutron/Proton EDM

- Results:

$$d_n(\bar{\theta}) \sim 3 \times 10^{-16} \bar{\theta} \text{ ecm}$$

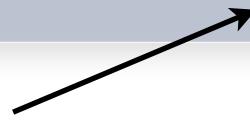
$$d_p(\bar{\theta}) \sim -4 \times 10^{-16} \bar{\theta} \text{ ecm}$$

- If the axion relaxes  $\theta$ , the CEDM sources shift the minimum of the axion potential  $V(\theta)$  away from zero

$$\theta_{ind} = \frac{1}{2} m_0^2 \sum_{q=u,d,s} \frac{\tilde{d}_q}{m_q}$$

$$d_p^{(PQ)} \sim (0.4 \pm 0.2) [4d_u - d_d - 5.3e(\tilde{d}_u + 0.13\tilde{d}_d) + \dots] + \mathcal{O}(d_s, w, C_{qq})$$

$$d_n^{(PQ)} \sim (0.4 \pm 0.2) [4d_d - d_u + 2.7e(\tilde{d}_d + 0.5\tilde{d}_u) + \dots] + \mathcal{O}(d_s, w, C_{qq})$$



Appearance of the same relative coefficients as the NQM appears accidental, as it depends (at  $\sim 30\%$ ) on the choice of  $\beta \in [0, 1]$

# Neutron EDM

- Precision?
  - numerical coefficients are consistent with NDA, NQM (for  $d_q$ ), and the chiral log (for  $\theta$ )
  - another test for  $d_n(d_q)$  via (LQCD) nucleon tensor charge [e.g. Falk et al '99]

$$\langle N | \frac{1}{2} d_q \bar{q} \tilde{F} \sigma q | N \rangle = \frac{1}{2} d_q \tilde{F}^{\mu\nu} \langle N | \sigma_{\mu\nu} | N \rangle = \frac{1}{2} g_T^q d_q \bar{N} \tilde{F} \sigma N$$

$$\implies d_n(d_q) = g_T^u d_d + g_T^d d_u \sim 0.8d_d - 0.25d_u$$



In the isospin-symmetric limit,  
inserting (connected) LQCD results  
[Hagler '09; Bhattacharya et al '11, '13]

(NB: Recent extractions from  
transversity slightly lower)

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$$\implies d_n(d_q) = g_T^u d_d + g_T^d d_u \sim 0.8d_d - 0.25d_u$$
  - sum-rules fixes ( $d_n \sim \langle \bar{q}q \rangle / \lambda^2$ ), so the normalization of the nucleon coupling matters

$$\lambda \sim 0.025 \text{ GeV}^3$$

from analysis of CP-even sum rules  
for  $m_n$ ,  $\sigma_N$ , etc (or lattice result for  
tensor charge above)

[Pospelov & AR '99, '00]

$$\lambda \sim 0.044 \pm 0.01 \text{ GeV}^3$$

from LQCD [Y. Aoki et al '08] run  
down from 2 GeV, \*BUT\*  $\langle \bar{q}q \rangle$  is  
also larger with LQCD values for  $m_q$   
so may be consistent [Hisano et al '12,  
Fuyuto et al '12]

# Neutron EDM

- Precision?
  - numerical coefficients are consistent with NDA, NQM (for  $d_q$ ), and the chiral log (for  $\theta$ )
  - another test for  $d_n(d_q)$  via (LQCD) nucleon tensor charge [e.g. Falk et al '99]
$$\langle N | \frac{1}{2} d_q \bar{q} \tilde{F} \sigma q | N \rangle = \frac{1}{2} d_q \tilde{F}^{\mu\nu} \langle N | \sigma_{\mu\nu} | N \rangle = \frac{1}{2} g_T^q d_q \bar{N} \tilde{F} \sigma N$$
$$\implies d_n(d_q) = g_T^u d_d + g_T^d d_u \sim 0.8d_d - 0.25d_u$$

- sum-rules fixes ( $d_n \sim \langle \bar{q}q \rangle / \lambda^2$ ), so the normalization of the nucleon coupling matters

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so may be consistent [Hisano et al '12,  
Fuyuto et al '12]

- higher order dependence on s-quark EDM?

# Pion-nucleon couplings

- Can follow a similar approach for the pion-nucleon couplings
  - focus on the isovector coupling [Pospelov '01]

(a)

$$\bar{g}_{\pi NN}^{(1)}(\tilde{d}_q) = \frac{\tilde{d}_u - \tilde{d}_d}{2f_\pi} \left\langle N \left| \sum_{q=u,d} \bar{q} g_s G \sigma q \right| N \right\rangle + \dots$$

# Pion-nucleon couplings

- Can follow a similar approach for the pion-nucleon couplings
  - focus on the isovector coupling [Pospelov '01]

The figure shows two Feynman diagrams labeled (a) and (b). Diagram (a) illustrates a loop correction to the pion nucleon vertex. A nucleon line (N) enters from the left, followed by a pion line (π). The vertex is labeled with a crossed circle symbol (⊗) and the operator  $\mathcal{O}_{CP}$ . A curved blue arrow points from this diagram down to the corresponding term in the equation below. Diagram (b) shows a more complex loop correction where a pion line (π) enters from the left, and the vertex is also labeled with a crossed circle symbol (⊗) and the operator  $\mathcal{O}_{CP}$ .

$$\bar{g}_{\pi NN}^{(1)}(\tilde{d}_q) = \frac{\tilde{d}_u - \tilde{d}_d}{2f_\pi} \left\langle N \left| \sum_{q=u,d} \bar{q} g_s G \sigma q - m_0^2 \bar{q} q \right| N \right\rangle + \dots$$

cancelation in vacuum

# Pion-nucleon couplings

- Using QCD sum rules

$$\int d^4x e^{ip \cdot x} \langle \bar{j}_n(x), j_n(0) \rangle_{\tilde{d}_q H_q} \sim p \left( \frac{2\lambda^2 \bar{g}_{\pi NN} m_N}{(p^2 - m_N^2)^2} + \frac{A}{p^2 - m_N^2} + \dots \right) + \dots$$

↓  
isolate chirally invt structure

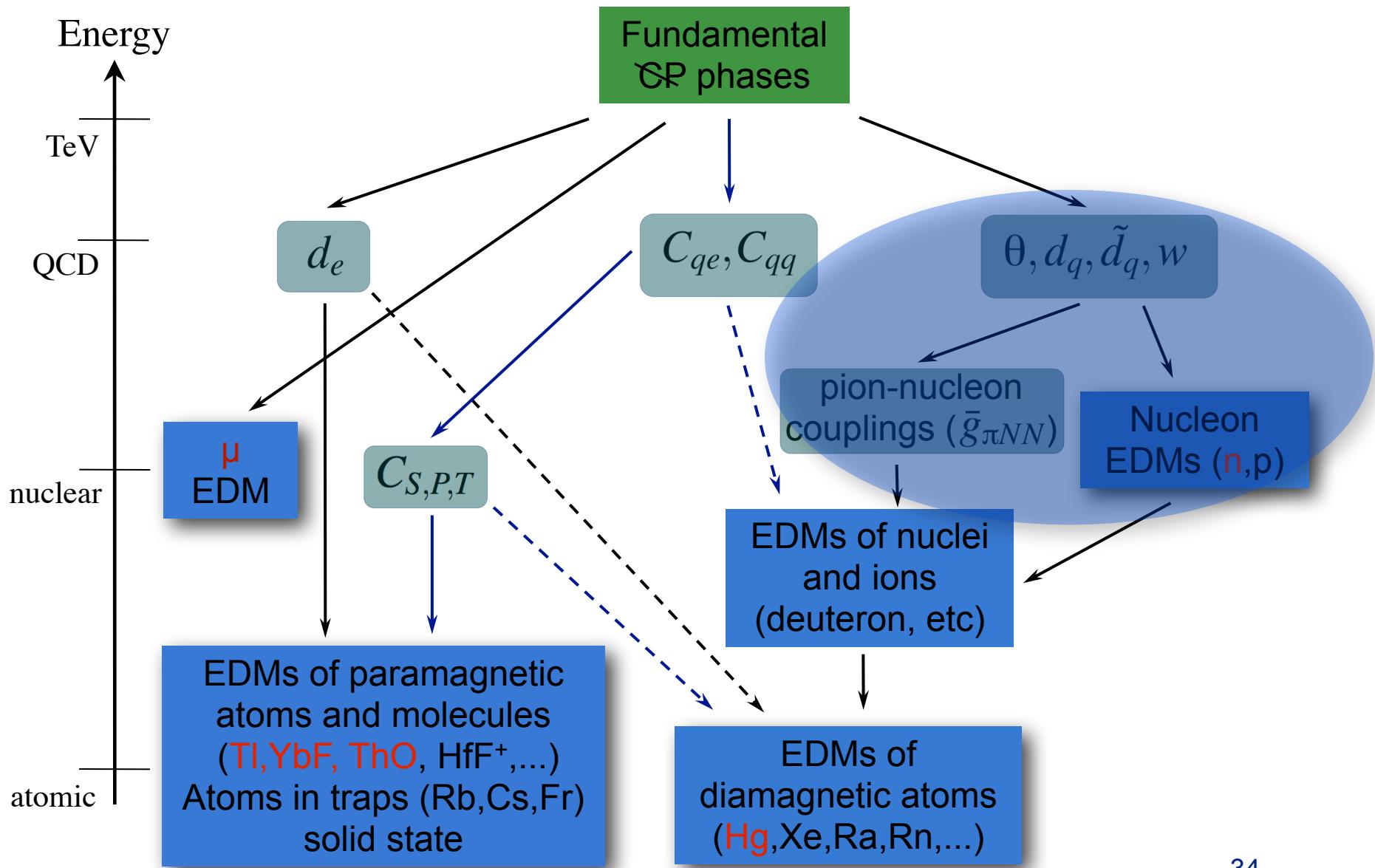
$$\bar{g}_{\pi NN}^{(1)}(\tilde{d}_q) \sim (2 \leftrightarrow 12) \text{GeV} \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3} (\tilde{d}_u - \tilde{d}_d) + \mathcal{O}(\tilde{d}_s, w)$$

$$\bar{g}_{\pi NN}^{(0)}(\tilde{d}_q) \sim (-1 \leftrightarrow 3) \text{GeV} \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3} (\tilde{d}_u + \tilde{d}_d) + \mathcal{O}(\tilde{d}_s, w)$$

[Pospelov '01]

- normalization again consistent with NDA, but larger errors due to cancelations between direct & rescattering terms  
[result slightly smaller than estimates using LETs: Falk et al '99; Hisano & Shimizu '04]
- dependence on quark EDMs suppressed by  $\alpha_{\text{em}}$

# EFT hierarchy



# Resulting Bounds on fermion EDMs & CEDMs

ThO “EDM”	$\left  d_e + e(26 \text{ MeV})^2 \left( 3\frac{C_{ed}}{m_d} + 11\frac{C_{es}}{m_s} + 5\frac{C_{eb}}{m_b} \right) \right  < 8.7 \times 10^{-29} e \text{ cm}$
YbF “EDM”	$\left  d_e + e(21 \text{ MeV})^2 \left( 3\frac{C_{ed}}{m_d} + 11\frac{C_{es}}{m_s} + 5\frac{C_{eb}}{m_b} \right) \right  < 1.1 \times 10^{-27} e \text{ cm}$
TI EDM [ $\pm 20\%$ ]	$\left  d_e + e(26 \text{ MeV})^2 \left( 3\frac{C_{ed}}{m_d} + 11\frac{C_{es}}{m_s} + 5\frac{C_{eb}}{m_b} \right) \right  < 1.6 \times 10^{-27} e \text{ cm}$
n EDM [ $\pm 50\%?$ ]	$e(\tilde{d}_d + 0.5\tilde{d}_u) + 1.3(d_d - 0.25d_u) + \mathcal{O}(\tilde{d}_s, w, C_{qq}) \left  < 2 \times 10^{-26} e \text{ cm}\right.$
Hg EDM [ $\pm \mathcal{O}(\text{few})?$ ]	$e \tilde{d}_d - \tilde{d}_u + \mathcal{O}(d_e, \tilde{d}_s, C_{qq}, C_{qe})  < 6 \times 10^{-27} e \text{ cm}$

Generic scaling:  $d_f \sim (\text{couplings}) \times \frac{m_f}{\Lambda_{CP}^2}$

See also recent compilation of limits: [Engel, Ramsey-Musolf, van Kolck '13 ]

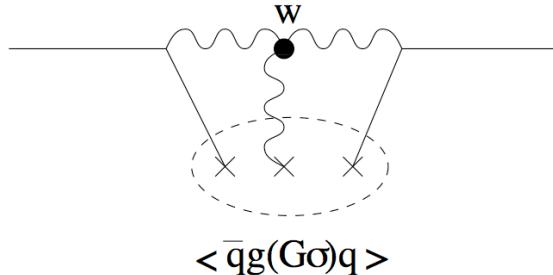
# Concluding Remarks

- EDM computations require a multi-scale approach:  
$$d_{\text{exp}}(C_{\text{atomic}}(C_{\text{nuclear}}(C_{\text{QCD}}(C_{\text{new physics}}))))$$
  - Reviewed the sum rules approach to QCD-scale calculations of CP-odd nucleon matrix elements
    - nucleon correlators illustrate new sources of mixing in CP-violating backgrounds
    - improving precision is hard without further input on (i) excited state mixing, and (ii) interpolating current ambiguity
  - Examples that can benefit from lattice input:
    - $d_N(\langle N | \bar{q} F \sigma \gamma_5 q | N \rangle)$  ✓ via tensor charges
    - $d_N(\langle N | G \tilde{G} | N \rangle), d_N(\langle N | \bar{q} G \sigma \gamma_5 q | N \rangle), \bar{g}_{\pi NN} (\langle N | \bar{q} g_s G \sigma q - m_0^2 \bar{q} q | N \rangle)$
    - s-quark matrix elements
- CP-even, relevant  
for all nuclear EDMs  
[➡ see also Emanuele's talk]

# Extra slides

# Further operators

- Weinberg operator:



$$d_n \sim \mu_n \frac{\langle N | \mathcal{O}_{CP} | N \rangle}{m_n \bar{N} i \gamma_5 N} \sim \mu_n \frac{3g_s m_0^2}{32\pi^2} w \ln(M^2/\mu_{IR}^2) \sim e 2 \times 10^{-2} \text{ GeV} w(1 \text{ GeV})$$

[Demir, Pospelov, AR '02]

$$\bar{g}_{\pi NN}^{(1)} = \bar{g}_{\pi NN}^{(1)}(w) \quad \text{suppressed by light quark masses}$$

- 4-quark (factorizable) operators:

$$d_n(C_{qq}) \sim (\text{few}) \times 10^{-2} \text{ GeV } C_{qq}$$

[Khatsimovsky et al '88;  
Hamzaoui & Pospelov '99;  
An, Ji & Xu '09]

$$\bar{g}_{\pi NN}^{(1)}(C_{ij}) = C_{ij} \frac{\langle q_i \bar{q}_i \rangle}{2f_\pi} \langle N | q_j \bar{q}_j | N \rangle$$

via PCAC, vacuum  
saturation [Demir et al '03]