

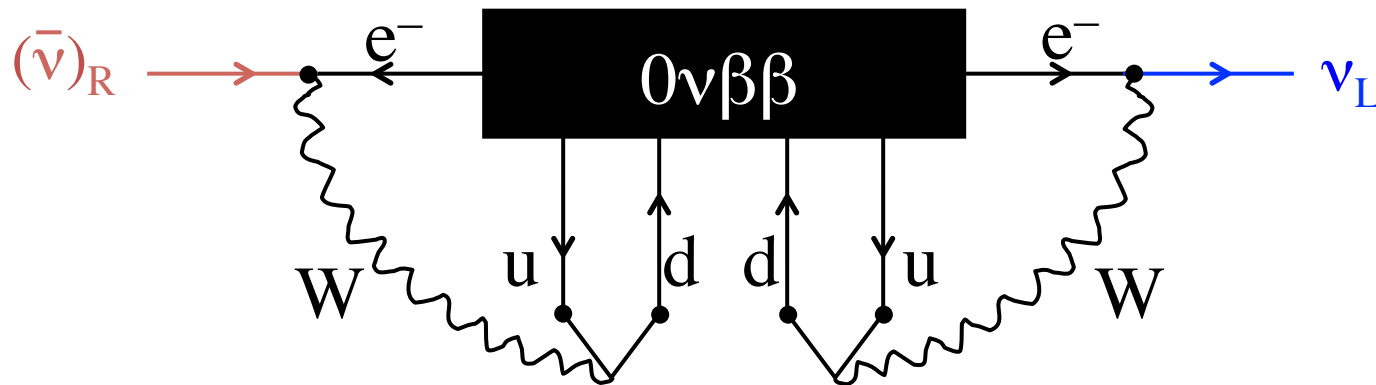
Lecture III: Majorana neutrinos

Petr Vogel, Caltech

NLDBD school, October 31, 2017

Whatever processes cause $0\nu\beta\beta$, its observation would imply the existence of a **Majorana mass term** and thus would represent ``**New Physics**'' :

Schechter and Valle,82



By adding only Standard model interactions we obtain

$$(\bar{\nu})_R \rightarrow (\nu)_L \text{ **Majorana mass term**}$$

Hence observing the $0\nu\beta\beta$ decay guaranties that ν are massive Majorana particles. But the relation between the decay rate and neutrino mass might be complicated, not just as in the see-saw type I.

The Black Box in the multiloop graph is an effective operator for neutrinoless double beta decay which arises from some underlying New Physics. It implies that neutrinoless double beta decay induces a non-zero effective Majorana mass for the electron neutrino, no matter which is the mechanism of the decay.

However, the diagram is almost certainly not the only one that generates a non-zero effective Majorana mass for the electron neutrino.

Duerr, Lindner and Merle in arXiv:1105.0901 have shown that evaluation of the graph, using $T_{1/2} > 10^{25}$ years implies that $\delta m_\nu = 5 \times 10^{-28}$ eV. This is clearly much too small given what we know from oscillation data. Therefore, other operators must give leading contribution to the neutrino masses.

QM of Majorana particles

Weyl, Dirac and Majorana relativistic equations:

Free fermions obey the Dirac equation: $(\gamma^\mu p_\mu - m)\Psi = 0$

Lets use the following representation of the γ matrices:

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Dirac equations can be then rewritten as two coupled two-component equations

$$\begin{aligned} -m\psi_- + (E - \vec{\sigma}\vec{p})\psi_+ &= 0 \\ (E + \vec{\sigma}\vec{p})\psi_- - m\psi_+ &= 0 \end{aligned}$$

Here $\psi_- = (1 - \gamma_5)/2 \Psi = \psi_L$, and $\psi_+ = (1 + \gamma_5)/2 \Psi = \psi_R$ are the chiral projections.

In the limit of $m \rightarrow 0$ these two equations decouple and we obtain two states with definite chirality and helicity: $\vec{\sigma} \cdot \hat{p} \psi_{\pm} = \pm \psi_{\pm}$

Thus, massless fermions obey the two-component Weyl equations

$$(E - \vec{\sigma} \cdot \vec{p}) \psi_+ = 0$$

$$(E + \vec{\sigma} \cdot \vec{p}) \psi_- = 0$$

The states $\psi_+ = \psi_R$ and $\psi_- = \psi_L$, so-called Weyl spinors, (also called van der Waerden spinors) transform independently under the two nonequivalent simplest representations of the Lorentz group.

For massive fermions there are two possible relativistic equations of motion.

1) The four component Dirac equation, or its equivalent two coupled two-component equations, with $\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$

2) Alternatively, as suggested by Majorana, there can be two nonequivalent, relativistic two-component equations

$$\begin{aligned} (E - \vec{\sigma}\vec{p})\psi_R - m\epsilon\psi_R^* &= 0 \\ (E' + \vec{\sigma}\vec{p}')\psi_L + m'\epsilon\psi_L^* &= 0 \end{aligned} \quad \text{where } \epsilon = i\sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

These two Majorana equations are totally independent, as indicated by different energies, momenta and masses.

Lets compare once more the Dirac and Majorana equations

$$\text{D: } (E - \vec{\sigma} \cdot \vec{p})\psi_R - m\psi_L = 0$$

$$\text{M: } (E - \vec{\sigma} \cdot \vec{p})\psi_R - m\epsilon\psi_R^* = 0$$

It is easy to see that they become identical if $m = 0$ as well if $\psi_L = \epsilon \psi_R^*$. Similarly for the other pair and $\psi_R = -\epsilon \psi_L^*$

$$\text{D: } (E + \vec{\sigma} \cdot \vec{p})\psi_L - m\psi_R = 0$$

$$\text{M: } (E + \vec{\sigma} \cdot \vec{p})\psi_L + m\epsilon\psi_L^* = 0$$

The four-component Dirac field is therefore equivalent to two degenerate, $m = m'$, two-component Majorana fields, with the corresponding relation between ψ_L and $\epsilon \psi_R^*$

Charge conjugation transformation:

Dirac bispinor $\Psi = \begin{pmatrix} \chi \\ \epsilon \xi^* \end{pmatrix}$ transforms as $\Psi_D^c = C \gamma^0 \Psi_D^* = \begin{pmatrix} \xi \\ \epsilon \chi^* \end{pmatrix}$

Charge conjugation matrix C in Weyl representation is $C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}$.

Therefore the Dirac bispinor ψ_D cannot be an eigenstate of charge conjugation unless $m = 0$.

In contrast, Majorana bispinor $\Psi_M = \begin{pmatrix} \chi \\ \epsilon \chi^* \end{pmatrix}$ with only two independent components

transforms as $\Psi_M^c = C \gamma^0 \Psi^* = \Psi_M$

In other words, it transforms to itself under charge conjugation.

This is so-called Majorana condition, ψ_M is identical with ψ_M^c .

In general, the Majorana field can be defined as

$$\chi(x) = [\psi(x) + \eta_C \psi^c(x)] / \sqrt{2}$$

By appropriate choice of the phase we obtain a field that is an eigenstate of charge conjugation with $\lambda_c = \pm 1$.

Neutrinos interact with chiral projection eigenstates.

The chirality, i.e. the eigenvalues of operators $(1 \pm \gamma_5)/2$ is a conserved, Lorentz invariant quantity for massive or massless particles. On the other hand, the helicity, the projection of spin on momentum, is not conserved. For massive particle there is always a frame in which the momentum points in the opposite direction. For massless particles chirality and helicity are identical.

But the chiral projections ψ_L and ψ_R do not obey the Dirac eq. unless $m=0$.

If ψ is a chirality eigenstate, i.e. $\gamma_5\psi = \lambda\psi$ then the charge conjugation state ψ^c is also an eigenstate, but with $\lambda^c = -\lambda$.

Also, if the Majorana state χ has the charge conjugation eigenvalue λ^c , the state $\gamma_5\chi$ has the opposite eigenvalue $-\lambda^c$.

Therefore, the charge conjugation eigenstates (Majorana states) cannot be simultaneously eigenstates of chirality.

The Majorana mass term is $m_L \overline{\nu_L} \nu_L^c$.

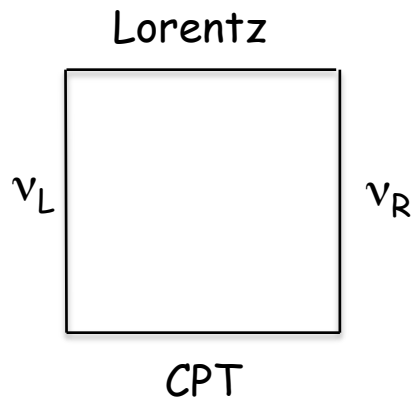
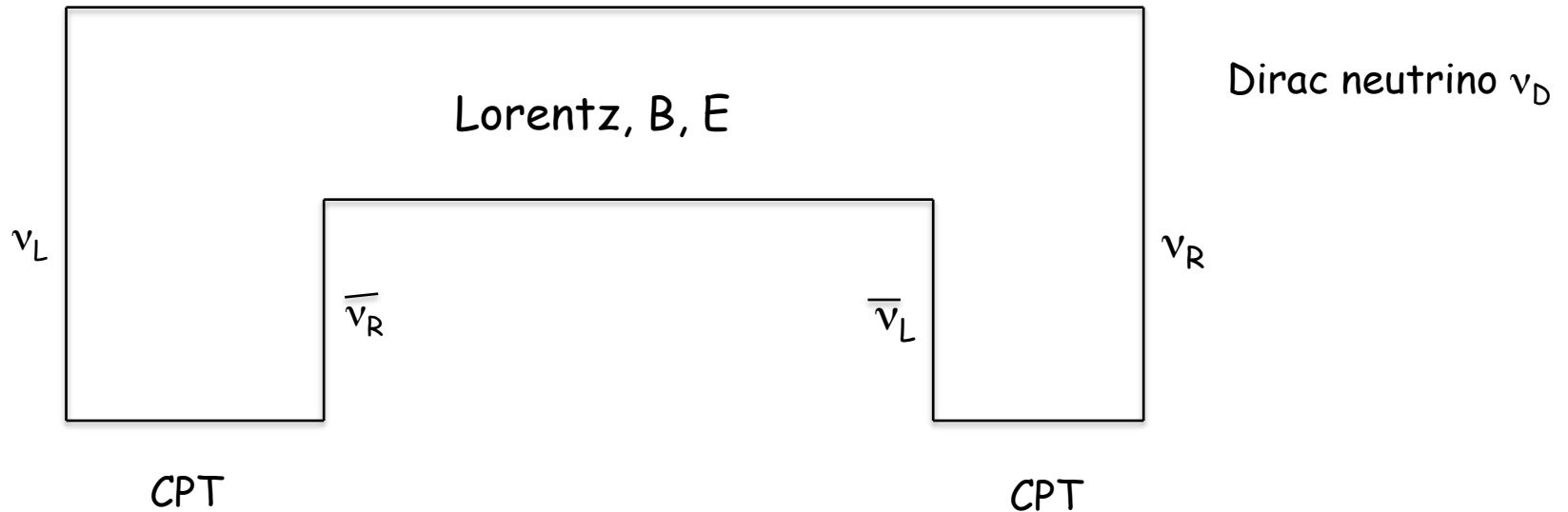
However, the objects ν_L and ν_L^c are not the mass eigenstates, i.e. the particles with definite mass. They are just the neutrinos in terms of which the model is constructed. The mass term $m_L \overline{\nu_L} \nu_L^c$ induces mixing of ν_L and ν_L^c .

As a result of the $\nu_L \leftrightarrow \nu_L^c$ mixing, the neutrino

mass eigenstates are $\nu_i = \nu_L + \nu_L^c = \nu + \overline{\nu}$.

These mass eigenstates are explicitly charge conjugation eigenstates. However, they do not have fixed chirality.

Four distinct states of a massive Dirac neutrino and the transformation among them. ν_L can be converted into the opposite helicity state by a Lorentz transformation, or by the torque exerted by an external B or E field.



There are only two distinct states of a Majorana neutrino ν_M . Under the Lorentz transformation ν_L is transformed into the same state ν_R as by the Lorentz transformation. The dipole magnetic and electric moments must vanish.

Number of parameters in the mixing matrix. Why there are more CP phases for Majorana neutrinos?

CKM matrix for quarks: In the quark mass eigenstate basis one can make a phase rotation of the **u-type** and **d-type** quarks, thus $V \rightarrow e^{i\Phi(u)} V e^{-i\Phi(d)}$, where $\Phi(u) = \text{diag}(\phi_u, \phi_c, \phi_t)$, etc.

The $N \times N$ unitary matrix V has N^2 parameters. There are $N(N-1)/2$ CP-even angles and $N(N+1)/2$ CP-odd phases. The rephasing invariance above removes $(2N-1)$ phases, thus $(N-1)(N-2)/2$ CP-odd phases are left.

So, for $N=3$ there are 3 angles and 1 CP phase. This is all one can determine in experiments that **do not violate** the lepton number conservation.

The usual convention is to have the angles θ_i in $[0, \pi/2]$ and the phases δ_i in $[0, 2\pi]$.

Now for Majorana neutrinos:

Consider N massive Majorana neutrinos that belong to the weak doublets L_i . In addition there are (presumably) also N weak singlet neutrinos, that in the see-saw mechanism are heavy (above the electroweak scale).

In the **low-energy effective theory** there are only the active neutrinos, with the mixing matrix U invariant under

$$U \rightarrow e^{-i\Phi(E)} U \eta_\nu$$

Here $\Phi(E)$ involves the free phases of the charged leptons and η_n is a diagonal matrix with allowed eigenvalues $+1$ and -1 . It takes into account the allowed rephasing for Majorana fields.

Thus U contains $N(N-1)/2$ angles in $[0, \pi/2]$, $(N-1)(N-2)/2$ 'Dirac' CP-odd phases and $(N-1)$ additional 'Majorana' CP-odd phases. ($N(N-1)/2$ phases altogether.) These phases are in $[0, 2\pi]$.

The matrix U (often called PMNS for $N=3$ generations) is responsible for neutrino oscillations in low-energy experiments.

How can we tell whether the total lepton number is conserved?

A partial list of processes where the lepton number would be violated:

Neutrinoless $\beta\beta$ decay: $(Z,A) \rightarrow (Z\pm 2,A) + 2e^{(\pm)}$, $T_{1/2} > \sim 10^{26} \text{ y}$

Muon conversion: $\mu^- + (Z,A) \rightarrow e^- + (Z-2,A)$, $\text{BR} < 10^{-12}$

Anomalous kaon decays: $K^+ \rightarrow \pi^- \mu^+ \mu^+$, $\text{BR} < 10^{-9}$

Observing any of these processes would mean that the lepton number is not conserved, and that neutrinos are massive Majorana particles.

In contrast production at LHC of a pair of the same charge leptons, with no missing energy, through production of doubly charged scalar that decays that way might test the lepton number violation at the corresponding scale, without the m_ν/E_ν suppression.

Lets look at this list some more.

The $0\nu\beta\beta$ decay $T_{1/2} \sim 10^{26}$ years for ^{136}Xe represents, in fact the branching ratio of only 2×10^{-5} , since the total lifetime of ^{136}Xe is determined by the very long lived $2\nu\beta\beta$ decay, with $T_{1/2} = 2 \times 10^{21}$ y. So, the branching ratio is not a good characteristic.

Muon conversion $\mu^- + (Z,A) \rightarrow e^- + (Z-2,A)$ with branching ratio 10^{-12} corresponds to the partial lifetime $T_{1/2} = 2.2 \times 10^3$ s, where I took just the free muon half-life as the total decay time.

Similarly, the kaon decay branch $K^+ \rightarrow \pi^0 \mu^+ \mu^-$, with branching ratio 10^{-9} corresponds to the partial decay time of 12 s.

Clearly, $0\nu\beta\beta$ decay dominates by a huge margin. That is so because many mols of the target can be studied for a long time, and the Avogadro number 6×10^{23} is much larger than typical beams. For example, Fermilab produces a few $\times 10^{20}$ protons per year on target.

How difficult it would be to observe the lepton number violation in other channels than the $0\nu\beta\beta$ decay can be illustrated by considering e.g. the process $e^- + A^Z \rightarrow \mu^+ + A^{Z-2}$, or related, $\mu^- + A^Z \rightarrow e^+ + A^{Z-2}$.

Lim, Takasugi and Yoshimura, *Progr. Theor. Phys.* **113**, 1367 (2005) evaluated the cross section assuming that neutrinos are Majorana fermions. As an example, for $Z=50$, $A=100$ they obtain

$$\sigma \sim 5 \times 10^{-65} \text{ cm}^2 (m_{e\mu}/100 \text{ eV})^2,$$

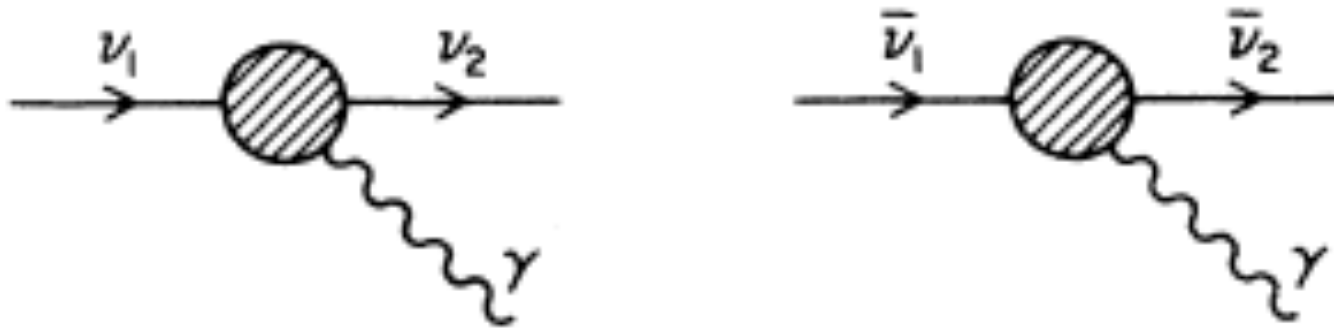
Reflecting the belief at that time that the electron neutrino mass could be ~ 30 eV. This is ~ 25 orders of magnitude less than the weak interaction cross section for a low energy neutrinos.

,

Are there any other possible way to distinguish Dirac from Majorana neutrinos? Yes, in principle.

Lets consider first a rather hypothetical example:

In the neutrino γ decay there are two graphs that may interfere for Majorana neutrinos, but only one for the Dirac neutrinos.



Angular distribution of photons in the lab system with respect to the neutrino beam direction is then

$$dN = \frac{1}{2}(1 + a \cos\theta)d \cos\theta ,$$

where $a = 0$ for Majorana and $a = -1$ for Dirac and left handed couplings. This process is unobservable in practice. Even if $m_{\nu_1} \gg m_{\nu_2}$ and the mixing angles are large, $1/\tau \sim (m_{\nu_1}/\text{eV})^5 \times 10^{-37} \text{ years}^{-1}$, more than 25 orders of magnitude longer than the age of the Universe.

Neutrino magnetic moments, and the distinction between the Dirac and Majorana neutrinos:

In the following I will describe a model independent constraint on the μ_ν that depends on the magnitude of m_ν and moreover depends on the charge conjugation properties of neutrinos, i.e. makes it possible, at least in principle, to decide between Dirac and Majorana nature of neutrinos.

But, before doing that I will describe how the neutrino magnetic moments μ_ν can be measured, what the present limits are, and what are the interesting related issues.

How can one measure μ_ν ?

Magnetic moment could be observed in ν -e scattering by looking at the electron recoil spectrum; the scattered neutrino is not observed. The electromagnetic cross section has a characteristic shape

$$d\sigma_{elm}/dT = \pi\alpha^2 \mu_\nu^2/m_e^2 (1-T/E_\nu)/T ,$$

where T is the recoil electron kinetic energy.

There is a singularity as the electron recoil kinetic energy $T \rightarrow 0$.

Nonvanishing μ_ν will be recognizable only if the σ_{elm} is comparable or larger in magnitude with the well understood weak interaction cross section, of magnitude $d\sigma_w/dT \sim 2G_F^2 m_e/\pi$ for small T/E_ν .

The magnitude of μ_ν which can be probed in this way is then given by, obtained by equating $\sigma_{elm} = \sigma_{weak}$

$$\frac{|\mu_\nu^{\text{exp}}|}{\mu_B} \simeq \frac{G_F m_e}{\sqrt{2}\pi\alpha} \sqrt{m_e T} \sim 10^{-10} \sqrt{\frac{T}{m_e}} .$$

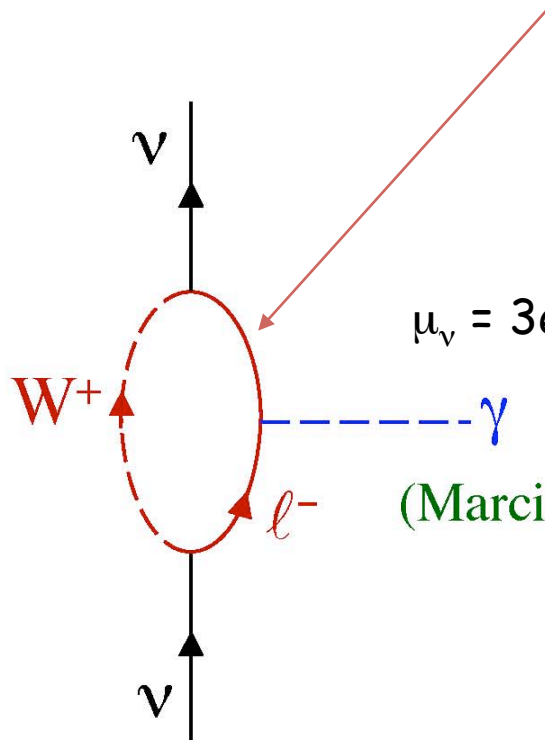
Considering realistic values of T it would be difficult to go beyond $\mu_\nu \sim 10^{-11} \mu_B$ this way. Present limits are indeed close to that value.

Limits on μ_ν can be also derived from bounds on unobserved energy loss in astrophysical objects. For sufficiently large μ_ν the rate of plasmon decay into $\nu \bar{\nu}$ pairs would conflict such bounds. However, since plasmons can also decay weakly into $\nu \bar{\nu}$ pairs the sensitivity of this probe is again limited by the size of the weak rate, leading to

$$\frac{|\mu_\nu^{\text{astro}}|}{\mu_B} \simeq \frac{G_F m_e}{\sqrt{2}\pi\alpha} (\hbar\omega_P)$$

where ω_p is the plasmon frequency. Since $(\hbar\omega_p)^2 \ll m_e T$, this bound is somewhat stronger than the limit from ν - e scattering, a few $\times 10^{-12} \mu_\nu/\mu_B$ but with less obvious model independence.

Neutrino mass and magnetic moment are intimately related. In the orthodox SM with massless neutrinos magnetic moments vanish. However, in the minimally extended SM with a Dirac neutrino of mass m_ν the loops like this produce an unobservably small, but nonvanishing dipole magnetic moment



$$\mu_\nu = \frac{3eG_F}{(2^{1/2} \pi^2 8)} m_\nu = 3 \times 10^{-19} m_\nu / eV \mu_B$$

It is customary to express μ_ν in units of the electron Bohr magneton.

(Marciano, Sanda; Lee, Shrock; Fujikawa, Shrock) (1977)

Magnetic moments are measured in magnetons, $eh/2mc$ with dimension $e \times \text{length}$. The expression above must be multiplied by hc to get it in the proper units. $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$, $hc = 2 \times 10^{-14} \text{ GeV cm}$, $\mu_\nu = 6.27 \times 10^{-30} e \text{ cm} (m_\nu/eV)$, $\mu_B = eh/2m_e c = 2 \times 10^{-11} e \text{ cm}$.

In analogy to the Schechter-Valle theorem, the existence of neutrino magnetic moment (coupling to elm. field), implies that neutrinos have mass. For the Dirac neutrino case the leading contribution to the m_ν is

$$m_\nu \sim \frac{\alpha}{16\pi} \frac{\Lambda^2}{m_e} \frac{\mu_\nu}{\mu_B} \sim \frac{\mu_\nu}{3 \times 10^{-15} \mu_B} [\Lambda(\text{TeV})]^2 \text{ eV.}$$

Where Λ is the scale of the new physics generating the μ_ν (divergent as Λ^2 arising from the dim = 4 neutrino mass operator).

Thus if $\Lambda \sim 1 \text{ TeV}$, $\mu_\nu \sim 10^{-15} \mu_B$, orders of magnitude below the current limits.

However, when Λ is not much larger than v ($v = 245 \text{ GeV}$), the contributions to m_ν from higher dimension operators can be important and should be considered.

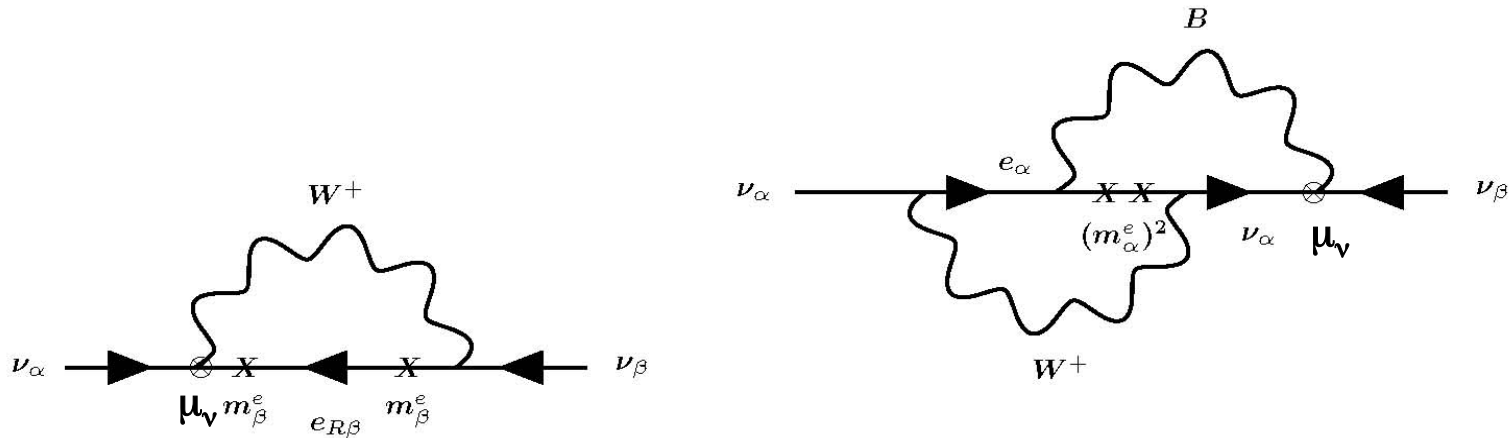
For details, including the contribution to the dimension 6 neutrino mass operator, see Bell *et al*, Phys. Rev. Lett. **95**, 151802(2005).
The final expression, in the absence of fine tuning (accidental cancellations) is

$$\mu_\nu \leq 8 \times 10^{-15} \mu_B (m_\nu/1 \text{ eV}) \quad \text{for } \Lambda \geq 1 \text{ TeV}$$

Thus, given the limits on m_ν , observation of μ_ν for Dirac neutrinos is unlikely.

The case of Majorana neutrinos is more subtle due to the relative flavor symmetries of m_ν (**symmetric**) and μ_ν (**antisymmetric**).

The one loop contributions to the Majorana neutrino mass associated with the neutrino magnetic moment sum to zero for them. (Davidson, Gorbahn and Santamaria, Phys. Lett. **B626**, 151 (2005))



In order to get a nonvanishing contribution to the Majorana neutrino mass associated with the magnetic moment one has to make charged lepton mass insertions X . The resulting δm_ν is smaller since it contains the differences between the (small) charged lepton Yukawa couplings (factor $m_\alpha^2 - m_\beta^2$). The most general bound on the transition magnetic moment of Majorana neutrino is

$$\mu_{\alpha\beta} \leq 4 \times 10^{-9} \mu_B \left(\frac{[m_\nu]_{\alpha\beta}}{1 \text{ eV}} \right) \left(\frac{1 \text{ TeV}}{\Lambda} \right)^2 \left| \frac{m_\tau^2}{m_\alpha^2 - m_\beta^2} \right|,$$

(N. Bell et al, Phys. Lett. **B642**, 377(2006))

Hence the constraints on the μ_ν of Majorana neutrinos are much weaker than for the Dirac neutrinos and easily compatible with the present experimental sensitivities.

Thus, if a neutrino magnetic moment is observed near its present experimental limit we would conclude that neutrinos are Majorana, and that the corresponding new scale $\Lambda < 100 \text{ TeV}$.

If we, further, could assume that all elements of the matrix $\mu_{\alpha\beta}$ are of similar magnitude, than a discovery of μ_ν at, say $10^{-11}\mu_B$ would imply $\Lambda < 10 \text{ TeV}$ with a possible implication for the mechanism of $0\nu\beta\beta$ decay.

Hence search for μ_ν is in some sense complementary to the search for $0\nu\beta\beta$ decay. But, unlike the $0\nu\beta\beta$ decay, we have just an upper bound, and not a clear map where to look.

spares

Basic formulae of the Standard Model:

The gauge bosons are W_μ^i , $i=1,2,3$ and B_μ .

After the spontaneous symmetry breaking, the massless photon field is

$$A = B \cos\theta_W + W^3 \sin\theta_W ,$$

and the massive neutral weak boson field is

$$Z = -B \sin\theta_W + W^3 \cos\theta_W ,$$

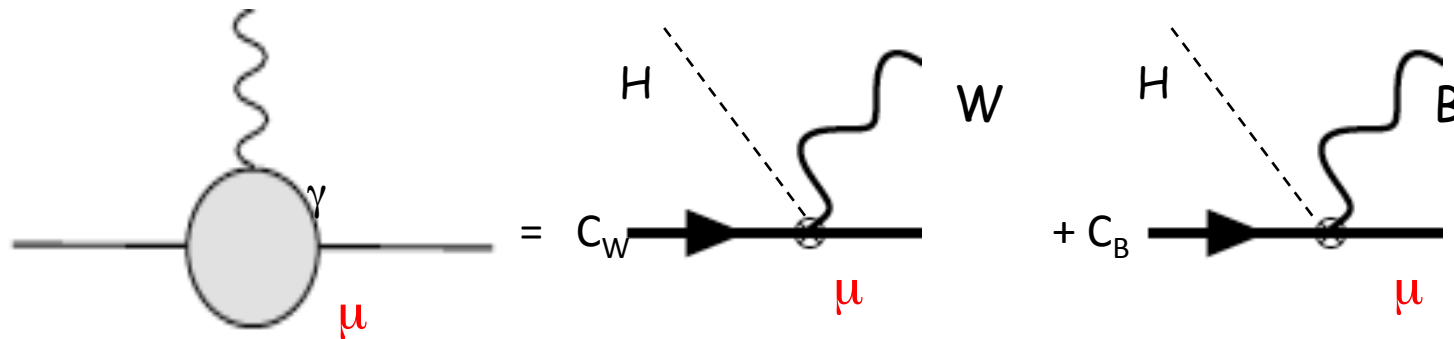
While the massive charged weak boson fields are

$$W^{\pm} = (W^1 \mp W^2) / \sqrt{2}$$

Masses of fermions are equal to the products of the Yukawa couplings and the Higgs vacuum expectation value $v = 246 \text{ GeV}$,
 $m = c_y v / \sqrt{2}$.

Thus for the electron, muon and tau $c_y = 2.9 \times 10^{-6}$, 6×10^{-4} , 0.010 , while for an 1 eV neutrino c_y would be 5.7×10^{-12} .

To sketch the derivation, note that the usual graph for μ_ν can be expressed in a gauge invariant form (H is the Higgs vacuum expectation value, and B is the isosinglet gauge field):



One can now close the loop and obtain a quadratically divergent contribution to the Dirac mass



For details, including the contribution to the dimension 6 neutrino mass operator, see Bell *et al*, Phys. Rev. Lett. **95**, 151802(2005).

The final expression, in the absence of fine tuning (accidental cancellations) is $\mu_\nu \leq 8 \times 10^{-15} \mu_B (m_\nu/1 \text{ eV})$ for $\Lambda \geq 1 \text{ TeV}$

Given the limits on m_ν , observation of μ_ν for Dirac neutrinos is unlikely.

It is often convenient to express the solutions of the Majorana equations in the four component form.

To do that use the fact that $-\epsilon\psi_L^*$ is also the solution of the Majorana equation for ψ_L .

In analogy, the $\epsilon\psi_R^*$ is the solution of the equation for ψ_R .

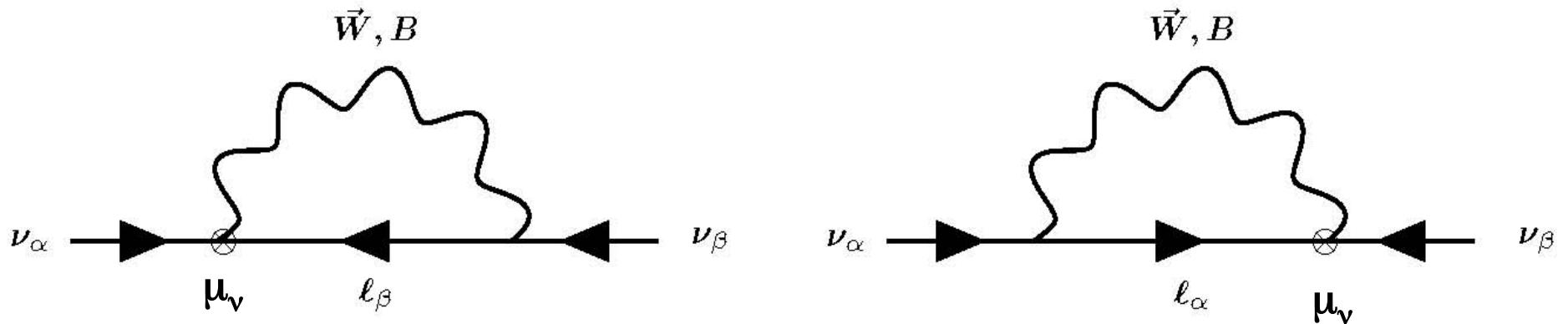
Thus, the four component solutions of Majorana equations (however, with only two independent components) are

$$\Psi_L = \begin{pmatrix} -\epsilon\psi_L^* \\ \psi_L \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} \psi_R \\ \epsilon\psi_R^* \end{pmatrix}$$

The four-component form of the Majorana fields formally obeys the Dirac equation, provided we use the relation

$$\psi_L = \epsilon \psi_R^* \text{ or } \psi_R = -\epsilon \psi_L^*.$$

The case of Majorana neutrinos is more subtle due to the relative flavor symmetries of m_ν (symmetric) and μ_ν (antisymmetric).



These one loop contributions to the Majorana neutrino mass associated with the neutrino magnetic moment sum to zero. (Davidson, Gorbahn and Santamaria, Phys. Lett. **B626**, 151 (2005))

Magnetic moments and distinction between the Dirac and Majorana neutrinos

A *Majorana* neutrino cannot have a magnetic or electric dipole moment:

$$\vec{\mu} \left[\begin{array}{c} \uparrow \\ e^+ \end{array} \right] = - \vec{\mu} \left[\begin{array}{c} \uparrow \\ e^- \end{array} \right]$$

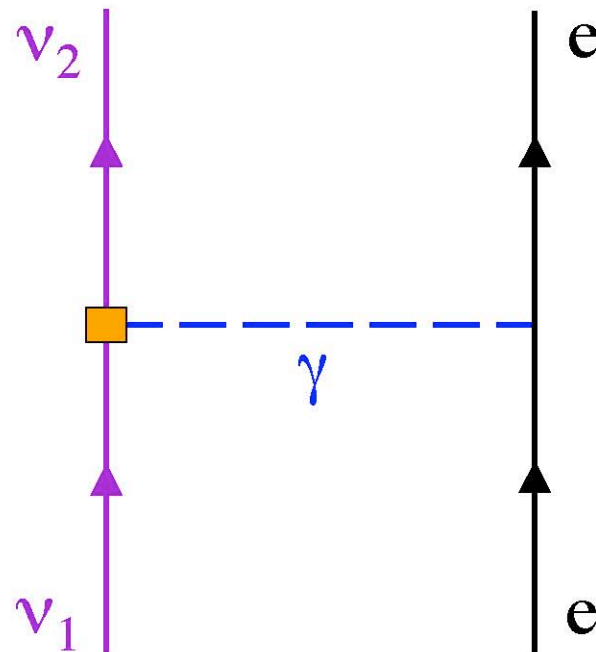
For Dirac fermions mag.mom. changes sign under the charge conjugation.

But for a Majorana neutrino, $\bar{\nu}_i = \nu_i$

$$\text{Therefore for them } \vec{\mu} \left[\bar{\nu}_i \right] = \vec{\mu} \left[\nu_i \right] = 0$$

Thus, only *Dirac* neutrino can have diagonal magnetic moments, *Majorana* neutrinos cannot have them, however both can have transition magnetic Moments.

Both *Dirac* and *Majorana* neutrinos can have *transition* dipole moments, leading to —

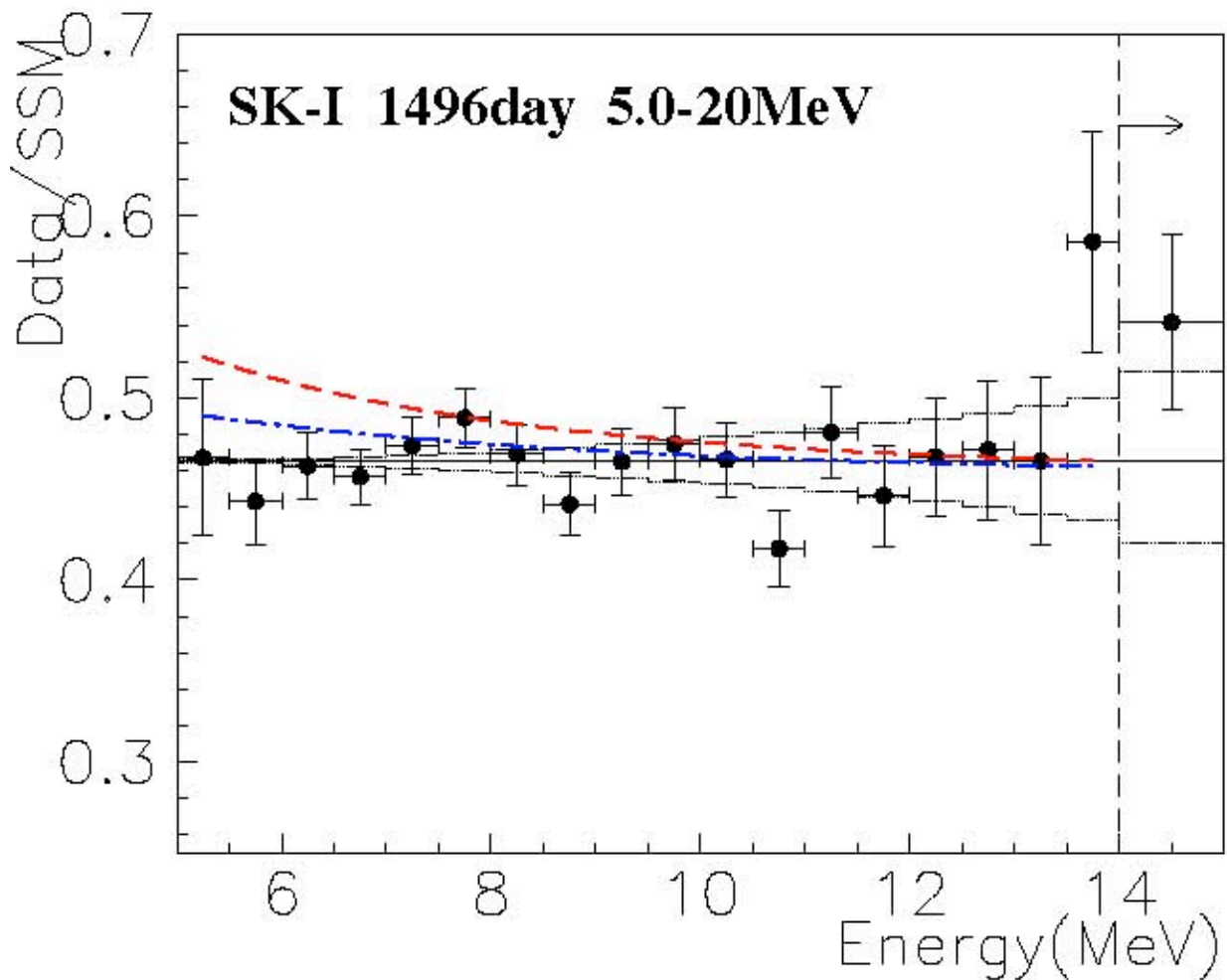


In ν - e scattering when the scattered neutrino is not observed, one cannot separate the effects of diagonal and transition magnetic dipole moments.

One can look for the dipole moments this way.

To be visible, they would have to *vastly exceed* Standard Model predictions.

Another way to obtain a limit on the μ_ν is from the analysis of the scattering of solar neutrinos on electrons observed in the SuperKamiokande experiment. There are subtleties in that problem, because solar neutrinos are affected by the matter effects, the resulting μ_ν is not necessarily the same one as for the reactor neutrinos (see Beacom and Vogel, Phys. Rev. Lett. **83**, 5222 (1999)).



Plotted is the ratio of
The observed rate to the
expected rate with no
oscillations.
The dashed red line is
obtained when $\mu_\nu = 1.1 \times 10^{-10}$
 μ_B is added to the
oscillation signal.

Even somewhat better
limit, $\mu_\nu < 5.4 \times 10^{-11} \mu_B$, was
obtained from the
analysis of Borexino data,
dominated by the lower
energy signal from the
 ${}^7\text{Be}$ decay (Phys. Rev. Lett.
101, 091302 (2008)).