Statistical Tests of Symmetry Violation

Steve Tomsovic

Washington State University, Pullman, WA USA

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Collaborators:

P-odd, T-even

J. David Bowman
Anna C. Hayes
Mikkel B. Johnson
Phys. Rev. C, 2000
unpublished work, 2003

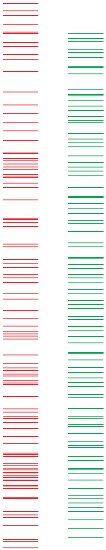
T-odd, P-even

J. Bruce French
Akhilesh Pandey
V. K. B. Kota
Ann. Phys. 1988

Outline

- Observables
  - a) fluctuation properties
    - universality
    - symmetry breaking - transition parameter
  - b) P-odd, T-even – longitudinal asymmetry
  - c) T-odd, P-even – spectral fluctuations, transition strengths
- Statistical Spectroscopy - French and collaborators
  - a) relating the transition parameter to interaction parameters
  - b) secular quantities - level densities, strength functions
    - central limit theorems
    - moment methods, partitioning
    - angular momentum decomposition
  - c) example calculations
- Perspectives

Fluctuation properties – universality



Poisson RMT

- Wigner introduces random matrix theory (RMT) for slow neutron resonances
- Bohigas-Gianonni-Schmit conjecture
  - fluctuation properties of quantum chaotic systems are universal and follow from RMT
  - integrable systems display Poissonian level statistics - Berry & Tabor

★ For chaotic systems, symmetry breaking transitions are governed by a transition parameter, Λ, (Pandey-Mehta, French et al.)

$$\begin{aligned} \Lambda &= \frac{v^2}{D^2} \\ v^2 &= local\ variance\ of\ many-body\ matrix\ element \\ D &= local\ mean\ energy\ spacing \end{aligned}$$

P-odd T-even statistical test

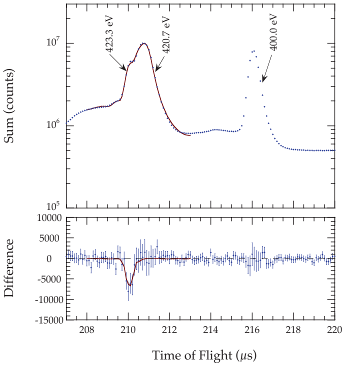
★ $\vec{\sigma}_n \cdot \vec{k}_n$ dependent neutron resonance cross-sections

- The longitudinal asymmetry is given by (Sushkov-Flambaum)

$$A_L^n = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \approx 2\sum_s \frac{\langle\psi_s|V_{pnc}|\psi_p\rangle}{E_p - E_s}\sqrt{\frac{\Gamma_s^n}{\Gamma_p^n}}$$

- Γ_s^n, Γ_p^n – slow neutron s- and p-wave partial width amplitudes, respectively.
- E_s, E_p – s- and p-wave resonance energies
- $\langle\psi_s|V_{pnc}|\psi_p\rangle$ – parity odd interaction many-body matrix element connecting the resonances

Cross-section and asymmetry



TRIPLE collaboration data

The P-odd T-even transition parameter

- From each measured parity asymmetry, one can extract a value of $\langle\psi_s|V_{pnc}|\psi_p\rangle$.
- The TRIPLE collaboration calls the many-body parity non-conserving matrix element variance M^2:

$$M^2 = \overline{|\langle\psi_s|V_{pnc}|\psi_p\rangle|^2} - \overline{|\langle\psi_s|V_{pnc}|\psi_p\rangle|}^2$$

- Thus, the extent of symmetry violation is governed by the transition parameter, Λ, which is essentially a locally smoothed strength function,

$$\Lambda = \frac{M^2}{D_{j+}D_{j-}} = \overline{Tr\left[\tilde{V}_{pnc}\delta(E_p - H_{nucl})\tilde{V}_{pnc}\delta(E_s - H_{nucl})\right]_{j+}}$$

where the density of states, $\rho_{j\pm}(E) = \dfrac{1}{D_{j\pm}} = \overline{Tr\left[\delta(E - H_{nucl})\right]_{j\pm}}$,

- In the neutron resonance region, $D_{j+} \approx D_{j-}$.

Remarks

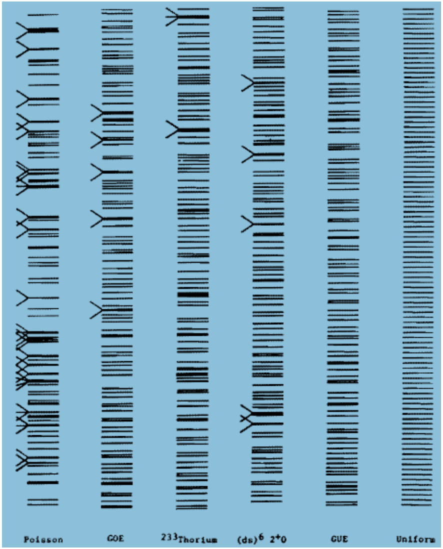
★ A theory for calculating the secular behavior of strength functions is a key to analyzing statistical tests of symmetry violation. The statistical spectroscopy of French and collaborators is designed for this purpose.

- It is also necessary to calculate fixed angular momentum-parity smoothed level densities within statistical spectroscopy.
- Strength densities increase exponentially with excitation energy, but spreading widths are algebraic (almost constant),

$$\Gamma_w = 2\pi \frac{M^2}{D_{j+}}$$

- Γ_w is the more natural quantity for combining data from different nuclei (for greater stastistical significance).

Wigner's suggested T-odd P-even statistical tests



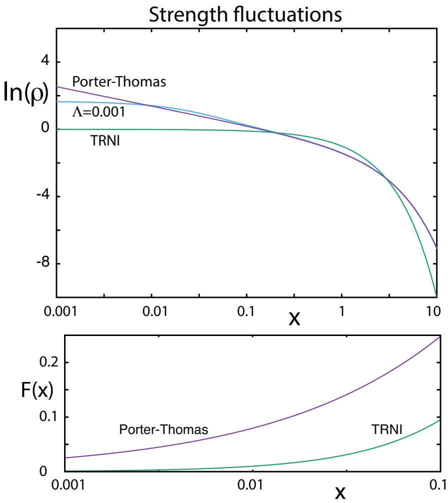
★ Use suitable fluctuation measure of level statistics.

- One of the best is the number variance, $\Sigma^2(r)$.
- Consider: nuclear data ensemble - 1407 resonance energies in 30 sequences of 27 different nuclei (Haq, Pandey, Bohigas)
- Perturbation theory gives

$$\Sigma^2(r,\Lambda) = \Sigma^2(r,0) - 4\Lambda R_2(r)$$

- Use to bound or determine Λ.
- Complete spectra, untainted by misidentified levels, are very important.

Wigner's suggested statistical tests: no. 1



- Transition strength densities,

$$\rho(x) = \frac{1}{\sqrt{2\pi x}} \exp\left(-\frac{x}{2}\right) \quad TRI$$
$$= \exp(-x) \quad TRNI$$

- For weak T-violation

$$\rho(x) \approx \frac{1}{\sqrt{a}} \exp\left(-\frac{x}{a}\right) I_0\left(\frac{\sqrt{1-a}}{a} x\right)$$

- the variance is $\sigma^2 = 2 - a$,

$$a = -\frac{2\pi^2\Lambda}{3}\left[\ln\left(2\pi^2\Lambda\right) + \gamma - 2\right]$$

★ A weak T-odd interaction only affects the smallest transition strengths.

Remarks

- ★ For the nuclear data ensemble, one would expect to find 70-80 transition strengths $< 10^{-3}$ of the mean.
- With perfect data, it could be possible to see effects at about the level of $\Lambda = 10^{-4}$; i.e. the imaginary part of the many-body matrix element $\approx 1/100$ of a mean spacing.
- The same remarks apply about Λ being essentially a strength function and about a T-odd interaction spreading width.
- Statistical spectroscopy calculations follow the same sequences of steps as for the P-odd T-even case.
- The data imitate a small TRNI breaking ($\Lambda = 10^{-3}$), but we didn't believe the quality of experimental data on very small strengths allowed for any strong claims.

Remarks: cont.

An approximate form for a P-odd interaction could be taken to be

$$V_w = f_\pi V_\pi + h^1_\rho V_\rho + h^2_\rho V_{\rho'} + h^1_\omega V_\omega + h^0_\rho V_{\rho''} + h^0_\omega V_{\omega'}$$

The question statistical spectroscopy is designed to answer can be formulated as,

★ "What can one infer about the set $\{f_\pi, h^1_\rho, h^2_\rho, h^1_\omega, h^0_\rho, h^0_\omega\}$ given a measured value of M^2 (or Γ_w)?"

For the time reversal tests, we just quoted an overall bound on, α, the relative size of the two-body matrix element coupling constant (< 2 parts in 1000):

$$H_{nucl} = h_1 + V_2 + i\alpha V_{TRNI}$$

Linking Λ to symmetry breaking two-body interactions

- Statistical spectroscopy gives a method of linking Λ to interaction parameters, ie. two-body interaction matrix elements.
- It is based on central limit theorems whose parameters are fixed by low order moment calculations (Mon and French).
- The expressions for these moments derive from unitary group decompositions (Chang, French, Thio), e.g.

$$Tr(V_k)_m = \binom{N-k}{N-m} Tr(V_k)_k$$

(V_k =k-body operator, m particles, and N sing. part. states) which gives the relationship of expecation values

$$\langle V_k \rangle_m = \binom{m}{k} \langle V_k \rangle_k$$

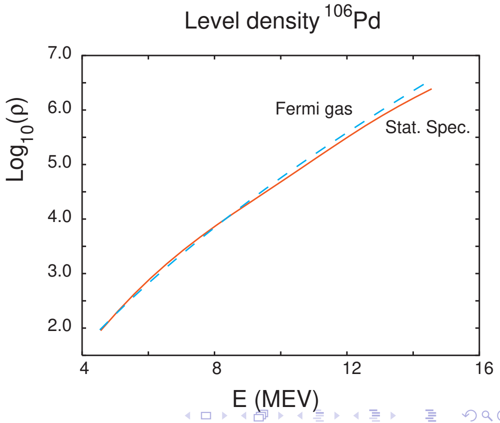
Secular quantities

The two necessary secular quantities are the smoothed level densities and strength functions. Both are subject to central limit theorems.

- The smoothed level density

$$\rho(E) = \overline{Tr\left[\delta(E - H_{nucl})\right]}$$

- Example level density: stat. spectroscopy $\Longrightarrow$
- Fermi gas parameters from von Egidy...



Smoothed level densities and moments

Consider a nuclear Hamiltonian of the form

$$H_{nucl} = h_1 + V_2 \qquad h_1 = \sum_j \epsilon_j a_j^\dagger a_j \qquad V_2 = \sum_{j<l,k<m} V_{jkml} a_j^\dagger a_k^\dagger a_l a_m$$

Imagine the set of two-body matrix elements, $\{V_{jklm}\}$, appear somewhat random and fluctuate about zero.

- ★ Moments of the interaction are dominated by pairwise correlations (operator form of CLT!). If $v^2 =< VV >$, then

$$\begin{aligned} < V^3 > &\approx 0 \qquad \textit{all odd moments vanish} \\ < V^4 > &= < VVVV > + < VVVV > + < VVVV > \approx 3v^4 \quad m >> 2 \\ & \quad \textit{or} \\ < V^4 > &= < VVVV > + < VVVV > \approx 2v^4 \qquad m = 2 \end{aligned}$$

dilute case: $m >> 2$ - moments of Gaussian

m=2 case - moments of semicircle

Model spaces and partitions

For a given nucleus far from magic, identify all proton and neutron orbitals above the core up to and a bit beyond the intruders (e.g. consult Bohr & Mottelson).

- For the level density of ^{106}Pd, one possibility is to consider
  proton orbitals – $7/2^-, 5/2^-, 3/2^-, 1/2^-, 9/2^+, 5/2^+, 7/2^+$
  neutrons – $9/2^+, 5/2^+, 7/2^+, 1/2^+, 3/2^+, 11/2^-, 7/2^-, 9/2^-$
- That leaves 26 valence protons and 20 valence neutrons outside a 20 proton, 40 neutron core.
- Enumerate all partitions of valence particles in these orbitals up to maybe 20 MEV above lowest energy.
  - Calculate single particle energies and dimensionalities for each partition. Let $\mathbf{m}$ denote a particular partition.
- Operator CLT says each partition is spread as a Gaussian over the eigenstates, centered at the partition energy, spread $< V^2 >$, and multiplied by partition dimension.

Partition width

The actual variance used for the partition Gaussians is given by (Chang, French, and Thio, 1970)

$$\sigma^2(\mathbf{m}) = \sum_{\substack{r\leq s\\ t\leq u}} \frac{(N_r - m_r)(N_s - m_s - \delta_{rs})m_t(m_u - \delta_{tu})}{(N_r - \delta_{rt} - \delta_{ru})(N_s - \delta_{rs} - \delta_{su} - \delta_{rs})N_t(N_u - \delta_{tu})} \times \sum_j (2j+1)(\mathcal{V}^{(j)}_{rstu})^2$$

Partitions carry a good parity quantum number and restricting level density or strength density calculations to a particular parity amounts to separating out positive and negative parity partitions.

Fixed-j moments

One approximation is to use energy dependent spin cutoff factors for each partition.

- For an energy and partition dependent angular momentum density, a series can be truncated at the first Hermite polynomial (Bethe, French et al.)

$$\begin{aligned}\rho_{\mathbf{m}}(J/E) &= \frac{2j+1}{\sqrt{8\pi\sigma_{\mathbf{m}}^6}}\exp\left[-\frac{(2j+1)^2}{8\sigma_{\mathbf{m}}^2}\right]\\ 3\sigma_{\mathbf{m}}^2 &\approx \langle J^2\rangle_{\mathbf{m}} + \langle J^2 P_1(H_{nucl})\rangle_{\mathbf{m}} P_1(E)\\ P_1(E) &= E - E_{\mathbf{m}} \qquad P_1(H_{nucl}) = H_{nucl} - \langle H_{nucl}\rangle_{\mathbf{m}}\end{aligned}$$

- This is one part of the calculation where it would be a worthwhile improvement to go to the second order polynomial correction. This would give a stronger energy dependence to the spin cutoff factor.

Strength functions

The central limit theorem applied to strength functions leads to a bivariate Gaussian density:

$$\hat{S}(E,E') = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\xi^2}} \exp\left[-\frac{1}{2(1-\xi^2)}\left(\frac{E^2}{\sigma_1^2} - \frac{2\xi EE'}{\sigma_1\sigma_2} + \frac{E'^2}{\sigma_2^2}\right)\right]$$

$$E \to E - E_{\mathbf{m}} \qquad E \to E' - E_{\mathbf{m}'}$$

- This expression is unit normalized and must be multiplied by an intensity, e.g. $\langle V_{pnc}^2 \rangle_{\mathbf{m}}$.
- The 5 parameters (2 centroids, 2 variances, and a correlation coefficient) can be determined by moments; e.g. $\langle V_{pnc}^2 \rangle_{\mathbf{m}}, \langle V_{pnc}^2 H_{nucl} \rangle_{\mathbf{m}}, \langle V_{pnc} H_{nucl} V_{pnc} \rangle_{\mathbf{m}}, \langle V_{pnc} H_{nucl} V_{pnc} H_{nucl} \rangle_{\mathbf{m}}, \langle H_{nucl}^2 \rangle_{\mathbf{m}}, \langle V_{pnc}^2 H_{nucl}^2 \rangle_{\mathbf{m}}, \langle V_{pnc} H_{nucl}^2 V_{pnc} \rangle_{\mathbf{m}}$

Additional technical details

- For strength functions, fixing the angular momentum follows similarly to the level density case.
- The description here is quite oversimplified. The one-body part of the nuclear Hamiltonian is dealt with exactly and the resulting δ-spikes are convoluted with bivariate Gaussians accounting for the residual strong interaction.
- Correlation coefficients are mostly in the range $[0.7 - 0.9]$ across the partitions for heavy nuclei and the final results are not heavily dependent on their exact values.

Parity violation and the covariance matrix

★ For heavy nuclei, the interactions multiplied by the parameters $\{f_\pi, h_\rho^1, h_\rho^2, h_\omega^1, h_\rho^0, h_\omega^0\}$ are strongly correlated.

- Construct a covariance matrix, $\mathcal{C}$, whose many-body matrix elements, $\mathcal{C}_{jk}$, are given by the spreading widths derived using the pairs V_j, V_k.
- Using the notation for eigenvalues and vectors, $\{\lambda_j, \chi_j\}$ respectively, the total spreading width is given by

$$\Gamma_w = \sum_{j=1}^{6} \lambda_j \chi_j^2$$

but there aren't really 6 independent parameters.

Spreading widths

- The matrix elements in the model spaces associated with $\{h^1_\rho, h^2_\rho, h^1_\omega\}$ are correlated at the 95% level consistently across the measured nuclei.
- They are correlated with f_π at the 85% level.
- $\{h^0_\rho, h^0_\omega\}$ are correlated with each other at the 75% level.

★ As a result 4 of the eigenvalues are completely negligible. The leading eigenvalue is associated with the eigenvector

$$\chi = f_\pi - 0.25h^1_\rho + 0.10h^2_\rho + 0.50h^1_\omega$$

and 60-70 times greater than the second eigenvalue.

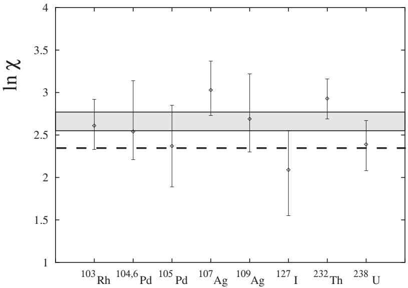
TRIPLE experiment and spreading widths

Table: Target nuclei parameters: number of asymmetries measured; neutron threshold energy (MeV); target ground state angular momentum; level density (eV); best values Fermi gas level density parameters $\{a(MeV^{-1}), \Delta(MeV)\}$; weak spreading width $(10^{-7}\ MeV)$.

Target	Asym.	S_n	I	D_J	a	Δ	Γ_w
^{103}Rh	4	7.000	$\frac{1}{2}$	61 ± 5	12.33	-1.40	$1.4^{+1.2}_{-0.6}$
^{104}Pd	1	7.094	0	220 ± 65	11.91	-0.78	$1.0^{+2.4}_{-0.5}$
^{105}Pd	3	9.562	$\frac{5}{2}$	22 ± 2	12.68	0.93	$0.8^{+1.3}_{-0.5}$
^{106}Pd	2	6.531	0	217 ± 61	13.28	-0.33	$1.0^{+2.4}_{-0.5}$
^{107}Ag	8	7.269	$\frac{1}{2}$	34 ± 4	13.22	-1.05	$2.7^{+2.6}_{-1.2}$
^{109}Ag	4	6.806	$\frac{1}{2}$	28 ± 3	13.98	-1.13	$1.3^{+2.5}_{-0.7}$
^{127}I	7	6.826	$\frac{5}{2}$	42 ± 3	12.82	-1.43	$0.6^{+0.9}_{-0.4}$
^{232}Th	16	4.786	0	19 ± 2	23.86	-0.59	$4.7^{+2.7}_{-1.8}$
^{238}U	5	4.806	0	21 ± 3	25.69	-0.07	$1.3^{+1.0}_{-0.6}$

TRIPLE experiment

- Effectively, one could account for the 2 greatest eigenvalues and vectors.
  - with exptl. uncertainties on M^2, the measurement, gives an ellipse of consistent values with the two eigenvectors.
- At right is plotted the TRIPLE data accounting for just the largest one ($\chi = f_\pi - 0.25h^1_\rho + 0.10h^2_\rho + 0.50h^1_\omega$)



- Cumulatively, TRIPLE coupled with stat. spec. implies $\chi = 14.3^{+1.7}_{-1.5}$, compared to the DDH(?) reasonable range $[-2.0, 10.3]$.
  - DDH underestimates couplings or statistical spectroscopy analysis needs improvement?

Some things to contemplate

- There is really only 1, possibly 2, parameters determining the scale of parity violation. This is a good thing for the TVPV ratio idea.
- A measurement of f_π (or some equivalent) could be used to benchmark the statistical calculations and guide which of the possible improvements must be incorporated:
  - better treatment of angular momentum
  - higher order moment corrections more generally
  - choice of spaces, single particle energies
  - use of effective operator theory or apply perturbative approach
- Statistical laws can be rather robust and forgiving. Matrix element errors that preserve interaction variances and covariances correct change nothing.