Calculation of the D=4 contribution to the nEDM using lattice QCD

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ACFI Workshop, January 22-24, 2015

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Outline I

Theory

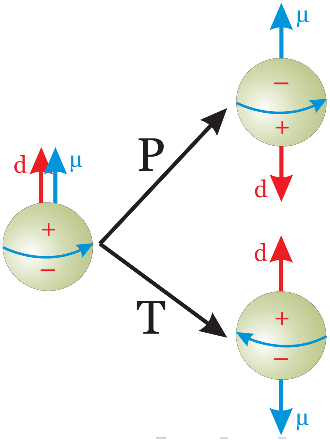
spin 1/2 particle has interaction with electric (magnetic) field:

$$H = -d\,\mathbf{E}\cdot\frac{\mathbf{S}}{S} \quad\rightarrow\quad \mathcal{L} = \frac{-id}{2}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}$$

- mdm odd under P
- edm odd under T
- both even under CPT
- $h\nu = 2\mu B \pm 2dE$

$$d = \frac{h\Delta\nu}{4E}$$

- measure with ultra-cold neutrons in E, B fields



Status of experiments

- Current value(limit) $d_n = 0.2(1.5)(0.7)10^{-26}$e cm (2.9×10^{-26}) (Sussex-RAL-ILL, 2006)
- New UCN experiments
  - PSI (CH): taking data, $\sim 1 \times 10^{-26}$ e cm by 2016
  - SNS (ORNL): goal 3-5$\times 10^{-28}$ e cm
  - ...
- New pEDM storage-ring experiment in development
  - USA/COSY/KAST/... $(1 \times 10^{-29}$ e cm $\rightarrow 1 \times 10^{-30}$ e cm$)$

(see talks at Lepton Moments 2014, Cape Cod, by Philipp Schmidt-Wellenburg, Steve Clayton, and Yannis Semertzidis, http://g2pc1.bu.edu/lept14/program.html)

The electric dipole moment of the nucleon in the SM

- Weak interactions: CKM quark mixing matrix (CP violation)
  - three loops
  - 4-5 orders smaller than current bound
- Topological charge (θ parameter) in QCD
  - In principle θ is $O(1)$, but in Nature $\theta \ll 1$, "Strong CP problem"
- quark electric and chromo-electric edms (higher dimension) appear in BSM theories (see talk by Boram Yoon tomorrow)

θ term in QCD

$$
\begin{aligned}
S_{\rm QCD} &= -S(\mathcal{A}_\mu) - i\theta \int d^4x \, \frac{g^2}{32\pi^2} \mathrm{tr}\left[G(x)\tilde{G}(x)\right] \\
&= -S(\mathcal{A}_\mu) - i\theta Q
\end{aligned}
$$

where Q is the (integer) topological charge, or winding number of the gauge field configuration $\{\mathcal{A}_\mu(x)\}$

θ term renormalizable, Lorentz and gauge invariant but CP (T) odd

$$
G(x)\tilde{G}(x) = 1/2\epsilon^{\mu\nu\delta\rho} G^{\delta\rho} G^{\mu\nu} \sim \mathbf{E}\cdot\mathbf{B}
$$

θ term in QCD

If the quarks were massless, the θ term could be removed from the action by doing chiral rotations on the quark fields,

$$\psi(x) \quad \rightarrow \quad (1 + i\alpha(x)\gamma_5)\,\psi(x)$$

because the measure in the path integral is *not* invariant under this change of variables,

$$\mathcal{D}\bar{\psi}\mathcal{D}\psi \quad \rightarrow \quad \exp\left[i\int d^4x\alpha(x)\frac{g^2}{8\pi^2}G(x)\tilde{G}(x)\right]\mathcal{D}\bar{\psi}\mathcal{D}\psi$$

so $\alpha(x) = \theta/2$ kills the θ term

θ term in QCD

Of course, quarks are not massless, so $\alpha(x) = \theta/2$ may kill the θ term but makes quark masses complex

Convention is to define

$$\bar{\theta} \quad = \quad \theta + \mathrm{Arg}\,\mathrm{Det} M_q$$

as the physical value, and if quark masses are real, $\bar{\theta} = \theta$
So for us, $\bar{\theta}$ is coefficient of iQ term in the action

Exp. then requires $\bar{\theta} \lesssim 10^{-10}$ which is Strong CP problem

The electric dipole moment of the nucleon

Let nucleon interact with external field in $\theta \neq 0$ vacuum

$$
\begin{aligned}
\langle N|J_\mu|N\rangle_\theta &= \bar{u}_\theta(\vec{p}', s')\left(F_1(q^2)\gamma_\mu + \frac{iF_2(q^2)}{2m_N}\sigma_{\mu\nu}q_\nu\right. \\
&+ \left.\frac{F_3(q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}q_\nu\right)u_\theta(\vec{p}, s) \\
q &= p' - p
\end{aligned}
$$

$q \to 0$ limit yields dipole moment(s)

In lattice gauge theory, compute correlation functions of fields in Euclidean space-time,

$$G^\mu(t', t) = \langle \chi_N(t', \vec{p}')\, J^\mu(t, q)\, \chi_N^\dagger(0, \vec{p})\rangle.$$

The electric dipole moment of the nucleon

Project onto ground states by separating interpolating fields and currents in Euclidean time (LSZ analog)

$$
\begin{aligned}
G^\mu(t',t) &= \sum_{s,s'} \langle 0|\chi_N|p',s'\rangle\langle p',s'|J^\mu|p,s\rangle\langle p,s|\chi_N^\dagger|0\rangle \frac{e^{-E'(t'-t)}e^{-Et}}{2E\,2E'} + \dots \\
&= G^\mu(q)\times f(t,t',E,E') + \dots,
\end{aligned}
$$

Appropriate projectors give form factors (*e.g.* in the CP even case)

$$
\begin{aligned}
\mathrm{tr}\frac{i}{4}\frac{1+\gamma^t}{2}\gamma^y\gamma^x\, G^x(q^2) &= p_y\, m(F_1(q^2)+F_2(q^2)) = p_y\, mG_M(q^2) \\
\mathrm{tr}\frac{i}{4}\frac{1+\gamma^t}{2}\gamma^y\gamma^x\, G^y(q^2) &= -p_x\, m(F_1(q^2)+F_2(q^2)) = -p_x\, mG_M(q^2) \\
\mathrm{tr}\frac{1}{4}\frac{1+\gamma^t}{2}\, G^t(q^2) &= m\,(E+m)\left(F_1(q^2) - \frac{q^2}{(2m)^2}F_2(q^2)\right) \\
&= m\,(E+m)G_E(q^2)
\end{aligned}
$$

The electric dipole moment of the nucleon

In the CP broken vacuum, we have (for example)

$$\begin{aligned}\mathrm{tr}\mathcal{P}^{xy}\, G^t(q^2) &= ip_z\left(\alpha m F_1(q^2) + \alpha\frac{E+3m}{2}F_2(q^2) + \frac{E+m}{2}F_3(q^2)\right) \\ &+ \mathcal{O}(\theta^2)\end{aligned}$$

where the mixing of even and odd FF comes from the nucleon spinors, which are no longer eigenstates of CP (Pospelov, Ritz 1998)

$$\begin{aligned}\sum_{s,s'} u_{s',\theta}(\vec{p})\bar{u}_{s,\theta}(\vec{p}) &= E(\vec{p})\gamma_t - i\vec{\gamma}\cdot\vec{p} + me^{2i\alpha\gamma_5}, \\ &\approx E(\vec{p})\gamma_t - i\vec{\gamma}\cdot\vec{p} + m(1+2i\alpha\gamma_5)\end{aligned}$$

where $u_\theta = \exp i\alpha\gamma_5 u$.

Need to subtract α-terms to get physical edm (F_3)

Computing with $\theta \neq 0$

The $\theta \neq 0$ action, being complex, is difficult to simulate with conventional lattice methods. However, this problem can be avoided by working in the small θ limit,

$$\begin{aligned} \langle \mathcal{O} \rangle_\theta &= \frac{1}{Z(\theta)} \int \mathcal{D}\mathcal{A}_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi \mathcal{O}\, e^{-S(\mathcal{A}_\mu) - i\theta \int d^4x\ \frac{g^2}{32\pi^2} \mathrm{tr}[G(x)\tilde{G}(x)]} \\ &\approx \frac{1}{Z(0)} \int \mathcal{D}\mathcal{A}_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi (1 - i\theta Q) \mathcal{O}\, e^{-S(\mathcal{A}_\mu)} \\ &= \langle \mathcal{O} \rangle - i\theta \langle Q\mathcal{O} \rangle \end{aligned}$$

Generate usual CP-even gauge field ensemble, re-weight with topological charge to get CP-odd part of correlation function

Outline I

2+1f DWF + Iwasaki (DSDR) gluons

Ensembles generated by the RBC/UKQCD Collaboration

- $a^{-1} = 1.73$ GeV (0.114 fm)
- lattice size $24^3 \times 64 \times 16$
- $V = (2.7\ \mathrm{fm})^3$
- $m_l = 0.005, 0.01$, $m_s = 0.04$, $m_{\rm res} = 0.00316$
  - $m_\pi = 330, 400$ MeV
- measurements on ~ 750 configs for each mass, separated by 10 MC time units

- $a^{-1} = 1.37$ GeV (0.144 fm)
- lattice size $32^3 \times 64 \times 32$
- $V = (4.6\ \mathrm{fm})^3$
- $m_l = 0.001$, $m_s = 0.04$, $m_{\rm res} = 0.0018$
  - $m_\pi = 170$ MeV
- measurements on 39 configurations, separated by 20 MC time units

[Blum, Izubuchi, Shintani PRD88 (2013) 9, 094503, arXiv:1208.4349

Lattice Symmetry

$$\mathcal{O} = \mathcal{O}^{(\mathrm{appx})} + \mathcal{O}^{(\mathrm{rest})}$$

Original

unbiased improved

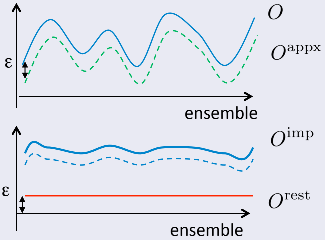
$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

Expensive : infrequently measured

Cheap : frequently measured

- $O^{(appx)}$: cheap & not to be too accurate
- N_G average suppresses the bulk part of noise cheaply

→ $O^{(imp)}$ has smaller error



Measurement details

330, 400 MeV Pions

- gaussian-smeared quark sources, APE smeared links
- (zero momentum) sequential propagators at the sink
- spatial momentum inserted at the operator (up to 4 units)
- Use all-mode-averaging (AMA) with
  - 400 (180) exact low-modes for 0.005 (0.01) (Implicitly-restarted Lanczos)
  - "sloppy" conjugate gradient stopping residual 10^{-4}
  - $N_G = 2^3 \times 4 = 32$ approximate measurements / config
  - 1 exact measurement (10^{-8} stopping residual)

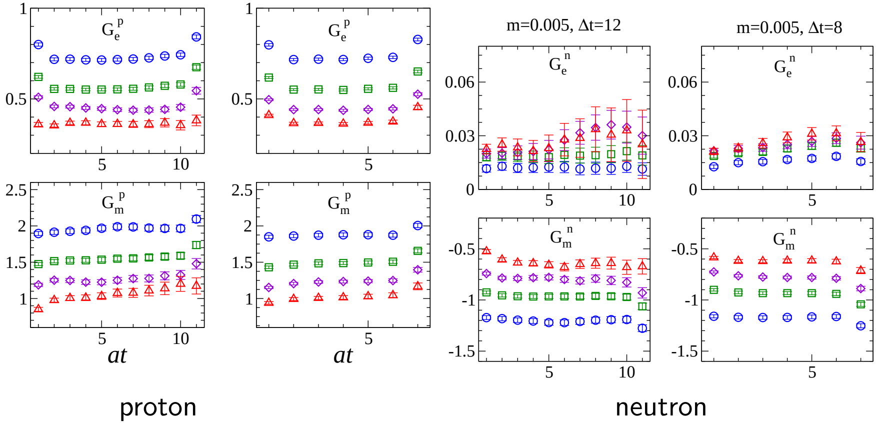
Measurement details

170 MeV pion

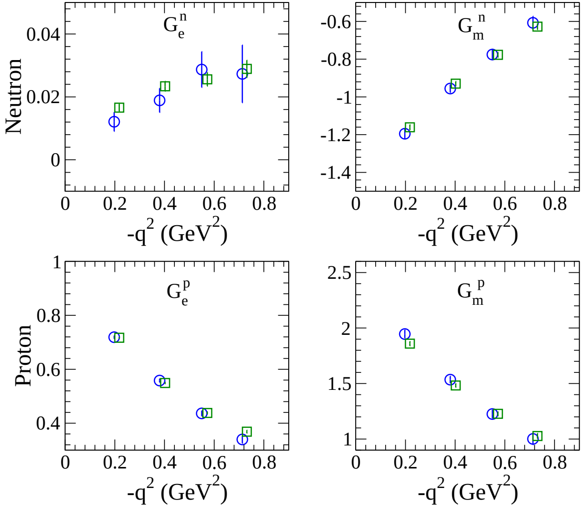
- gaussian-smeared quark sources, APE smeared links
- (zero momentum) sequential propagators at the sink
- spatial momentum inserted at the operator (up to 4 units)
- Use all-mode-averaging (AMA) with
  - 1000 exact low-modes (Implicitly-restarted Lanczos)
  - "sloppy" conjugate gradient: 125 iterations
  - $N_G = 2 \times 2^3 \times 7 = 116$ approximate measurements / config
  - 4 exact measurements/config (10^{-8} stopping residual)

Outline I

CP even Sach's form factors (different source-sink separations, 8 and 12 time units)

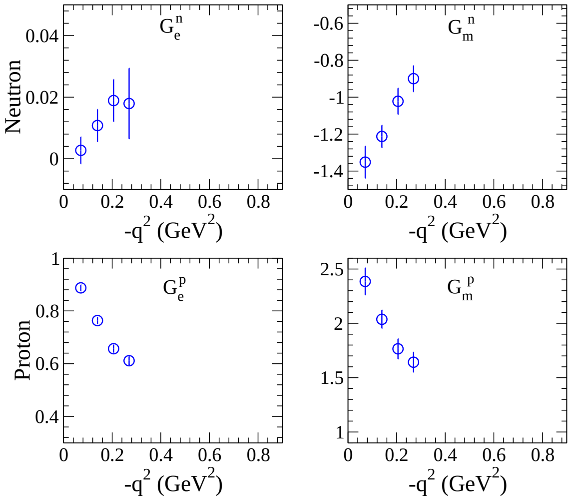


CP even Sach's form factors ($m_l = 0.005$, $m_\pi = 330$ MeV)



- stat errors $\sim$ 1-few%
- Excited state effects visible, but small

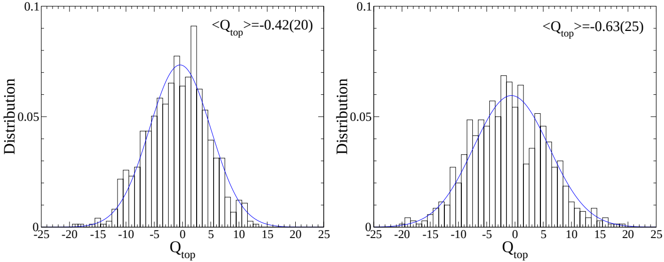
CP even Sach's form factors ($m_l = 0.001$, $m_\pi = 170$ MeV)



- stat errors ~ 1-few% here too! AMA is effective

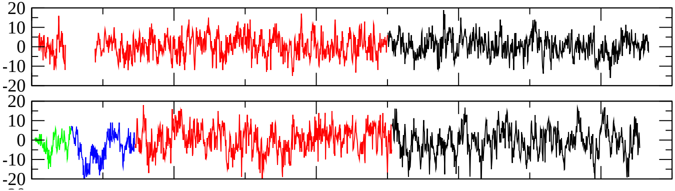
Topological Charge (330, 400 MeV pions)

$m_l = 0.005$ $m_l = 0.01$



5Li Q

(deForcrand, etal)

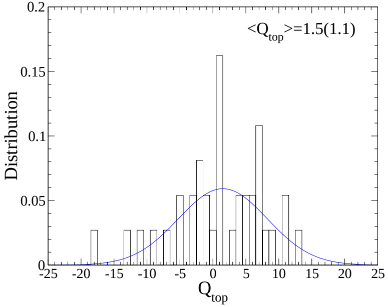


$m_l = 0.005$

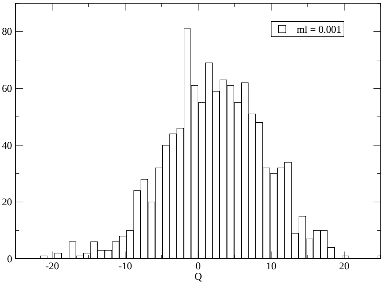
$m_l = 0.01$

Topological Charge (170 MeV pion)

Charge distribution a bit sketchy!



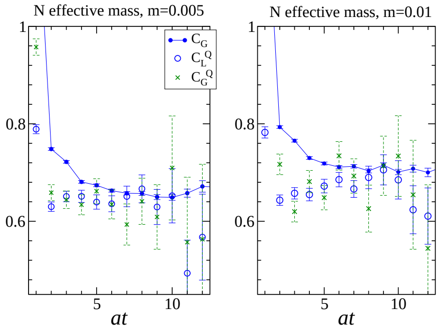
Used



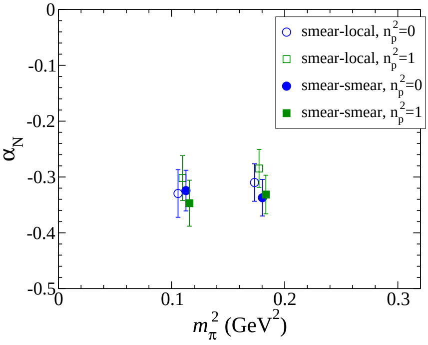
Full ensemble

Mixing coefficient from odd/even 2pt function

effective nucleon mass

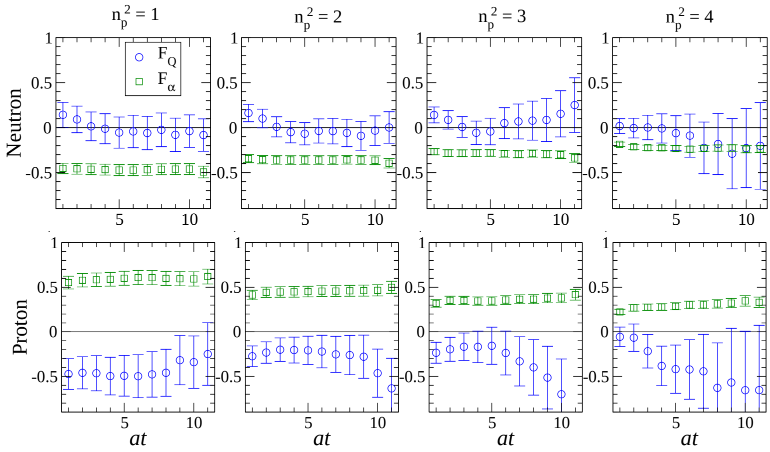


mixing coefficient

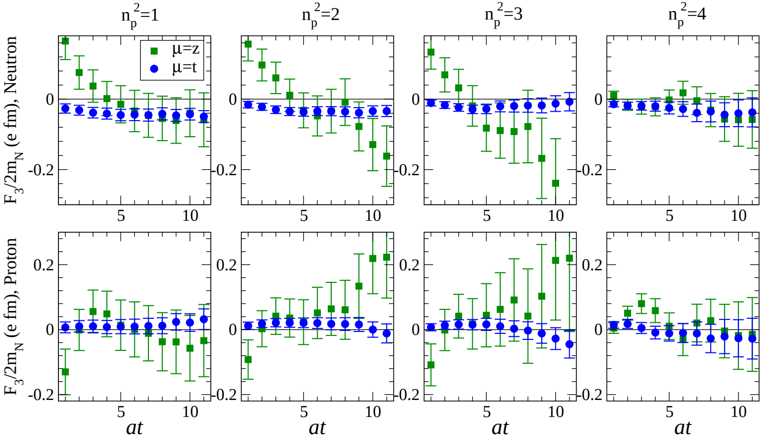


Fit CP even and odd parts to common mass
find mixing is momentum, mass (?) independent

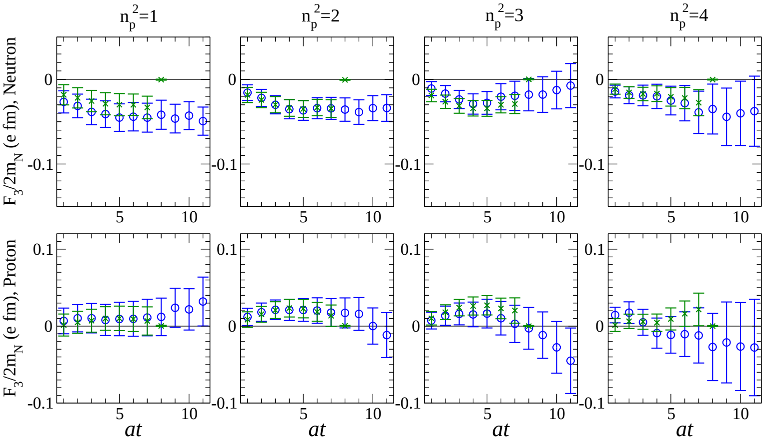
F_3 form factor, unsubtracted ($m_l = 0.005$, $m_\pi = 330$ MeV)



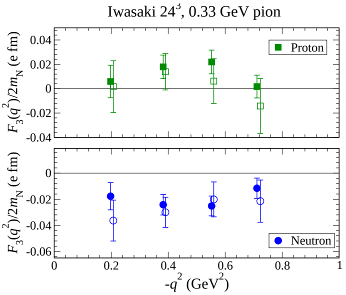
F_3 form factor, subtracted. $J_\mu = J_z, J_t$

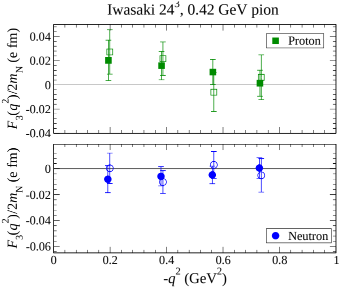


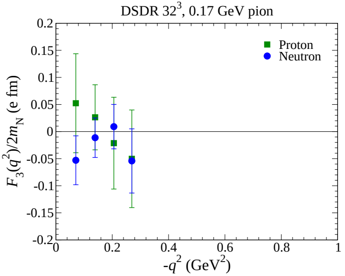
F_3 form factor, subtracted. Excited state systematics



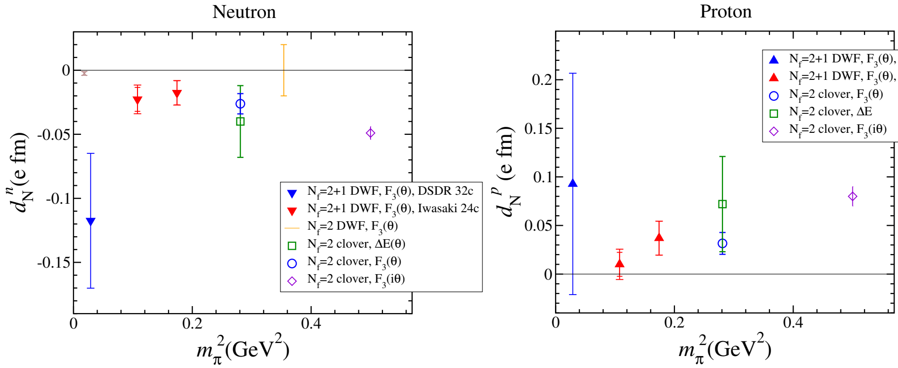
F_3 form factor, subtracted. Mild q^2 dependence







Nucleon electric dipole moment (in units of $\bar{\theta}$)



d should vanish in the chiral limit

Neutron has "wrong" mass dependence? Maybe just statistics

An idea

- despite AMA, still statistics challenged
- Correlating noisy, would-be-zero measurement with topological charge seems like a bad idea
- After all, nucleon correlation function measured in one "corner" is completely unrelated (has no overlap) with topological fluctuations in another corner
- we may be amplifying noise!
- maybe a more local correlation, *i.e.* with local topological charge density would work better
- This is not correct, but we may learn something, and as $V \to \infty$ it is correct
- could make it arbitrarily complicated, so start of simple and see if signal/noise improves

An idea

- Try on 24^3, $m_f = 0.005$ ensemble
- already summed over spatial location of operator (FT)
- Can break up Q on time slices
- Correlate nucleon 2, 3 pt functions with $Q(t)$

$$\begin{aligned} \alpha = -0.178(12) &\rightarrow -0.0217(6) \\ F_3/2m = 0.021(19) &\rightarrow 0.0045(12) \\ F_3/2m = -0.040(14) &\rightarrow -0.0054(8) \end{aligned}$$

Larger sums over time slices under investigation, also spatial variation interesting too

Outline I

Summary/Outlook

- Nucleon EDM calculations important to distinguish source in case of discovery(!)
- Signal for p,n EDM's emerging– AMA important
- Statistical errors still relatively large, work still to do
- Current DWF calculations on RBC/UKQCD ensembles
  - 32^3 $((4.6\,\mathrm{fm})^3)$, $m_\pi = 170$ MeV
  - 48^3 $((5.5\,\mathrm{fm})^3)$, $m_\pi = 140$ MeV (underway, RBC/LANL)

Acknowledgments

- This research is supported in part by the US DOE
- Computational resources provided by the RIKEN BNL Research Center, RIKEN, and USQCD Collaboration
- Lattice computations done on
  - Ds cluster at FNAL (USQCD)
  - RICC at RIKEN
  - Gordon cluster at SDSC (XSEDE)