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Nuclear-Matter Theory: Introduction to chiral effective field theory

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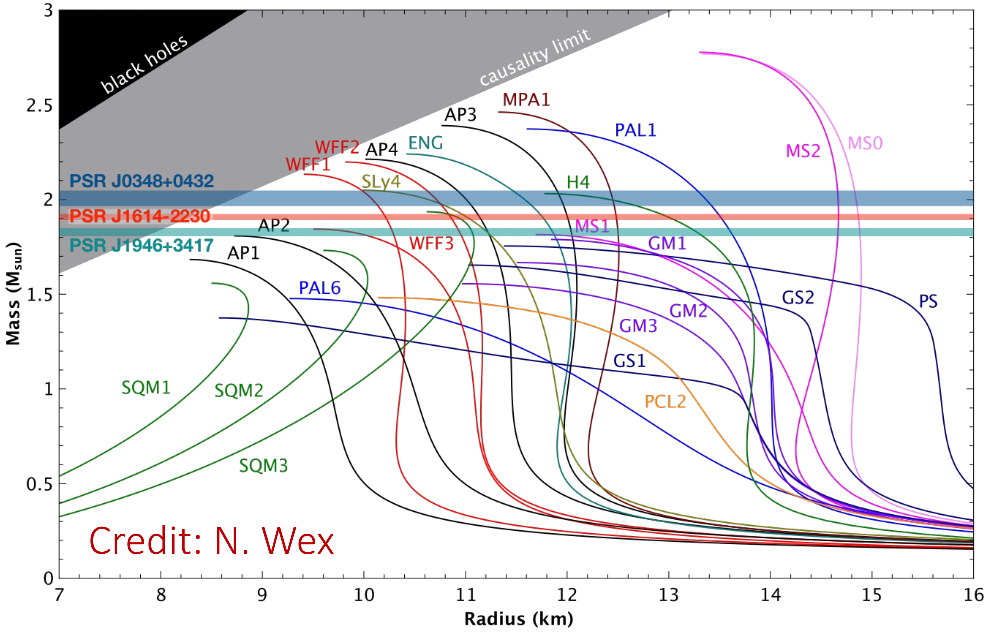
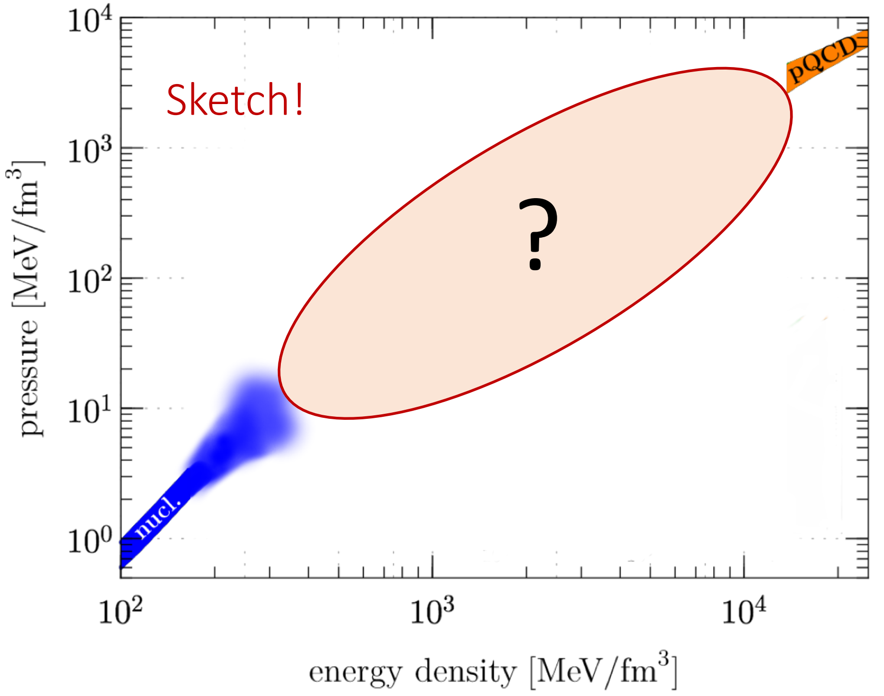
The equation of state

Large number of neutron-star equations of state available in the literature, but:

- They do **not provide any theoretical uncertainty** estimates.
- They are not constructed based on some fundamental guiding principle; hence, it is **not clear how to improve them** systematically.





Constraints:

- At low densities from **nuclear theory** and experiment.
- At very high density from pQCD, please ask me later.

see, e.g., Kurkela, Vuorinen et al.

The equation of state

Many different approaches to calculate EOS but here, we focus on **microscopic calculations: Solve** $\mathcal{H}|\psi\rangle = E|\psi\rangle$

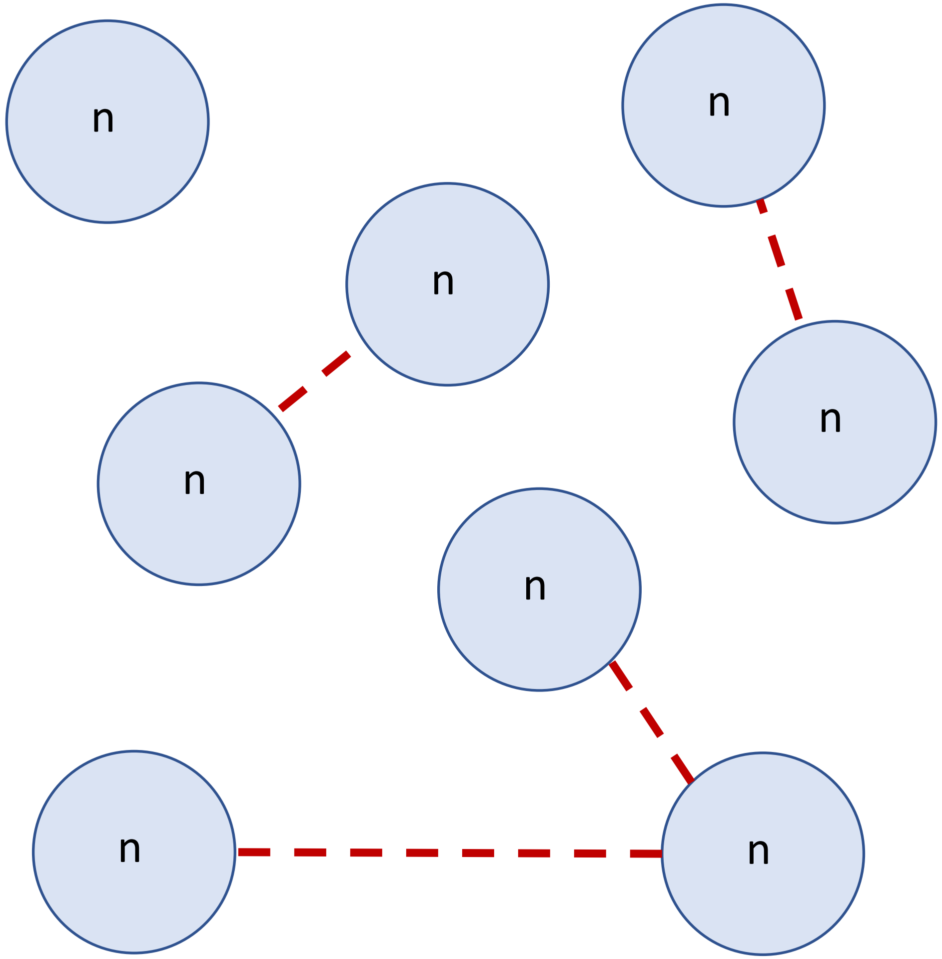
We need:

- A theory for the strong interactions among nucleons

  Chiral Effective Field Theory

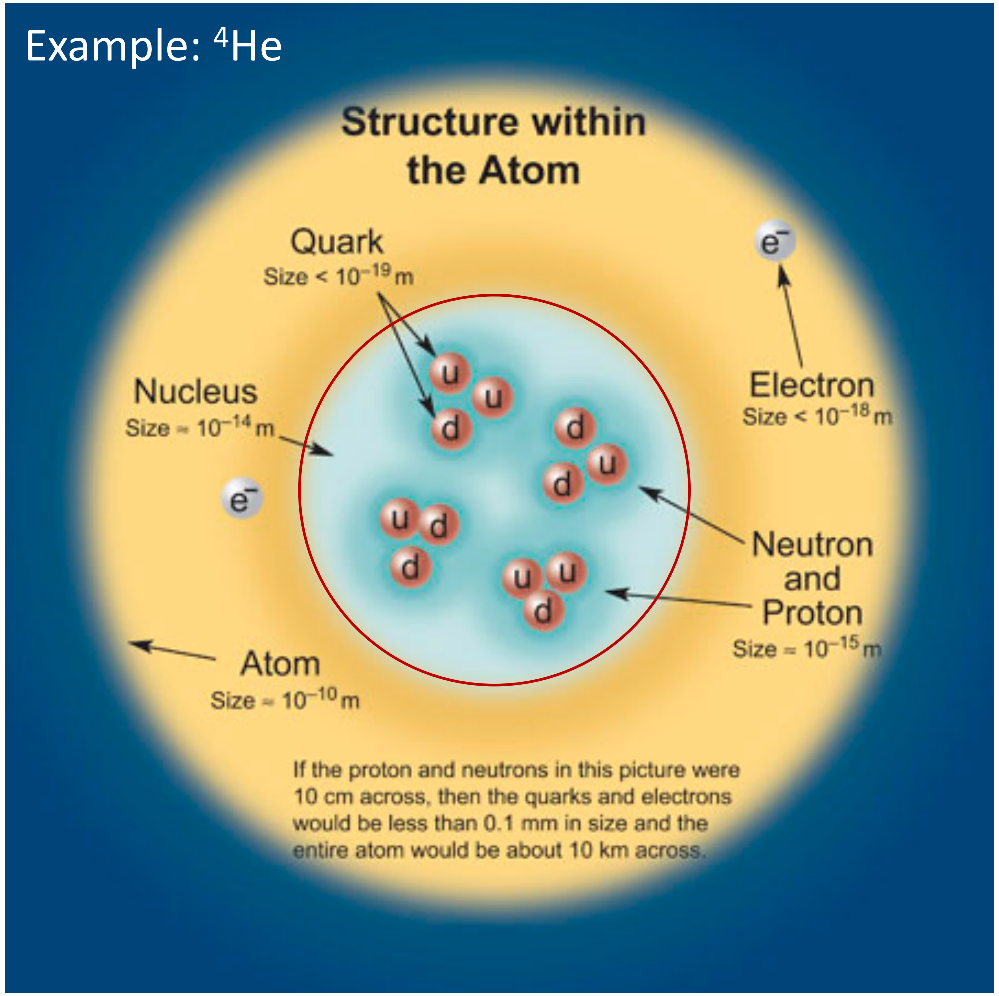
- A computational method to solve the many-body Schrödinger equation.

  e.g., many-body perturbation theory, quantum Monte Carlo, coupled cluster, self-consistent Green's function, …



Low-energy QCD

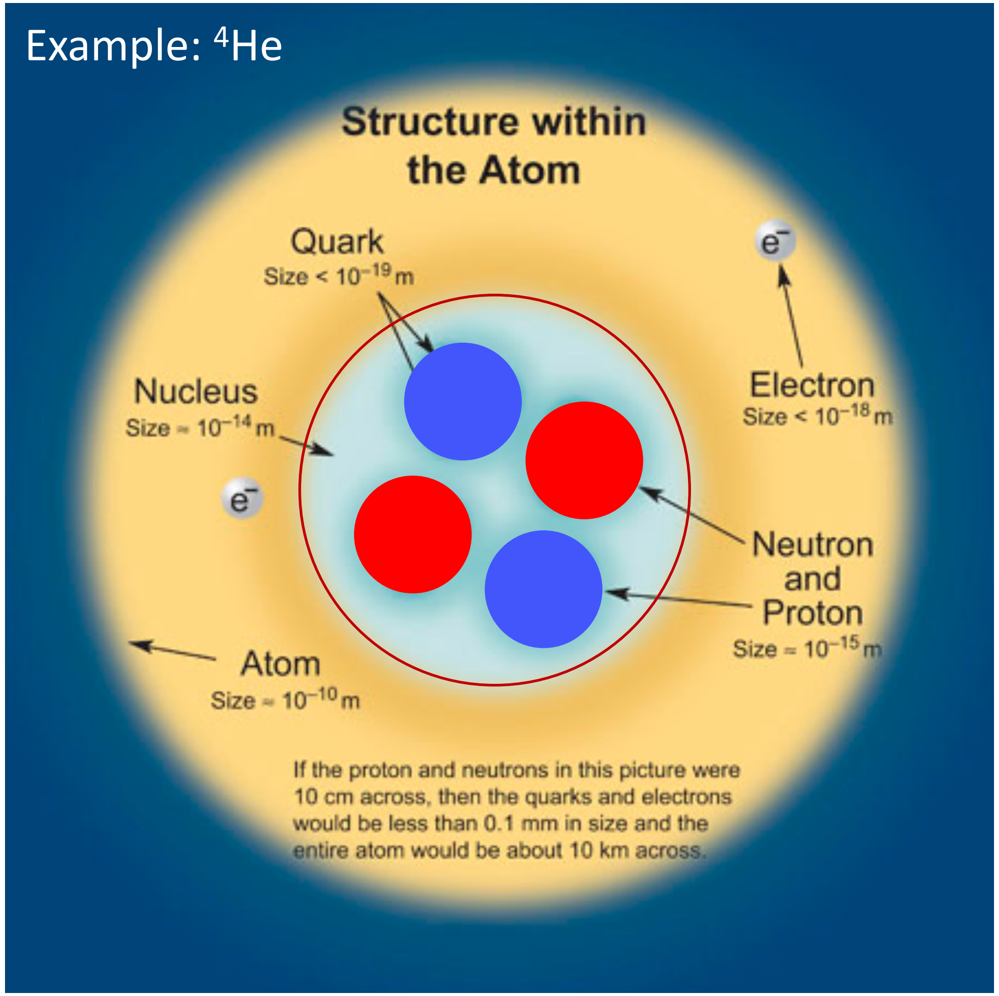
- Atomic nucleus consists of strongly interacting matter.
- Made up by quarks and gluons (Quantum Chromodynamics).
- Extremely complicated to solve!



Low-energy QCD

- Atomic nucleus consists of strongly interacting matter.
- Made up by quarks and gluons (Quantum Chromodynamics).
- Extremely complicated to solve!

- Probing a nucleus at low energies does not resolve quark substructure of nucleons!
- We can describe the nucleus in terms of neutrons (udd) and protons (uud).



Nuclear interactions and EFT

Solve **Schrödinger equation** using imaginary time projection $\mathcal{H}|\psi\rangle = E|\psi\rangle$

Hamiltonian is sum of kinetic and interaction parts: $\mathcal{H} = \sum_i \mathcal{T}_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$

Two-nucleon forces

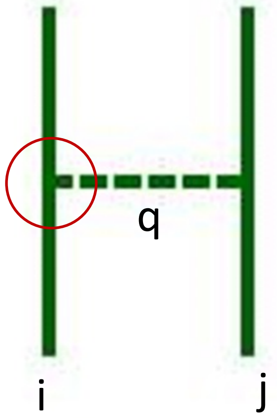
Three-nucleon forces

- V is hermitian, because the Hamiltonian is hermitian,
- V is symmetric under the permutation of identical particles, i.e., $V_{ij} = V_{ji}$,
- V is translationally and rotationally invariant,
- V is invariant under translations in time, i.e., time-independent,
- V is Lorentz invariant (for nonrelativistic interactions this reduces to Galilean invariance),
- V is invariant under parity transformations and time reversal,
- V has to conserve baryon and lepton number,
- V has to be approximately isospin symmetric and charge independent,
- and V has to include the properties of spontaneously and explicitly broken chiral symmetry.

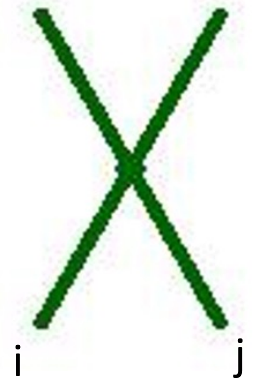
Nuclear interactions

Contact and pion-exchange interactions: $V(\mathbf{q},\mathbf{k}) = V_{\rm cont}(\mathbf{q},\mathbf{k}) + V_\pi(\mathbf{q},\mathbf{k})$

For example:



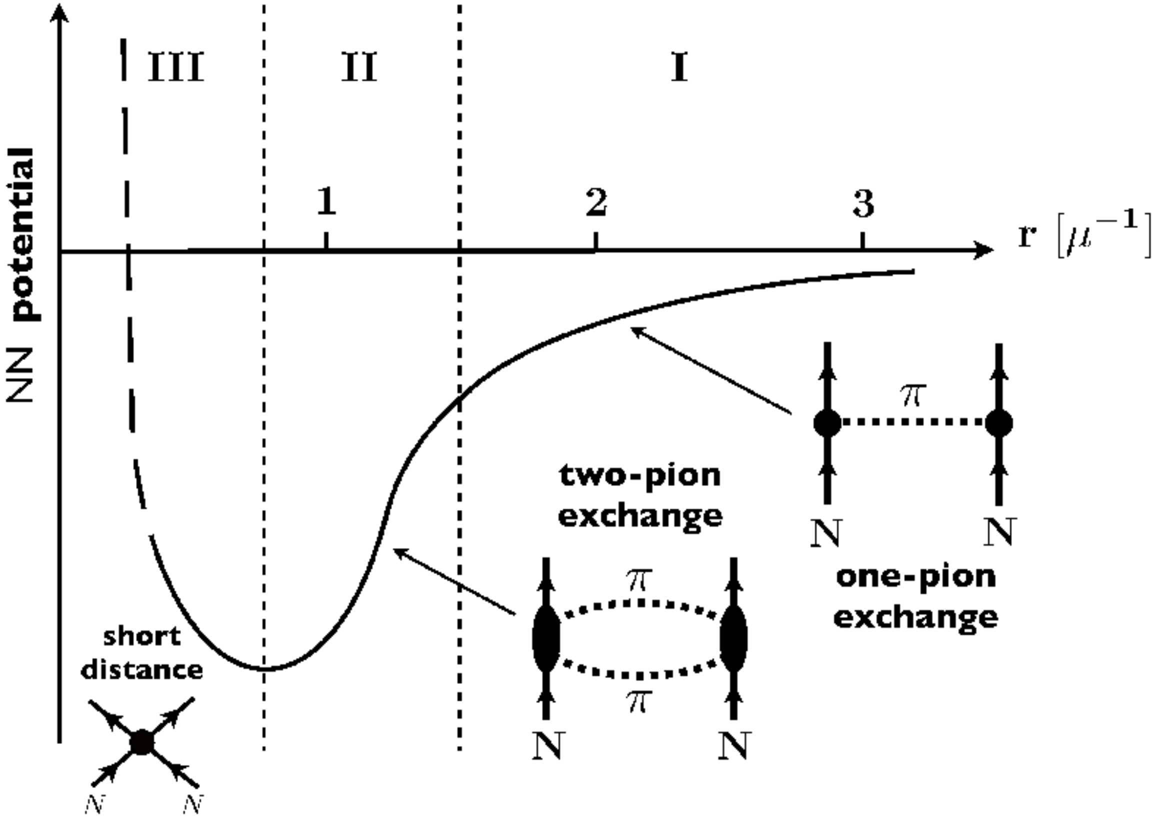
$$V^{(0)}_{\rm OPE}(\mathbf{q}) = -\frac{g_A^2}{4f_\pi^2}\frac{\boldsymbol{\sigma}_i\cdot\mathbf{q}\,\boldsymbol{\sigma}_j\cdot\mathbf{q}}{q^2+m_\pi^2}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$



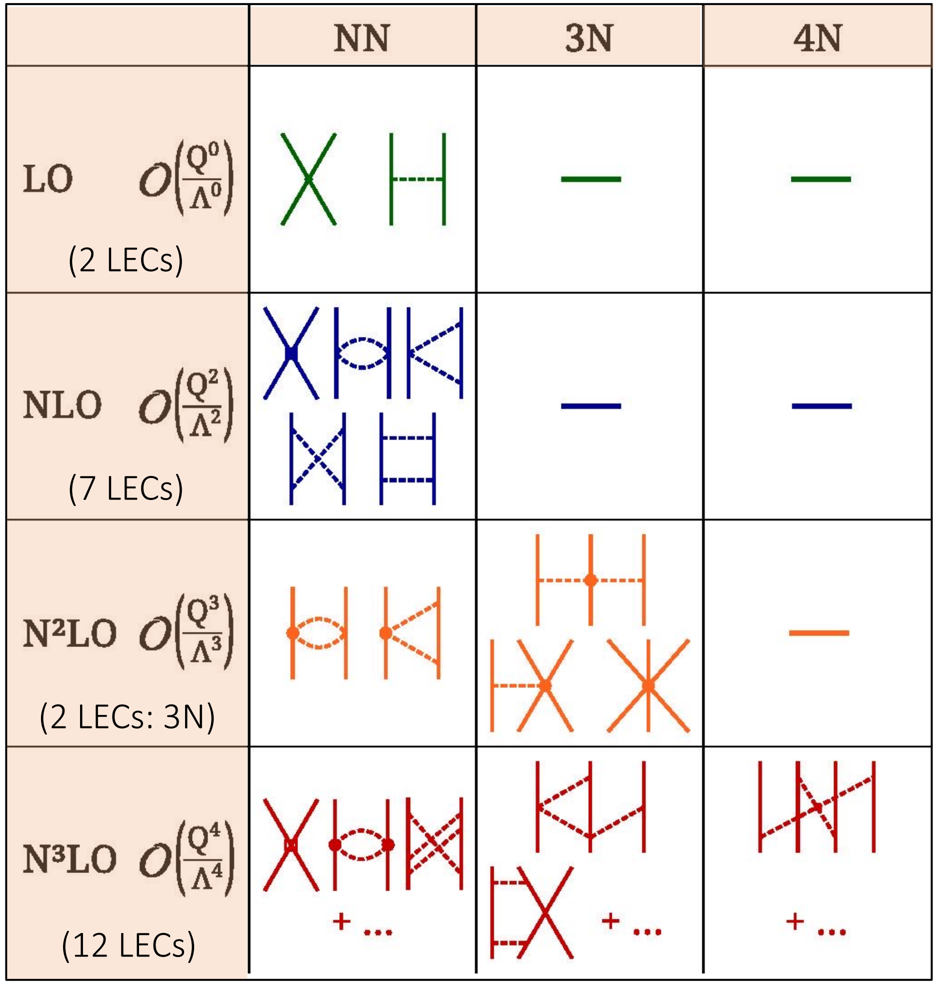
$$V^{(0)}_{\rm cont} = \alpha_1\mathbb{1} + \alpha_2\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + \alpha_3\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + \alpha_4\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$

$$\begin{aligned}V^{(2)}_{\rm cont} = \;& \gamma_1\,q^2 + \gamma_2\,q^2\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + \gamma_3\,q^2\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + \gamma_4\,q^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + \gamma_5\,k^2 + \gamma_6\,k^2\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\\ &+\gamma_7\,k^2\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + \gamma_8\,k^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + \gamma_9\,(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)(\mathbf{q}\times\mathbf{k}) + \gamma_{10}\,(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)(\mathbf{q}\times\mathbf{k})\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\\ &+\gamma_{11}(\boldsymbol{\sigma}_i\cdot\mathbf{q})(\boldsymbol{\sigma}_j\cdot\mathbf{q}) + \gamma_{12}(\boldsymbol{\sigma}_i\cdot\mathbf{q})(\boldsymbol{\sigma}_j\cdot\mathbf{q})\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + \gamma_{13}(\boldsymbol{\sigma}_i\cdot\mathbf{k})(\boldsymbol{\sigma}_j\cdot\mathbf{k})\\ &+\gamma_{14}(\boldsymbol{\sigma}_i\cdot\mathbf{k})(\boldsymbol{\sigma}_j\cdot\mathbf{k})\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\,.\end{aligned}$$

Chiral Effective Field Theory



Holt et al., PPNP 73 (2013)

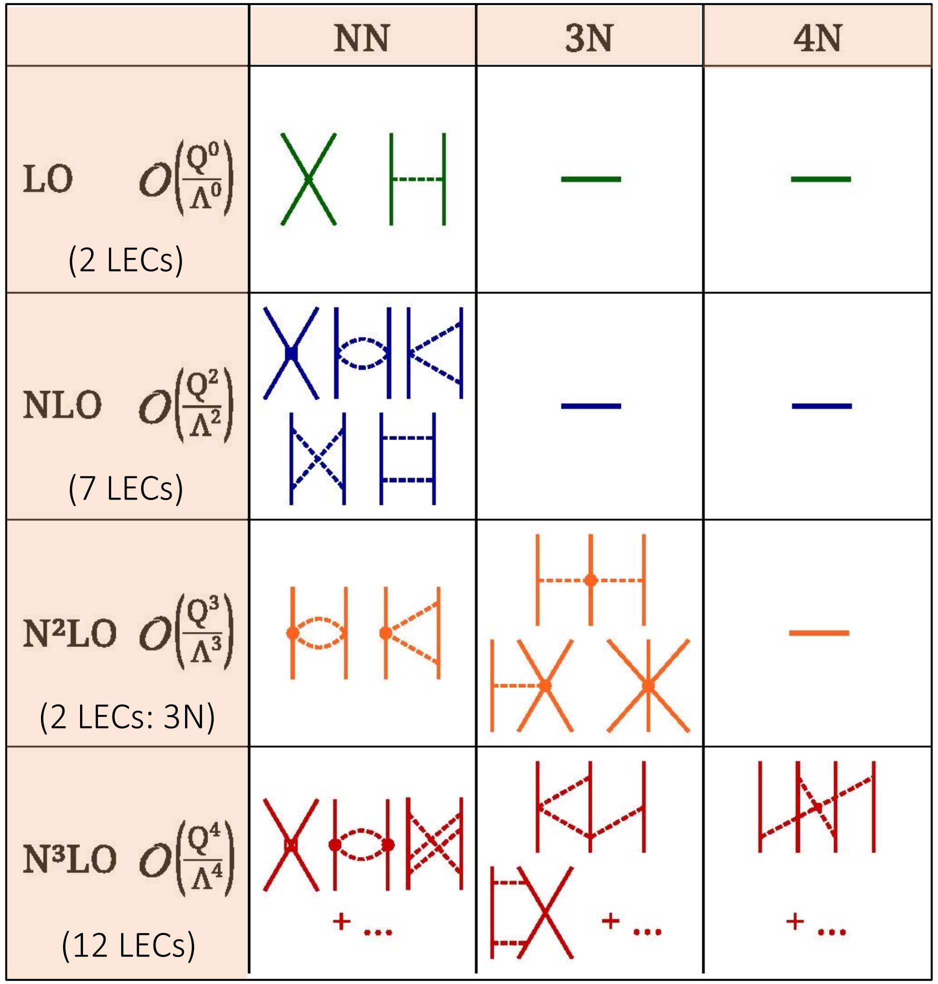


Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Chiral Effective Field Theory

Systematic expansion of nuclear forces in momentum Q over breakdown scale Λ_b :

- Based on symmetries of QCD
- Pions and nucleons as explicit degrees of freedom
- Power counting scheme results in systematic expansion, **enables uncertainty estimates!** **(see Christian‘s talk!)**
- Natural hierarchy of nuclear forces
- **Consistent interactions**: Same couplings for two-nucleon and many-body sector
- **Fitting:** NN forces in NN system (NN scattering), 3N forces in 3N/4N system (Binding energies, radii)



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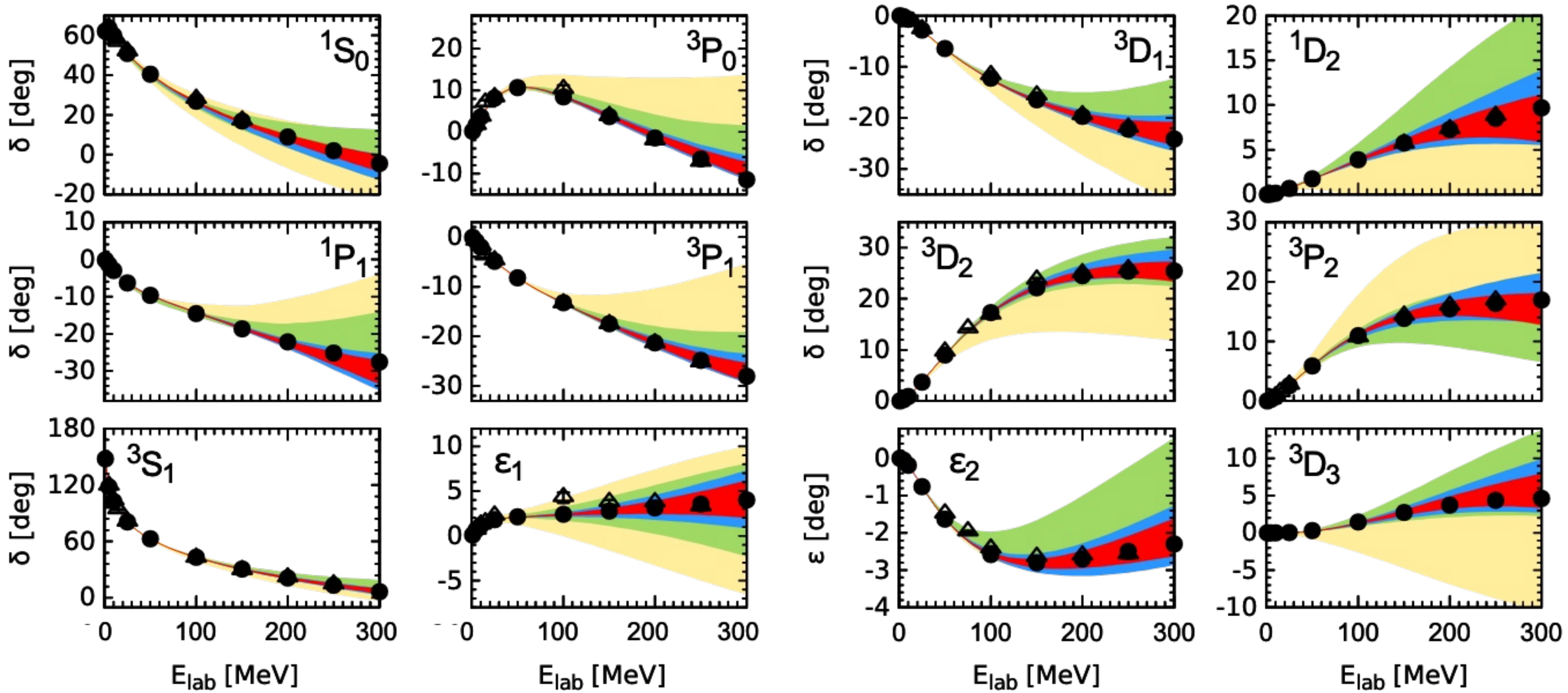
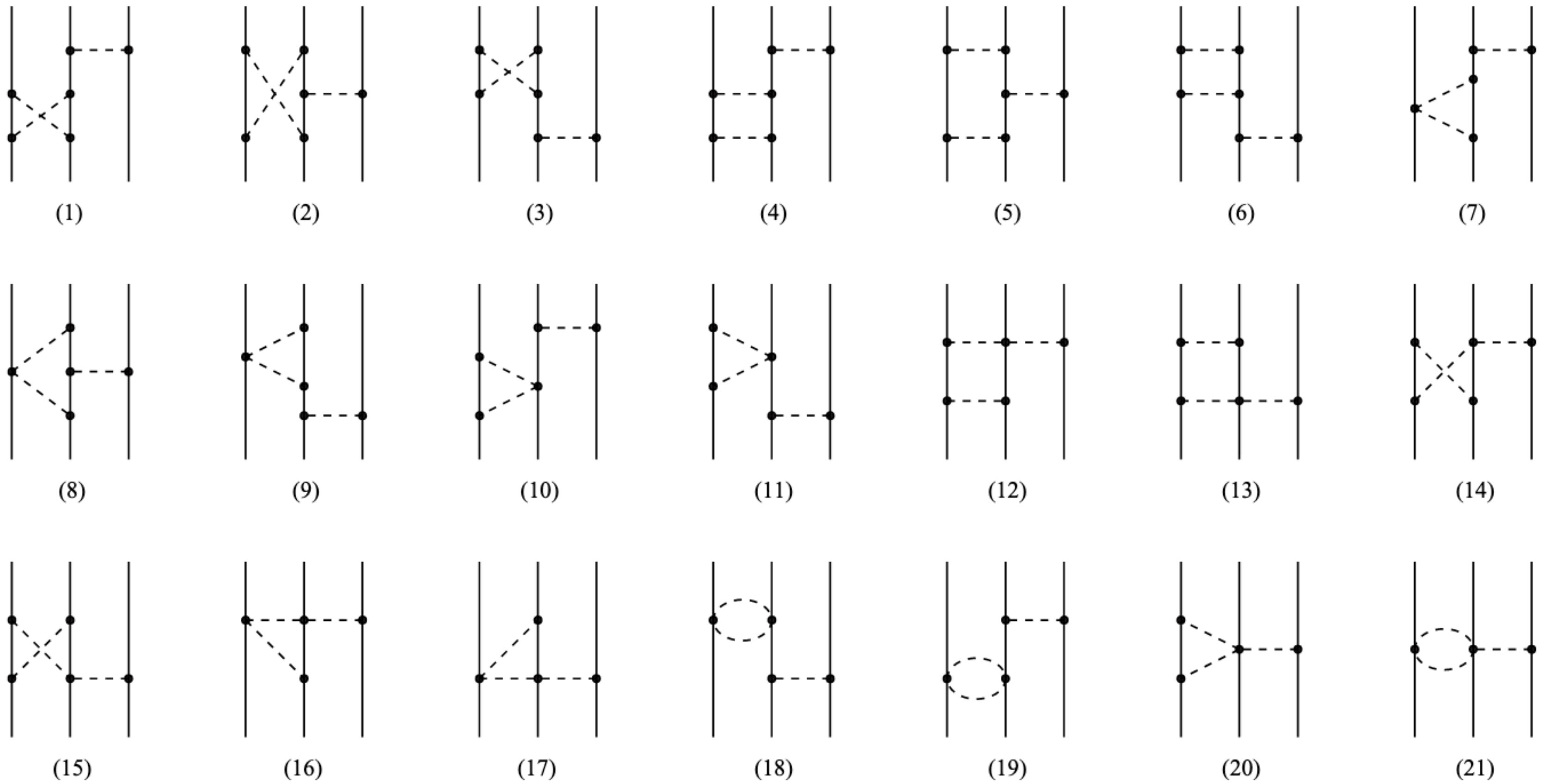
Neutron-proton scattering phase shifts

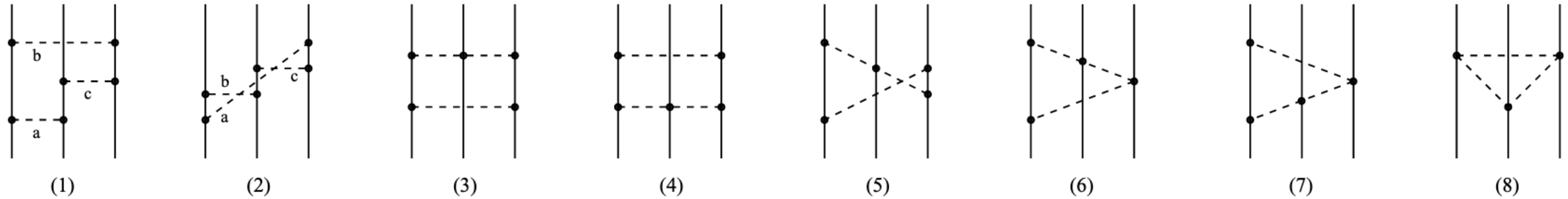


Epelbaum et al., PRL (2015)
See also Carlsson et al. PRX (2016)

Can work to desired accuracy with error estimates!

Subleading Three-nucleon forces





$$+\vec{\sigma}_1\cdot\vec{q}_3\vec{\sigma}_3\cdot\vec{q}_1\ R_9+\vec{\sigma}_1\cdot\vec{\sigma}_3\ R_{10}$$
$$+\vec{q}_1\cdot\vec{q}_3\times\vec{\sigma}_2\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3\ R_{11}, \qquad \text{(A1)}$$

where the functions $R_i \equiv R_i(q_1, q_3, z)$ with $z = \hat{q}_1\cdot\hat{q}_3$ are defined as follows:

$$R_1 = \frac{(-1+z^2)g_A^6 M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2(zq_1+q_3)\right)}{128F^6\pi\left(4(-1+z^2)M_\pi^2-q_2^2\right)\left(4M_\pi^2q_3+q_3^3\right)} - \frac{A(q_2)g_A^6q_2^2\left(2M_\pi^2(q_1+zq_3)+zq_3\left(-q_1^2+q_3^2\right)\right)}{128F^6\pi(-1+z^2)q_1q_3^2}$$
$$-\frac{A(q_3)g_A^6\left(zq_2^2(zq_1-q_3)q_3+2M_\pi^2\left(z(-2+z^2)q_1^2-(1+z^2)q_1q_3-zq_3^2\right)\right)}{128F^6\pi(-1+z^2)q_1q_3}$$
$$+\frac{A(q_1)g_A^6\left(2M_\pi^2q_2^2+q_3\left(-zq_1^3+(2-3z^2)q_1^2q_3-z(-2+z^2)q_1q_3^2+q_3^3\right)\right)}{128F^6\pi(-1+z^2)q_3^2}$$
$$-\frac{I(4:0,-q_1,q_3;0)g_A^6q_2^2}{32F^6(-1+z^2)\left(4(-1+z^2)M_\pi^2-q_2^2\right)q_3}\left(8(-1+z^2)M_\pi^4(2zq_1+(1+z^2)q_3)\right.$$
$$\left.+q_2^2q_3\left(z^2q_1^2+z(-1+z^2)q_1q_3-q_3^2\right)+2M_\pi^2\left(z(-2+z^2)q_1^3-(1+2z^2)q_1^2q_3+3z(-2+z^2)q_1q_3^2+(-3+2z^4)q_3^3\right)\right),$$

$$R_2 = \frac{A(q_2)g_A^6q_2^2\left(-2M_\pi^2((1+z^2)q_1+2zq_3)+zq_3\left((1+z^2)q_1^2-2q_3^2\right)\right)}{128F^6\pi(-1+z^2)^2q_1^3q_3^2}$$
$$+\frac{A(q_3)g_A^6\left(M_\pi^2\left(2zq_1^2+(1+3z^2)q_1q_3+2zq_3^2\right)+zq_3\left(-zq_1^3-z^2q_1^2q_3+zq_1q_3^2+q_3^3\right)\right)}{64F^6\pi(-1+z^2)^2q_1^3q_3}$$
$$+\frac{A(q_1)g_A^6}{128F^6\pi(-1+z^2)^2q_1^2q_3^2}\left(2M_\pi^2((1+z^2)q_1^2+z(3+z^2)q_1q_3+(1+z^2)q_3^2)\right.$$
$$\left.+q_3\left(-(z+z^3)q_1^3+(2-5z^2+z^4)q_1^2q_3+z(1+z^2)q_1q_3^2+(1+z^2)q_3^3\right)\right)$$
$$-\frac{I(4:0,-q_1,q_3;0)g_A^6}{32F^6(-1+z^2)^2q_1^2\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3}\left(q_2^4q_3\left(-2z^2q_1^2+(1+z^2)q_3^2\right)\right.$$
$$-8(-1+z)(1+z)M_\pi^4\left(z(2+z^2)q_1^3+(1+2z^2)^2q_1^2q_3+z(2+7z^2)q_1q_3^2+(1+2z^2)q_3^3\right)$$
$$\left.+2M_\pi^2q_2^2\left(2zq_1^3+(1-z^2+6z^4)q_1^2q_3-2z(-1-3z^2+z^4)q_1q_3^2+(3+3z^2-4z^4)q_3^3\right)\right)$$
$$+\frac{g_A^6M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2(zq_1+q_3)\right)}{128F^6\pi q_1^2\left(4(-1+z^2)M_\pi^2-q_2^2\right)\left(4M_\pi^2q_3+q_3^3\right)},$$

$$R_3 = -\frac{zA(q_2)g_A^6q_2^2\left(-4M_\pi^2(q_1+zq_3)+q_3\left(2zq_1^2+(-1+z^2)q_1q_3-2zq_3^2\right)\right)}{128F^6\pi(-1+z^2)^2q_1^2q_3^3}$$
$$-\frac{zA(q_3)g_A^6}{128F^6\pi(-1+z^2)^2q_1^2q_3^2}\left(M_\pi^2\left(-2z(-3+z^2)q_1^2+4(1+z^2)q_1q_3+4zq_3^2\right)+q_3\left(-(1+z^2)q_1^3\right.\right.$$
$$\left.\left.-2z^3q_1^2q_3+(1+z^2)q_1q_3^2+2zq_3^3\right)\right)$$
$$-\frac{zA(q_1)g_A^6\left(2M_\pi^2\left(2q_1^2+4zq_1q_3+(1+z^2)q_3^2\right)+q_3\left(-2zq_1^3+(1-3z^2)q_1^2q_3+2zq_1q_3^2+(1+z^2)q_3^3\right)\right)}{128F^6\pi(-1+z^2)^2q_1q_3^3}$$
$$-\frac{I(4:0,-q_1,q_3;0)zg_A^6}{32F^6(-1+z^2)^2q_1\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3^2}\left(q_2^4q_3\left((1+z^2)q_1^2+z(-1+z^2)q_1q_3-(1+z^2)q_3^2\right)\right.$$
$$+8(-1+z)(1+z)M_\pi^4\left(3zq_1^3+(-1+10z^2)q_1^2q_3+3z(1+2z^2)q_1q_3^2+(1+2z^2)q_3^3\right)$$
$$\left.+2M_\pi^2q_2^2\left(z(-3+z^2)q_1^3+(3-9z^2)q_1^2q_3-z(5+z^2)q_1q_3^2+(-3-3z^2+4z^4)q_3^3\right)\right)$$
$$+\frac{zg_A^6M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2(zq_1+q_3)\right)}{128F^6\pi q_1\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3^2\left(4M_\pi^2+q_3^2\right)},$$

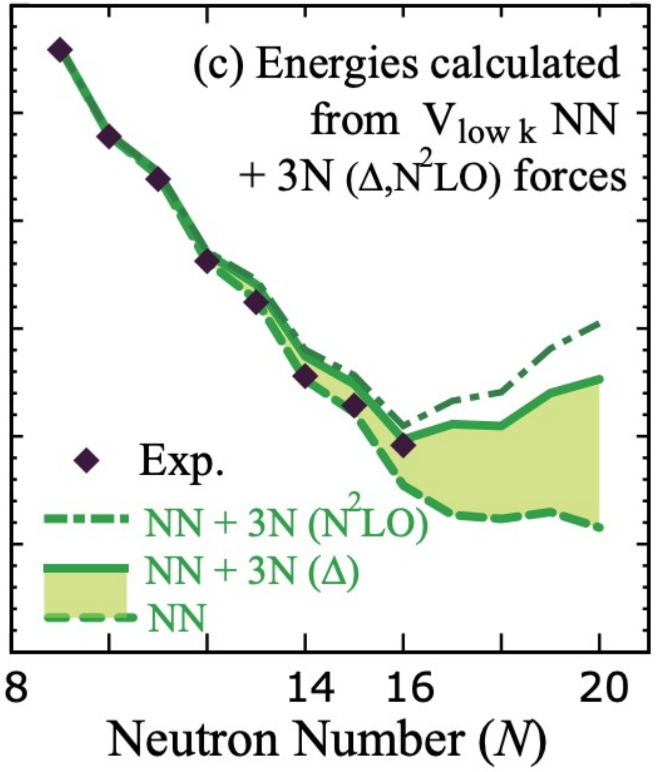
$$R_4 = \frac{A(q_2)g_A^6q_2^2\left(-2z^2q_1^2q_3+(1+z^2)q_3^3+2M_\pi^2\left(2zq_1+(1+z^2)q_3\right)\right)}{128F^6\pi(-1+z^2)^2q_1^2q_3^3}$$
$$+\frac{A(q_1)g_A^6\left(-2M_\pi^2\left(2zq_1^2+(1+3z^2)q_1q_3+2zq_3^2\right)+q_3\left(2z^2q_1^3+2z^3q_1^2q_3+(1-4z^2+z^4)q_1q_3^2-2zq_3^3\right)\right)}{128F^6\pi(-1+z^2)^2q_1q_3^3}$$
$$-\frac{A(q_3)g_A^6}{128F^6\pi(-1+z^2)^2q_1^2q_3^2}\left(2M_\pi^2\left(-z^2(-3+z^2)q_1^2+z(3+z^2)q_1q_3+(1+z^2)q_3^2\right)\right.$$
$$\left.+q_3(-(z+z^3)q_1^3-(1-z^2+2z^4)q_1^2q_3+z(1+z^2)q_1q_3^2+(1+z^2)q_3^3)\right)$$
$$-\frac{I(4:0,-q_1,q_3;0)g_A^6}{32F^6(-1+z^2)^2q_1\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3^2}\left(q_2^4q_3\left((z+z^3)q_1^2+(-1+z^2)^2q_1q_3-2zq_3^2\right)\right.$$
$$+8(-1+z)(1+z)M_\pi^4\left(3z^2q_1^3+9z^3q_1^2q_3+(-2+9z^2+2z^4)q_1q_3^2+z(2+z^2)q_3^3\right)$$
$$\left.+2M_\pi^2q_2^2\left(z^2(-3+z^2)q_1^3+(2z-8z^3)q_1^2q_3+(4+5z^2(-3+z^2))q_1q_3^2+2z(-3+z^2+z^4)q_3^3\right)\right)$$
$$+\frac{zg_A^6M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2(zq_1+q_3)\right)}{128F^6\pi q_1\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3^2\left(4M_\pi^2+q_3^2\right)},$$

$$R_5 = \frac{A(q_2)g_A^6q_2^2\left(-4M_\pi^2(q_1+zq_3)+q_3\left(2zq_1^2+(-1+z^2)q_1q_3-2zq_3^2\right)\right)}{128F^6\pi(-1+z^2)^2q_1q_3^4}$$
$$-\frac{A(q_3)g_A^6}{128F^6\pi(-1+z^2)^2q_1q_3^3}\left(2M_\pi^2\left(z(-3+z^2)q_1^2-2(1+z^2)q_1q_3-2zq_3^2\right)+q_3\left((1+z^2)q_1^3+2z^3q_1^2q_3-(1+z^2)q_1q_3^2\right.\right.$$
$$\left.\left.-2zq_3^3\right)\right)+\frac{A(q_1)g_A^6\left(2M_\pi^2\left(2q_1^2+4zq_1q_3+(1+z^2)q_3^2\right)+q_3\left(-2zq_1^3+(1-3z^2)q_1^2q_3+2zq_1q_3^2+(1+z^2)q_3^3\right)\right)}{128F^6\pi(-1+z^2)^2q_3^4}$$
$$+\frac{I(4:0,-q_1,q_3;0)g_A^6}{32F^6(-1+z^2)^2\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3^3}\left(q_2^4q_3((1+z^2)q_1^2+z(-1+z^2)q_1q_3-(1+z^2)q_3^2)\right.$$
$$+8(-1+z)(1+z)M_\pi^4\left(3zq_1^3+(-1+10z^2)q_1^2q_3+3z(1+2z^2)q_1q_3^2+(1+2z^2)q_3^3\right)$$
$$\left.+2M_\pi^2q_2^2(z(-3+z^2)q_1^3+(3-9z^2)q_1^2q_3-z(5+z^2)q_1q_3^2+(-3-3z^2+4z^4)q_3^3)\right)$$
$$-\frac{g_A^6M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2(zq_1+q_3)\right)}{128F^6\pi\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3^3\left(4M_\pi^2+q_3^2\right)},$$

$$R_6 = \frac{A(q_2)g_A^6\left(2M_\pi^2+q_2^2\right)}{128F^6\pi}+\frac{A(q_1)g_A^6\left(2z\left(M_\pi^2+q_1^2\right)q_3+q_1\left(8M_\pi^2+3q_1^2+q_3^2\right)\right)}{128F^6\pi q_1}$$
$$+\frac{A(q_3)g_A^6\left(2zq_1\left(M_\pi^2+q_3^2\right)+q_3\left(8M_\pi^2+q_1^2+3q_3^2\right)\right)}{128F^6\pi q_3}$$
$$-\frac{g_A^6M_\pi}{128F^6\pi q_1\left(4M_\pi^2+q_1^2\right)\left(4(-1+z^2)M_\pi^2-q_2^2\right)q_3\left(4M_\pi^2+q_3^2\right)}\left((5+z^2)q_1^3q_2^2q_3^3+8M_\pi^6\left(z(-3+4z^2)q_1^2\right.\right.$$
$$\left.+2(19-18z^2)q_1q_3+z(-3+4z^2)q_3^2\right)+2M_\pi^4\left(4z(-1+z^2)q_1^4+(77-36z^2)q_1^3q_3+2z(33+8z^2)q_1^2q_3^2\right.$$
$$\left.+(77-36z^2)q_1q_3^3+4z(-1+z^2)q_3^4\right)+2M_\pi^2q_1q_3\left((10+z^2)q_1^4+2z(9+2z^2)q_1^3q_3+(29-7z^2)q_1^2q_3^2\right.$$
$$\left.\left.+2z(9+2z^2)q_1q_3^3+(10+z^2)q_3^4\right)\right)-\frac{I(4:0,-q_1,q_3;0)g_A^6\left(2M_\pi^2+q_2^2\right)}{32F^6q_1\left(-4(-1+z^2)M_\pi^2+q_2^2\right)q_3}\left(q_1q_2^2q_3\left(q_1^2+zq_1q_3+q_3^2\right)\right.$$
$$\left.+4M_\pi^4\left(zq_1^2-2(-2+z^2)q_1q_3+zq_3^2\right)+2M_\pi^2\left(4q_1q_3\left(q_1^2+q_3^2\right)+z\left(q_1^4+6q_1^2q_3^2+q_3^4\right)\right)\right),$$

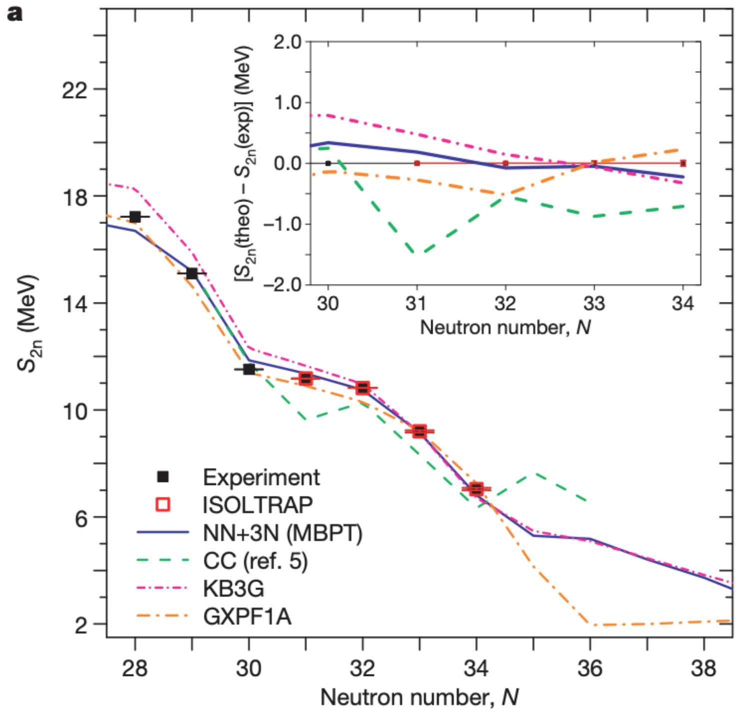
$$R_7 = \frac{3g_A^6M_\pi\left(2M_\pi^2+q_2^2\right)}{256F^6\pi q_1^2\left(-4(-1+z^2)M_\pi^2+q_2^2\right)}-\frac{3A(q_3)g_A^6\left(2M_\pi^2+q_2^2\right)((1+z^2)q_1+2zq_3)}{256F^6\pi(-1+z^2)^2q_1^3}$$
$$-\frac{3A(q_1)g_A^6\left(2M_\pi^2+q_2^2\right)(2zq_1+(1+z^2)q_3)}{256F^6\pi(-1+z^2)^2q_1^2q_3}+\frac{3A(q_2)g_A^6\left(2M_\pi^2+q_2^2\right)\left(2zq_1^2+(1+3z^2)q_1q_3+2zq_3^2\right)}{256F^6\pi(-1+z^2)^2q_1^3q_3}$$

Chiral Effective Field Theory - Successes



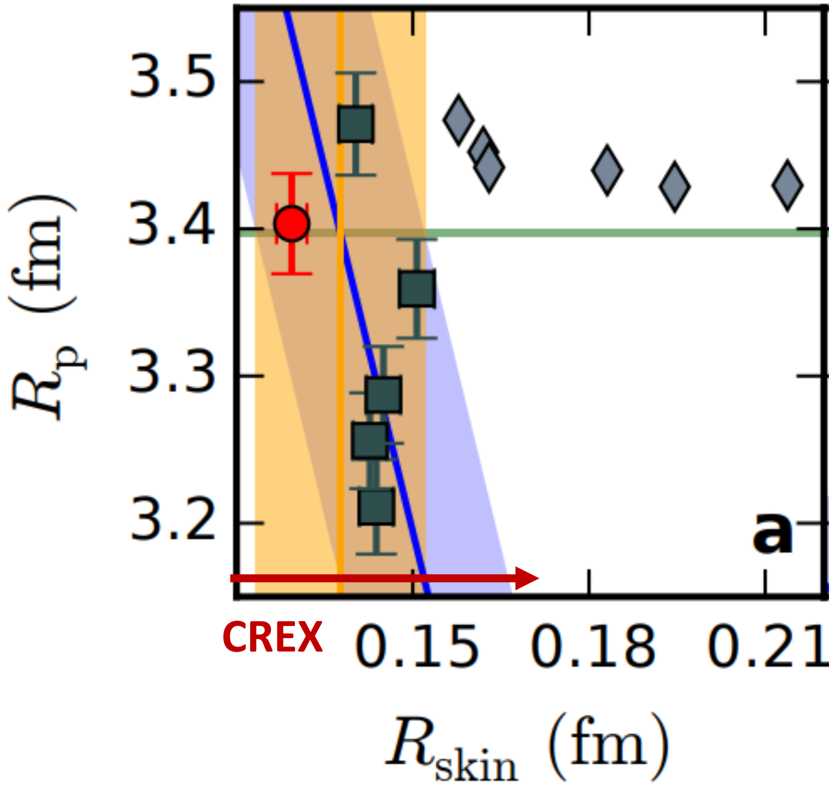
Oxygen anomaly explained

Otsuka et al., PRL 105 (2010)



Calcium 2n separation energies

Wienholtz et al., Nature 498 (2013)



Neutron skin of ^{48}Ca

Hagen et al., Nature Physics (2015)

Remember: Fits (only) to light systems!

See works by many others in the community, e.g., Hergert, Roth, Bogner, Holt, Stroberg and many more…

Quantum Monte Carlo method

Solve **Schrödinger equation:** $$\mathcal{H}|\psi\rangle = E|\psi\rangle$$

- Very precise method for strongly interacting systems.
- Treat many-body Schrödinger equation as diffusion in imaginary time:

$$\lim_{\tau\to\infty} e^{-H\tau}|\Psi_T\rangle \to |\Psi_0\rangle$$

Basic steps:

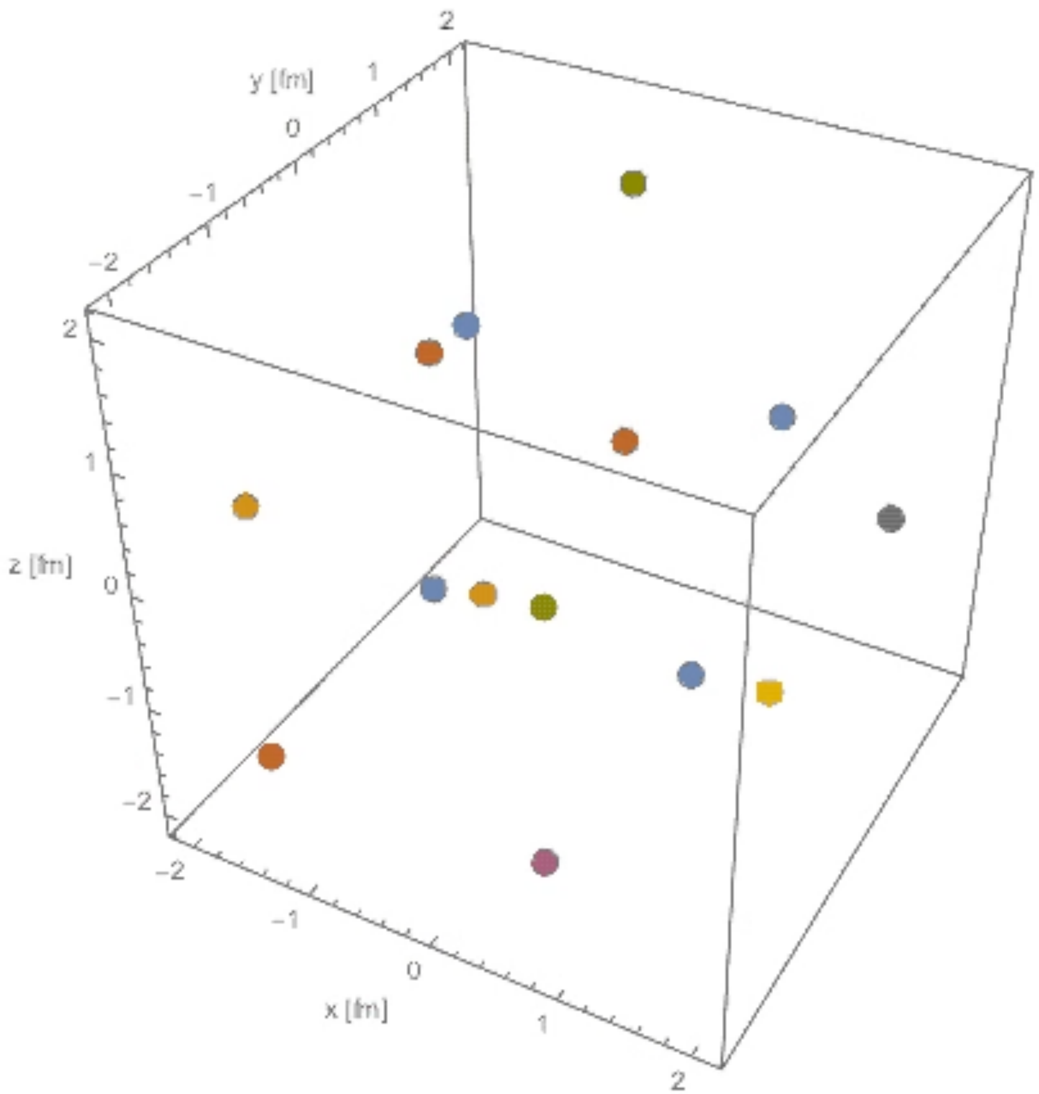
- Choose trial wavefunction which overlaps with the ground state:

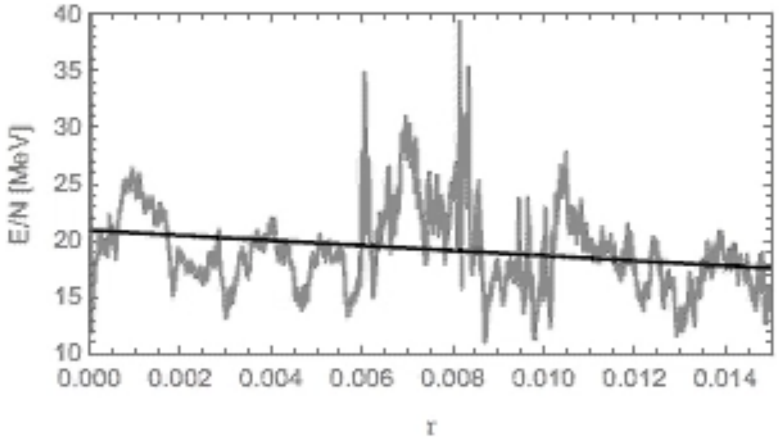
$$|\psi(R,0)\rangle = |\psi_T(R,0)\rangle = \sum_i c_i|\phi_i\rangle \to \sum_i c_i e^{-(E_i-E_0)\tau}|\phi_i\rangle$$

- Evaluate propagator for small timestep $\Delta\tau$
- Make consecutive small time steps using Monte Carlo techniques to project out ground state

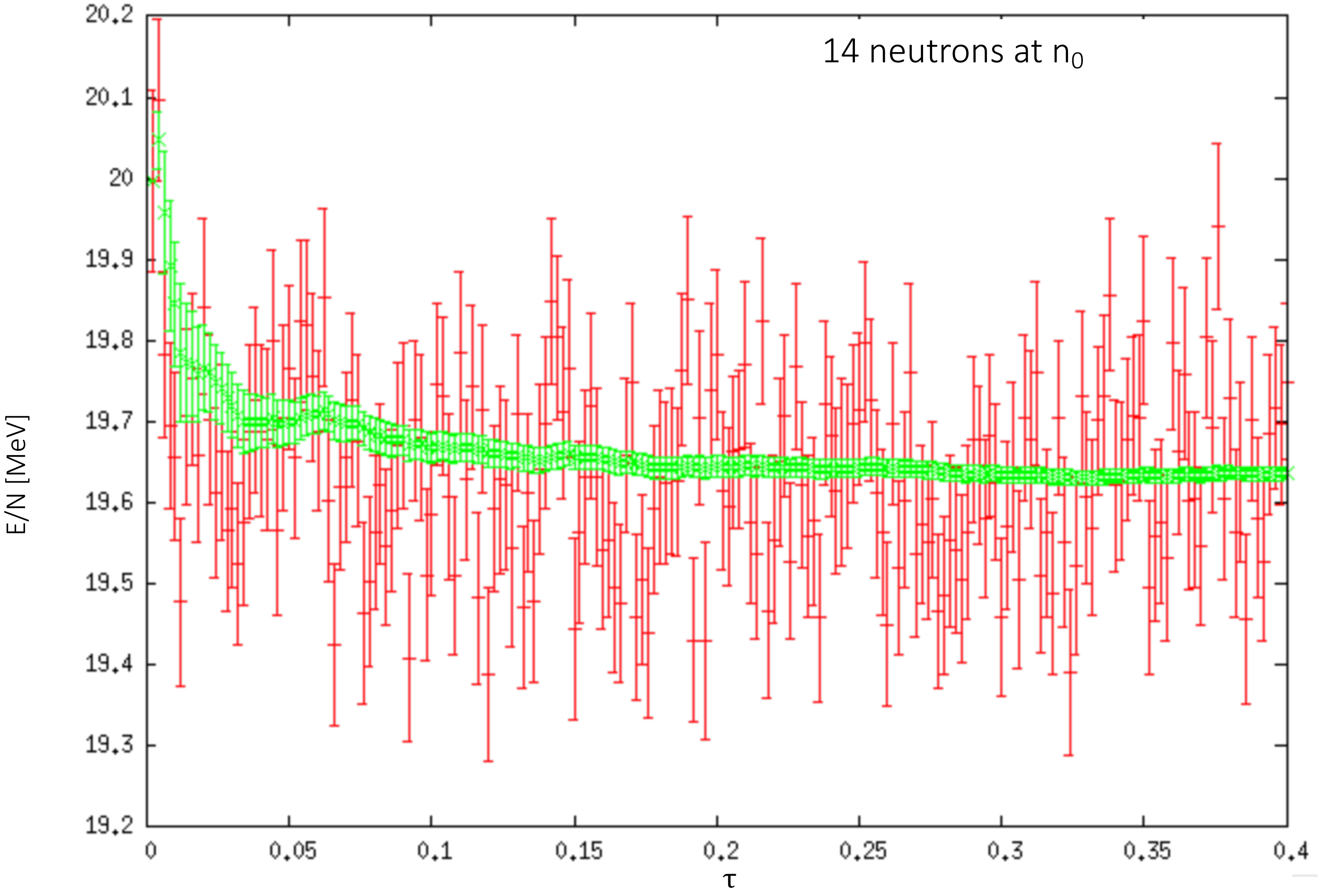
$$|\psi_T(R,\tau)\rangle \to |\phi_0\rangle \quad \text{for} \quad \tau\to\infty$$

14 neutrons at n_0





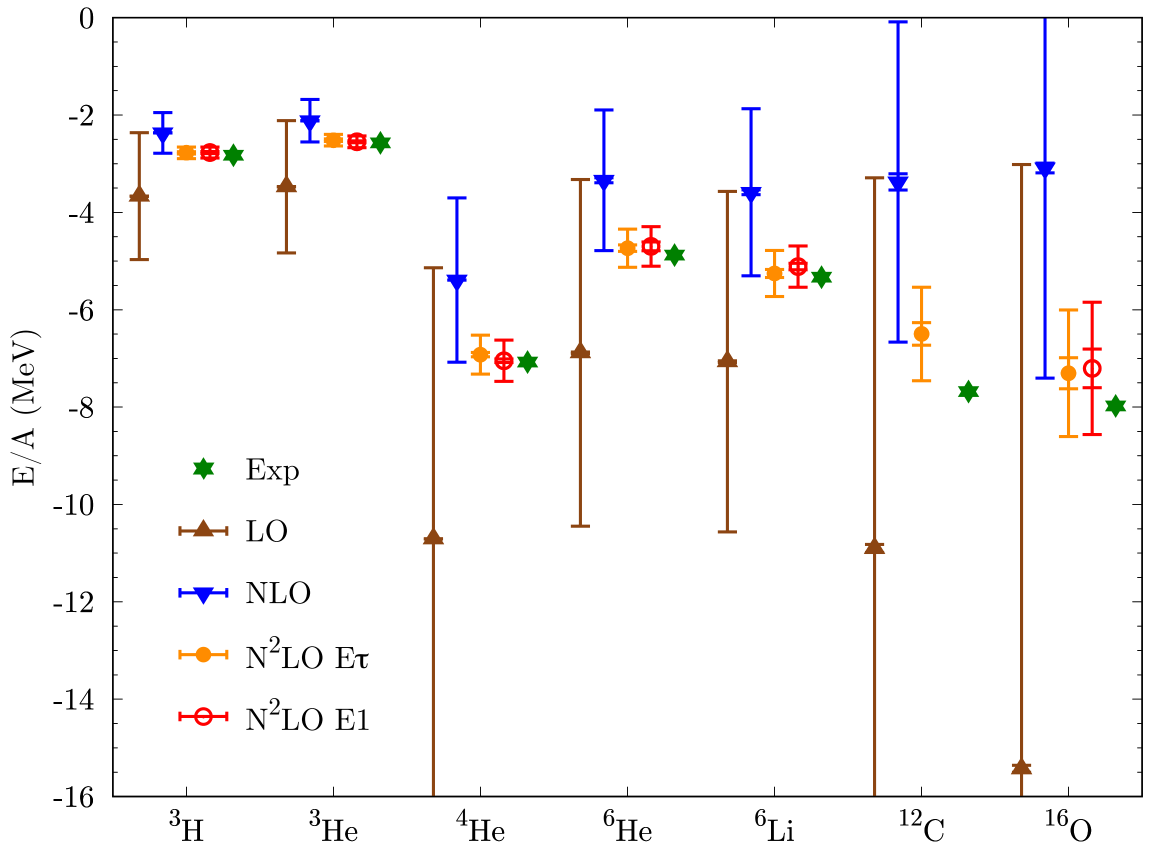
Quantum Monte Carlo method

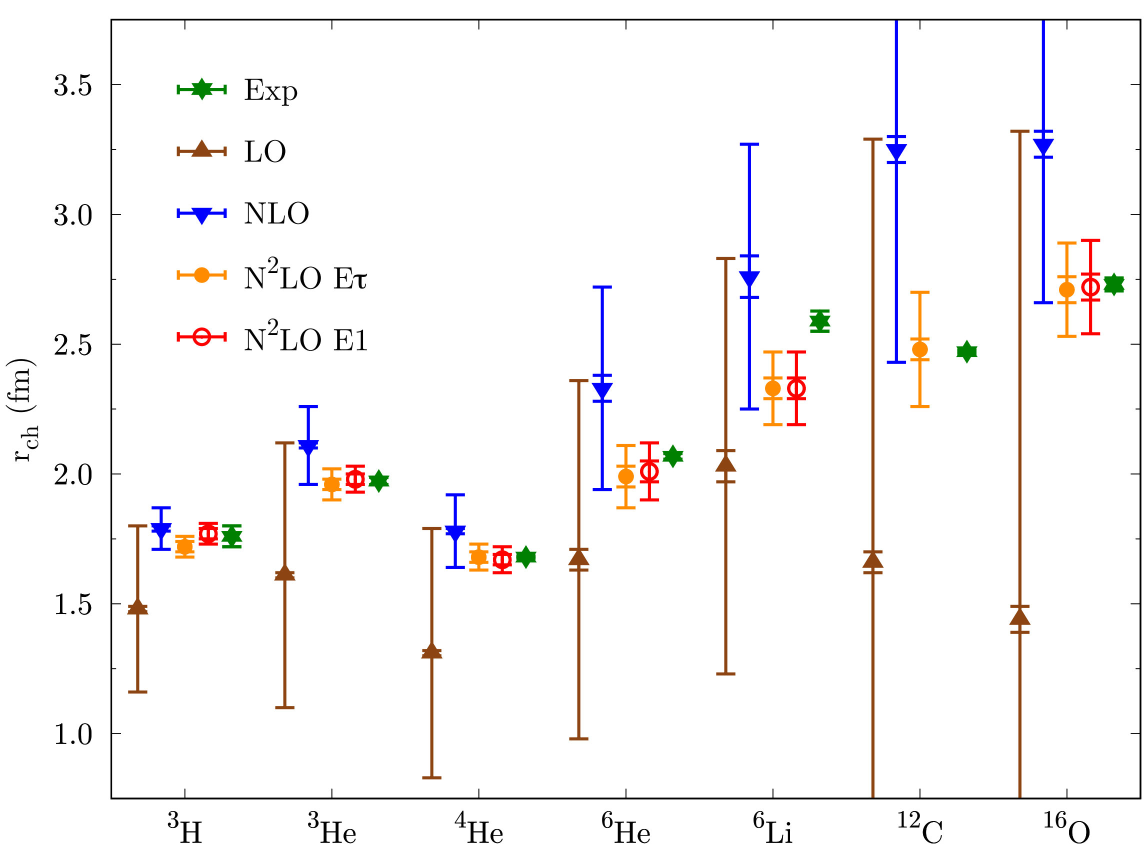


Credit: Diego Lonardoni

Results for nuclei

Results for chiral EFT calculations of nuclei with Quantum Monte Carlo (QMC) methods:

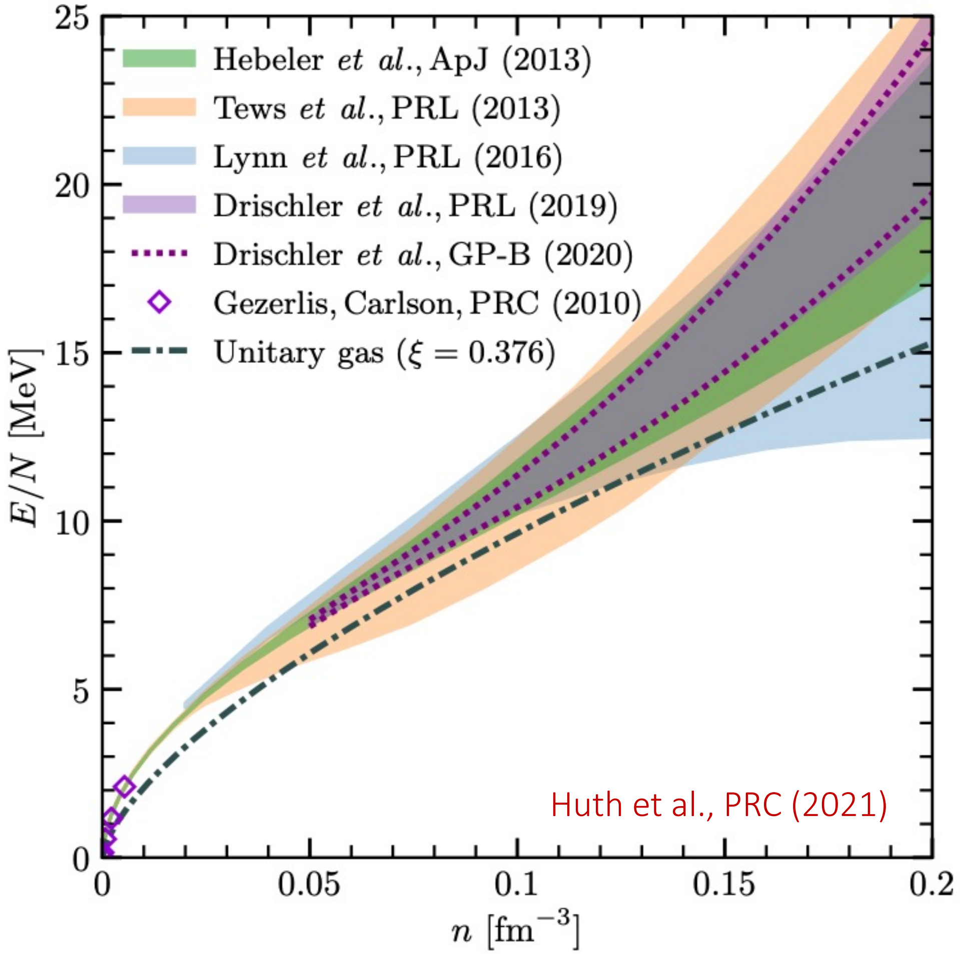


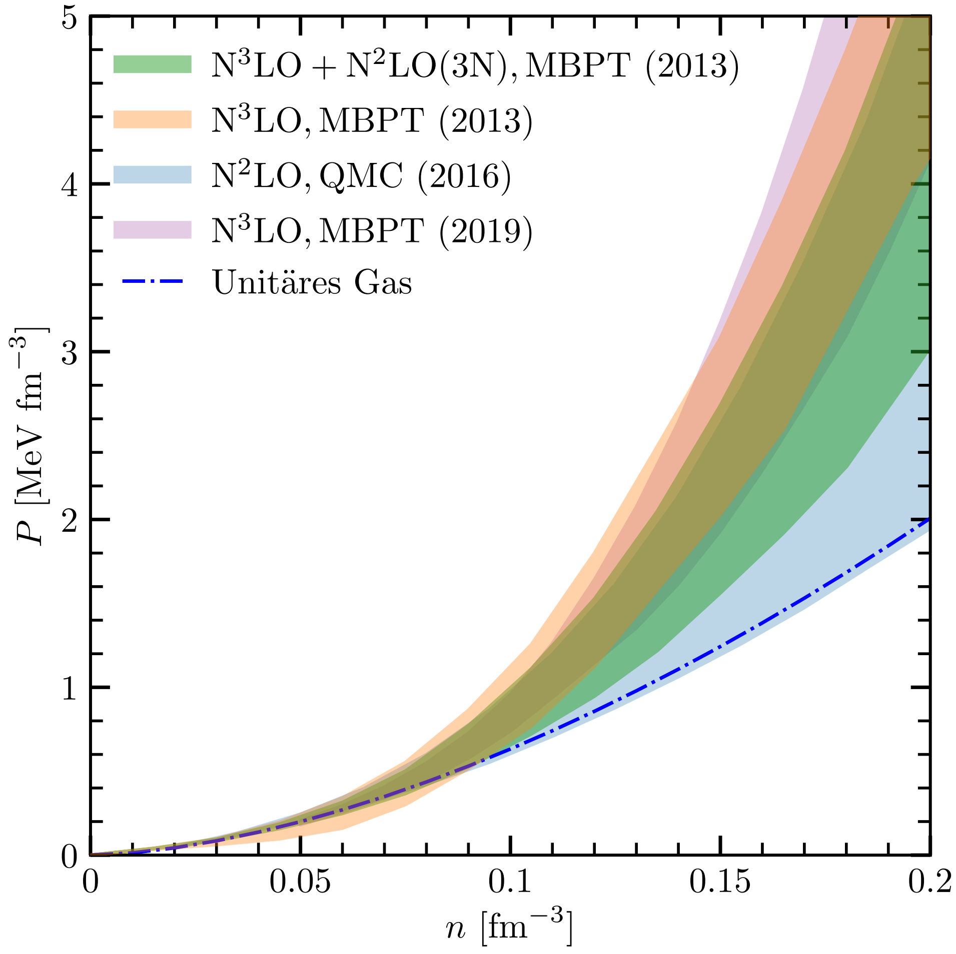


Excellent description of properties of nuclei up to the medium-mass region.

Lonardoni et al., PRL and PRC (2018)

Results for neutron matter





Known operator basis allows us to extrapolate in isospin asymmetry and density away from nuclei.

Excellent agreement for different many-body methods/EFT schemes, interactions main limitation!

Neutron-star EOS

Envelopes around all EOS that:

- Are **causal** ($c_S^2 \leq 1$) and **stable** ($c_S \geq 0$ inside NS).
- Are **consistent with low-density results** from chiral effective field theory (up to two different densities).
- Support at least **1.9 solar-mass** neutron stars.

Current nuclear-physics uncertainties remain sizable!

Extract information from NS observations.

IT, Margueron, Reddy, EPJ A (2019)