Two-photon exchange calculations versus data

Oleksandr Tomalak
Johannes Gutenberg University, Mainz, Germany
Scattering experiments and $2\chi$

- $2\chi$ is not among standard radiative corrections

$$\sigma^{\exp} \equiv \sigma_{1\gamma}(1 + \delta_{\text{rad}} + \delta_{\text{soft}} + \delta_{2\gamma})$$

- charge radius insensitive to $2\chi$ model

- magnetic radius depends on $2\chi$ model

J. C. Bernauer et al. (2014)
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- 2 % systematic deviation

MAMI vs. world data

magnetic form factor

J. C. Bernauer et al. (2014)
µH hyperfine splitting and $2\gamma$

1S HFS in µH
PSI, J-PARC, RIKEN-RAL
1 ppm accuracy

R. Pohl et al. (2016)

- leading theoretical uncertainty: 213 ppm from $2\gamma$, 109 ppm from $2\gamma$

- HFS in terms of forward lepton-proton scattering amplitudes

- traditional decomposition:

$$\Delta_{\text{HFS}} = \Delta_Z + \Delta^R + \Delta^\text{pol}$$

  Zemach term
  $G_E, G_M$

  recoil correction
  $G_E, G_M$

  polarizability
  $F_2, g_1, g_2$

- $A_1@\text{MAMI}$ fit allows to quantify $2\gamma$ uncertainty
  J. C. Bernauer et al. (2014)

- proton radii, form factors and spin structure are important
Zemach contribution

- Zemach correction expanding form factors

\[ \Delta_Z = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left( \frac{G_M(Q^2) G_E(Q^2)}{\mu_P} - 1 \right) + \frac{4\alpha m_r Q_0}{3\pi} \left( -r_E^2 - r_M^2 + \frac{r_E^2 r_M^2}{18} Q_0^2 \right) \]

- dependence on splitting: consistency check

- 95 ppm change for $\mu$H and ep radii with $Q_0 = 0.2$ GeV

- 3 times more precise: 140 ppm $\rightarrow$ 49 ppm

- magnetic radius is important
Hyperfine splitting and $2\gamma$

- compare with precise $1S$ HFS from eH
- achieved accuracy in 70th: $10^{-12}$

- dispersive evaluation and phenomenological extractions agree
Hyperfine splitting and $2\chi$

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- dispersive evaluation and phenomenological extractions agree

$\Delta_{\text{HFS}}$, ppm

- $\Delta_{\text{pol}}$, Faustov et al.
- $\Delta Z + \Delta R$, Bodwin et al.
- 1S HFS in eH

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Hyperfine splitting and $2\gamma$

- compare with precise 1S HFS from eH
  achieved accuracy in 70th: $10^{-12}$

- exploit eH HFS measurements
  scaled by a reduced mass $m_r$

\[
\Delta(\mu H) = \frac{m_r(m_\mu)}{m_r(m_e)} \Delta (\text{eH}) + \\
\Delta_{\text{HFS}}(m_\mu) = \frac{m_r(m_\mu)}{m_r(m_e)} \Delta_{\text{HFS}}(m_e)
\]

- uncertainty: 100 ppm $\rightarrow$ 16 ppm !!!

- dispersive evaluation and phenomenological extractions agree
Hyperfine splitting and $2\chi$

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- uncertainty: 100 ppm $\rightarrow$ 16 ppm !!!
Elastic lepton-proton scattering

- leading $2\gamma$ contribution: interference term

- $2\gamma$ correction to cross section is given by amplitudes real parts
Elastic lepton-proton scattering

\[ K = \frac{k + k'}{2} \]

\[ P = \frac{p + p'}{2} \]

- electron-proton scattering: 3 structure amplitudes

\[ T^{\text{non-flip}} = \frac{e^2}{Q^2} \bar{l} \gamma_\mu l \cdot \bar{N} \left( G_M(\nu, Q^2) \gamma^\mu - F_2(\nu, Q^2) \frac{P^\mu}{M} + F_3(\nu, Q^2) \frac{\hat{K} P^\mu}{M^2} \right) N \]


- muon-proton scattering: add helicity-flip amplitudes

\[ T^{\text{flip}} = \frac{e^2}{Q^2} \frac{m}{M} \bar{l} \cdot \bar{N} \left( F_4(\nu, Q^2) + F_5(\nu, Q^2) \frac{\hat{K}}{M} \right) N + \frac{e^2}{Q^2} \frac{m}{M} F_6(\nu, Q^2) \bar{l} \gamma_5 l \cdot \bar{N} \gamma_5 N \]


- 2\( \gamma \) correction to cross section is given by amplitudes real parts
non-forward scattering at low momentum transfer

photoproduction vertex or Compton tensor

box diagram

assumption about the vertex
non-forward scattering at low momentum transfer

photoproduction vertex or Compton tensor

assumption about the vertex

based on on-shell information
non-forward scattering
proton state

\[ \begin{align*}
\gamma & \quad \gamma \\
p & \quad p' \\
l & \quad l' 
\end{align*} \]

Dirac and Pauli form factors

assumption about the vertex
Blunden, Melnitchouk and Tjon (2003)

based on on-shell information
forward scattering

\[ l \gamma \gamma l \]
\[ p \gamma \gamma p \]

near-forward scattering
account for all inelastic $2\gamma$

\[ l \gamma \gamma l' \]
\[ p \gamma \gamma p' \]
Low-$Q^2$ inelastic $2\chi$ correction (e-p)

- $2\chi$ blob: near-forward virtual Compton scattering

$$\delta_{2\gamma} \approx a \sqrt{Q^2} + b Q^2 \ln Q^2 + c Q^2 \ln^2 Q^2$$

Feshbach inelastic elastic

unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)


- $2\chi$ at large $\varepsilon$ agrees with empirical fit

O. T. and M. Vanderhaeghen (2016)

\[ Q^2 = 0.25 \text{ GeV}^2 \]

\[ Q^2 = 0.05 \text{ GeV}^2 \]
Scattering experiments and $2\chi$

- charge radius extractions:

<table>
<thead>
<tr>
<th>eH, eD spectroscopy</th>
<th>ep scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu H, \mu D$ spectroscopy</td>
<td>$\mu p$ scattering ????</td>
</tr>
</tbody>
</table>

- $\mu p$ elastic scattering is planned by MUSE@PSI(2018-19) measure with both electron/muon charges

- $2\chi$ correction in MUSE ?
MUSE@PSI (2018-19) estimates ($\mu$-$p$)

- proton box diagram model + inelastic $2\gamma$

\begin{align*}
\delta_{2\gamma}, \% \quad &\text{box diagram model, } \mu^- p \\
\text{total, } \mu^- p \\
\text{total, } e^- p
\end{align*}

\begin{align*}
k &= 115 \text{ MeV} \\
&
\begin{array}{c}
0 \\
0.005 \\
0.010 \\
0.015 \\
0.020 \\
0.025
\end{array}
\end{align*}

\begin{align*}
\delta_{2\gamma}, \% \quad &\text{box diagram model, } \mu^- p \\
\text{total, } \mu^- p \\
\text{total, } e^- p
\end{align*}

\begin{align*}
k &= 210 \text{ MeV} \\
&
\begin{array}{c}
0 \\
0.020 \\
0.040 \\
0.060 \\
0.080
\end{array}
\end{align*}

O. T. and M. Vanderhaeghen (2014, 2016)
MUSE@PSI (2018-19) estimates ($\mu$-p)

- proton box diagram model + inelastic $2\chi$

- expected muon over electron ratio

  small inelastic $2\chi$

  small $2\chi$ uncertainty

- MUSE can test $r_E$ in one charge channel

K. Mesick talk (PAVI 2014), MUSE TDR (2016)
near-forward scattering
(large $\varepsilon$)

$p + \text{all inelastic}$

dispersion relations
(arbitrary $\varepsilon$)

$X = p + \pi N$
Fixed-$Q^2$ dispersion relation framework

on-shell $1\chi$ amplitudes

![Graph showing data points and fitted curves for $\sigma_\text{on-shell}$ vs. $\nu$ in GeV]

experimental data

unitarity

$\Re F(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{\text{min}}}^{\infty} \frac{\Im F(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$

$2\chi$ imaginary parts

disp. rel.

$2\chi$ real parts

cross section correction

$2\chi$ prediction

![Graph showing predicted $\delta_{\text{PPC}}(Q^2)$ vs. $Q^2$ in GeV$^2$]

Fixed-Q$^2$ dispersion relation framework
Mandelstam plot (ep)

- proton intermediate state is outside physical region for $Q^2 > 0$
- $\pi N$ intermediate state is outside physical region for $Q^2 > 0.064 \text{ GeV}^2$
Analytical continuation. Elastic state

- contour deformation method:

\[ \int d\Omega \rightarrow \text{angular integration to integration on curve in complex plane} \rightarrow \text{deform integration contour keeping poles inside going to unph. region} \]

- analytical continuation reproduces results in unphysical region

\[ Q^2 = 0.1 \text{ GeV}^2 \]

- central value: form factor fit of A1@MAMI (2014)
- uncertainty: difference to 2γ with dipole form factors

O. T. and M. Vanderhaeghen (2014), Blunden and Melnitchouk (2017)
Analytical continuation. πN states

- pion electroproduction amplitudes: MAID2007

- analytical continuation: fit of low-$Q^2$ expansion in physical region

\[ G_{1,2}(s, Q^2), \quad Q^2 F_3(s, Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \ldots \]

- uncertainty: extrapolation + large invariant masses
- $\pi N$ intermediate state is outside physical region for $Q^2 > 0.064$ GeV$^2$
- $\pi N$ intermediate state is outside physical region for $Q^2 > 0.064 \text{ GeV}^2$
- \( \pi N \) intermediate state is outside physical region for \( Q^2 > 0.064 \text{ GeV}^2 \)
Analytical continuation. $\pi N$ states

- pion electroproduction amplitudes: MAID$_{2007}$

- analytical continuation: fit of low-$Q^2$ expansion in physical region

$$G_{1,2}(s, Q^2), \quad Q^2 F_3(s, Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \ldots$$

- uncertainty: extrapolation + large invariant masses
\[ \pi N \text{ in dispersive framework (e-}\rho) \]

- dispersion relations agree with near-forward at large \( \varepsilon \)

\[ Q^2 = 0.005 \text{ GeV}^2 \]
$\pi N$ in dispersive framework (e-p)

$Q^2 = 0.005 \text{ GeV}^2$

- $\pi N$, unsubtracted dispersion relations
- $\pi N$, near-forward from structure functions

- dispersion relations agree with near-forward at large $\varepsilon$

$Q^2 = 0.05 \text{ GeV}^2$

- $\pi N$ is dominant inelastic $2\gamma$

O. T., B. Pasquini and M. Vanderhaeghen (2017)
Comparison with data

\[ R_{2\gamma} = \frac{\sigma(e^+ p)}{\sigma(e^- p)} \approx 1 - 2\delta_{2\gamma} \]

- Weighted \( \Delta \) is similar to narrow one of Blunden et al. (2017)
- \( \pi N \) contribution is closer to data than \( \Delta \) only
Comparison with data

\[ R_{2\gamma} = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 - 2\delta_{2\gamma} \]

- near-forward 2\(\gamma\) agree with data
- multi-particle 2\(\gamma\), e.g. \(\pi\piN\), is important

\[ k = 2.01 \text{ GeV} \]
uncorr. + corr. uncertainties

Maximon and Tjon IR prescription

OLYMPUS (2016)
Feshbach
elastic
elastic + \(\pi\)N
total 2\(\gamma\), near-forward

O. T., B. Pasquini and M. Vanderhaeghen (2017)
Comparison with data

- dispersion relations agree with CLAS data

Comparison with data

O. T., B. Pasquini and M. Vanderhaeghen (2017)
Conclusions

why $2\chi$?

largest theoretical uncertainty in low-energy proton structure
Conclusions

why $2\chi$?

largest theoretical uncertainty in low-energy proton structure

how to study?

box diagram

small scatt. angles
ep, $\mu p$ (all states)

 dispersion relations

all scatt. angles
ep ($p + \pi N$ states)

- multi-particle $2\chi$, e.g. $\pi\pi N$, within dispersion relations is important
Our best $2\gamma$ knowledge

- small $Q^2$: near-forward at large $\varepsilon$, all inelastic states
- $Q^2 \lesssim 1\text{ GeV}^2$: elastic+$\pi N$ within dispersion relations
- intermediate range: interpolation

$Q^2 = 0.1\text{ GeV}^2$
Outlook

- theoretical $2\chi$
  - JLAB data $g_1$, $g_2$
  - MUSE data

ep $\rightarrow$ magnetic radius extraction $\rightarrow$ application to 1S HFS exp $\rightarrow$ dispersive $2\chi$ evaluation

$\mu p$
Thanks for your attention !!!
Muon discrepancies: new physics?

anomalous magnetic moment

$3.6 \, \sigma$

theory vs exp.

hadronic uncertainty is dominant in theory

Our goals

decrease hadronic uncertainties

study the low energy proton structure

Pohl et al., Nature (2010)
Tool to explore the proton structure

\[ u(k, h) \rightarrow \bar{u}(k', h') \]

\[ N(p, \lambda) \rightarrow \bar{N}(p', \lambda') \]

\[ \frac{e^2}{Q^2} (\bar{u}(k', h') \gamma_\mu u(k, h)) \cdot (\bar{N}(p', \lambda') \Gamma_\mu(Q^2) N(p, \lambda)) \]

\[ \Gamma_\mu(Q^2) = \gamma_\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P(Q^2) \]

lepton energy

Dirac and Pauli form factors

momentum transfer

\[ Q^2 = -(k - k')^2 \]

\[ \omega \]

lp amplitude

\[ T = \frac{e^2}{Q^2} (\bar{u}(k', h') \gamma_\mu u(k, h)) \cdot (\bar{N}(p', \lambda') \Gamma_\mu(Q^2) N(p, \lambda)) \]
- Sachs electric and magnetic form factors:

\[ G_E = F_D - \tau F_P \quad G_M = F_D + F_P \]

- Rosenbluth separation:

\[ \frac{d\sigma^{unpol}}{d\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \]

\[ \tau \quad \varepsilon \quad \text{kinematical variables} \]

\[ G_M^2(Q^2) \]

\[ Q^2 = 2.64 \text{ GeV}^2 \]

- Rosenbluth slope is sensitive to corrections beyond 1\(\gamma\)
Form factors measurement

- Sachs electric and magnetic form factors:
  \[ G_E = F_D - \tau F_P \quad G_M = F_D + F_P \]

- polarization transfer method:
  \[ e + p \rightarrow e + p \]
  realized in 2000 at JLab

\[ P_T \sim G_E(Q^2)G_M(Q^2) \]
\[ P_L \sim G_M^2(Q^2) \]

\[ \frac{P_T}{P_L} \sim \frac{G_E(Q^2)}{G_M(Q^2)} \]
Polarization transfer
JLab (Hall A, C)

Rosenbluth separation
SLAC, JLab (Hall A, C)

\[ \frac{\mu_p G_{Ep}}{G_{Mp}} \]

\[ Q^2 \text{ (GeV}^2) \]

V. Punjabi et al. (2015)
Proton form factors puzzle

Polarization transfer vs.
JLab (Hall A, C)

Rosenbluth separation
SLAC, JLab (Hall A, C)

a possible explanation
two-photon exchange

2χ measurements
e⁺p/e⁻p cross section ratio

\[ R_{2\gamma} = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 - 2\delta_{2\gamma} \]

- discrepancy motivates model-independent study of 2χ
Proton charge radius

electric charge radius

\[ < r_E^2 > \equiv -6 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \]

ep elastic scattering

\[ r_E = 0.879 \pm 0.008 \text{ fm} \]
Proton charge radius

electric charge radius

\[<r_E^2> \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}\]

ep elastic scattering

\[r_E = 0.879 \pm 0.008 \text{ fm}\]

atomic spectroscopy

\[\Delta E_{nS} \sim m_r^3 <r_E^2>\]

H, D spectroscopy

\[r_E = 0.8758 \pm 0.0077 \text{ fm}\]

CODATA 2010

\[r_E = 0.8409 \pm 0.0004 \text{ fm}\]

CREMA (2010, 2013)

\[\mu H \text{ Lamb shift}\]
Proton radius puzzle

electric charge radius

\[ < r_E^2 > \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} \]

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CREMA (2010, 2013)

\[ 4 \% \text{ difference} \]
μH Lamb shift and 2𝛾

2P-2S transition in μH
charge radius discrepancy
310 μeV
μH experimental uncertainty
2.5 μeV

2𝛾 hadronic correction

ΔE_{2P-2S}^{2γ} = 33 ± 2 μeV


important to reduce ambiguities of 2γ
Dispersion relation framework

\[ f(z) \]

analyticity

experimental cross sections

energy levels correction

optical theorem

amplitudes: imaginary parts

\[ \Re \mathcal{F}(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{min}}^{\infty} \frac{\Im \mathcal{F}(\nu') + i0}{\nu'^2 - \nu^2} d\nu' \]

DR

amplitudes: real parts