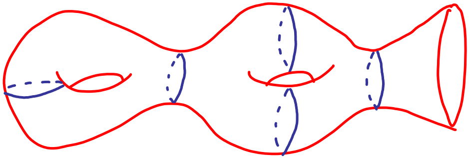
# Black Holes Microstates in Three Dimensional Gravity



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# The Puzzle:

Nature at low energies is well described by a local field theory:

- General Relativity + the Standard Model + ...

But the theories of quantum gravity we understand, like string theories, have vastly more degrees of freedom.

Are these extra degrees of freedom necessary?

- Do theories of pure (metric only) quantum gravity exist?
- Can you explain black hole entropy with only geometric degreres of freedom?

Let's study these questions in as simple a theory as possible.

# Three Dimensional Gravity

General Relativity in 2+1 dimensions has no local degrees of freedom but still has a rich structure (black holes, cosmology, AdS/CFT, etc.).

The two coupling constants

- Newton constant $G$
- Cosmological constant $\Lambda \sim -1/\ell^2 < 0$

can be combined into a dimensionless ratio $k = \frac{\ell}{16G} \sim \frac{1}{\hbar}$.

We are studying gravity in $AdS_3$, three dimensional Anti-de Sitter space.

AdS/CFT: A theory of quantum gravity in $AdS_3$ is dual to a two dimensional CFT with central charge $c = 24k$.

# A More Direct Approach:

We could study the landscape of CFTs with large central charge.

Instead, let us try to quantize gravity directly:

- Quantize a space of metrics to obtain a bulk Hilbert space.
- Compare to semi-classical expectations.

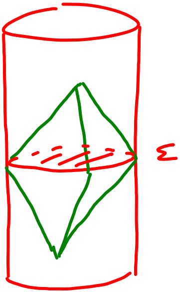
We will identify a class of black hole microstate geometries:

- Finite number of degrees of freedom coming from topology hidden behind the horizon.
- Count microstates explicitly.

The result will be compared with the semi-classical BH Entropy.

# Anti-de Sitter Space

$AdS_3$ is $\Sigma \times \mathbb{R}$, where $\Sigma = D^2$ is the disk.




The metric is

$$ds^2 = -dt^2 + \cos^2 t \; d\Sigma^2$$

where $d\Sigma^2$ is the negative curvature metric on the disk.

The boundary of the disk is at the boundary of AdS.

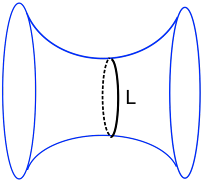
# The $AdS_3$ Black Hole

A simple family of solutions is of the form $\Sigma \times \mathbb{R}$:

$$ds^2 = -dt^2 + \cos^2 t \; d\Sigma^2$$

where $d\Sigma^2$ is the negative curvature metric on some surface $\Sigma$.

For example, when $\Sigma$ is a cylinder:

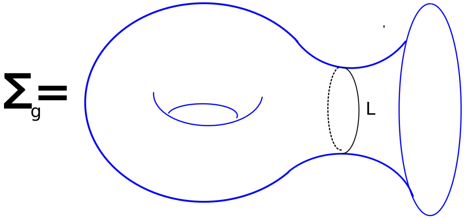
$\Sigma =$ 


the geometry is the AdS-black hole.

The two ends of the cylinder are the two asymptotic boundaries of the AdS black hole, separated by a horizon of size $L$.

# Microstate Geometries

Now take $\Sigma = \Sigma_g(L)$ to be a Riemann surface with one boundary:



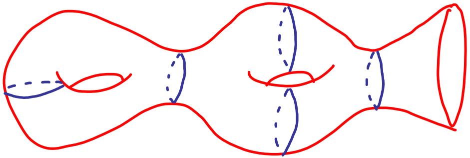

To an asymptotic observer, the geometry is *identical* to a black hole of area $L$.

But the other asymptotic region has been replaced by topology behind the horizon.

# The Phase Space

The configuration space is the Moduli space $\mathcal{M}_g(L)$ of Riemann surfaces $\Sigma_g(L)$.

Decomposing into pairs of pants, we have a length $L_i$ and a twist $\theta_i$ for each cuff.



So $\dim_{\mathbb{C}} \mathcal{M}_g(L) = 3g - 2$.

# Chiral Gravity

To quantize, we must consider a modification of general relativity known as Chiral Gravity, where we include a gravitational Chern-Simons coupling $\frac{1}{\mu} = \ell$.

Our microstate geometries are the most general known solutions of Chiral Gravity, up to gauge equivalence.

The symplectic structure is

$$\omega = k \left( dL \wedge d\theta + \sum_i dL_i \wedge d\theta_i \right)$$

where $L, \theta$ are the length and twist parameters of the horizon.

# Black Hole microstates

We can now quantize the phase space of black hole microstate geometries.

We will count the number of states as a function of $L$ and compare it to the Bekenstein-Hawking-Wald entropy:

$$S_{BH} = 2kL = 4\pi\sqrt{k\Delta} \ .$$

where

$$\Delta = \frac{1}{4\pi^2} kL^2 \in \mathbb{Z}$$

is the mass of the black hole.

It is quantized because $L$ is conjugate to a periodic variable $\theta$.

# Counting States

The number of states is (roughly) the volume of phase space:

$$N_g(k, \Delta) \approx \int_{\mathcal{M}_{g,1}} e^{k\kappa + \Delta\psi} = \sum_{d=0}^{3g-2} \frac{k^{3g-2-d}\Delta^d}{(3g-2-d)!\, d!}\, I_{g,d}$$

where

$$I_{g,d} \equiv \int_{\mathcal{M}_{g,1}} \kappa_1^{3g-2-d} \psi_1^d$$

is an intersection number on moduli space $\mathcal{M}_{g,1}$.

Algorithms for computing these intersection numbers were given by Witten-Kontsevich, Mirzakhani.

This allows us to understand their $g \to \infty$ asymptotics.

# The Exact Quantum States

The moduli space $\mathcal{M}_g(L)$ of *bordered* Riemann surfaces is symplectomorphic to the moduli space $\mathcal{M}_{g,1}$ of *punctured* Riemann surfaces, with

$$\omega = k\kappa_1 + \Delta\psi_1$$

where

- $\kappa_1$ is the Weil-Petersson class on $\mathcal{M}_{g,1}$
- $\psi_1$ is the Chern class of the cotangent at the puncture

The quantization of $k$ and $\Delta$ follow from the quantization condition: $[\omega] \in H^2(\mathcal{M}_{g,1}, \mathbb{Z})$.

A black hole microstate is a section of $\mathcal{L}_{k,\Delta}$ on $\mathcal{M}_{g,1}$, where

$$c_1(\mathcal{L}_{k,\Delta}) = k\kappa_1 + \Delta\psi_1 \ .$$

# The Results

The fixed genus result is too small to reproduce black hole entropy

$$N_g(k,\Delta) \approx \frac{1}{g!}\Delta^{3g-2} \ll e^{4\pi\sqrt{k\Delta}} .$$

so we must take $g = \mathcal{O}(\Delta)$.

The sum over genus

$$\begin{aligned} N(k,\Delta) &\equiv \sum_{g=0}^{\infty} N_g(k,\Delta) \\ &\approx \sum_{d=0}^{\infty} \frac{1}{d!(2d+1)!!}\left(\frac{\pi^2}{2}\frac{\Delta}{k}\right)^d \left(\sum_{g\gg d}(2g)!k^{3g} + \dots\right) \end{aligned}$$

is an asymptotic series.

# An Entropy Proportional to Area

Conjecture: The divergence is cured as usual, by resumming non-perturbative effects.

The result is an entropy linear in horizon area

$$N(k, \Delta) \approx C_g e^{\pi\sqrt{\Delta/k}} \approx e^{\pi L}$$

but with a coefficient which is too small:

- Entropy $\approx$ area in AdS units, not area in Planck units!

Quantizing geometry gives the spectrum of a CFT with $c = 6$.

# Conclusions

Quantizing topology behind the horizon leads to completely explicit, geometric black hole microstates.

At fixed genus we cannot reproduce black hole entropy.

- A large black hole can only be described by very complex topology behind the horizon.

The sum over genus gives an entropy proportional to horizon area

$$\log N(k, \Delta) \approx \pi\sqrt{\Delta/k} \ll 4\pi\sqrt{k\Delta} = S_{BH}$$

but with a coefficient which is too small.

Perhaps pure quantum gravity exists only when $k = 1/4$.