Lecture I: $0\nu\beta\beta$ decay and $m_{\beta\beta}$

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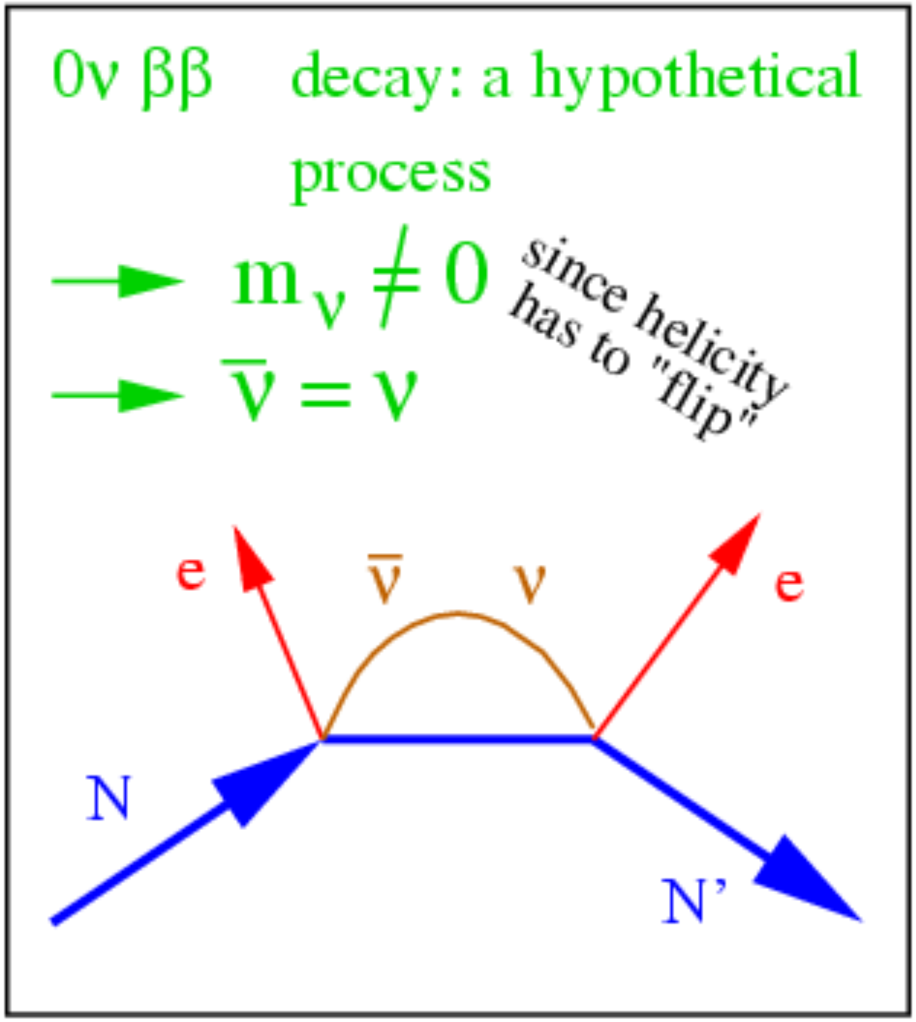
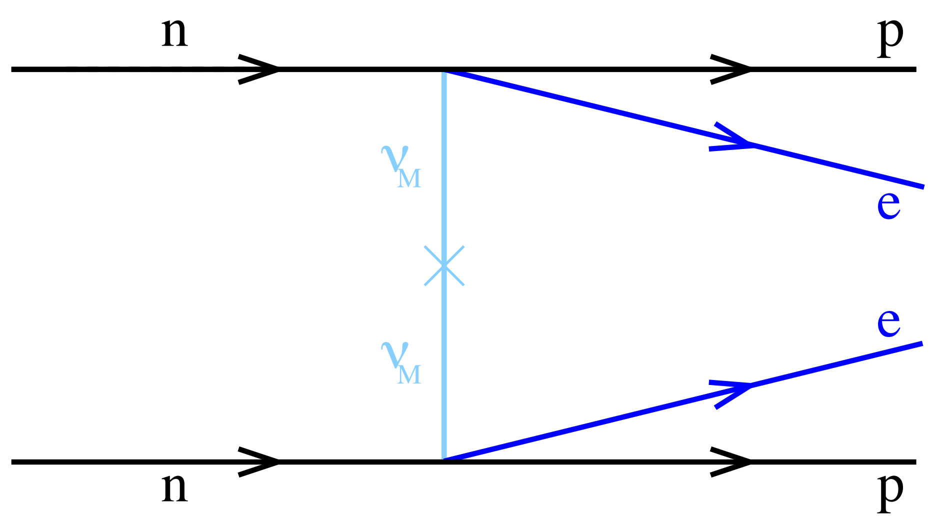
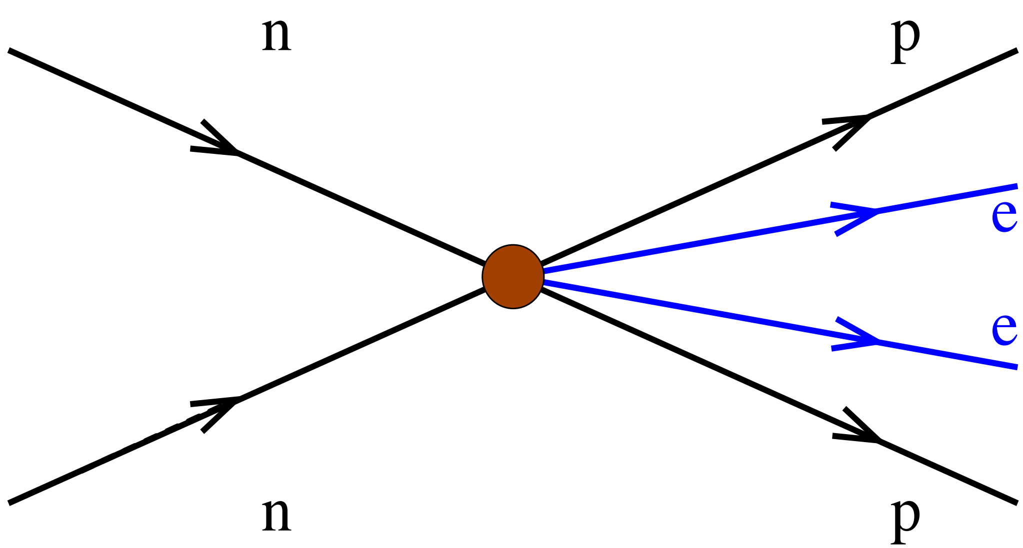


Schematic representation of the $0\nu\beta\beta$ decay. The exchanged virtual neutrino is supposed to have only the standard model weak interactions. The indicated properties immediately follow.



A more formal picture of the $0\nu\beta\beta$ decay. Since the exchanged neutrino is light, the corresponding range is long. Neutrino mass here is associated with The **See-saw type I** mechanism and **$m_\nu \sim v^2/M_N$,** where M_N is the very heavy sterile neutrino mass.



There is, however, another possibility. The short-range, involving an exchange of some heavy, often new, particle. This is therefore effectively a contact four nucleon vertex, represented by a dimension 9 operator. The physics of this type of lepton number violation is present in the **See-saw type II** or **type III** models. Depending on the parameters of these models, the corresponding half-life could be as low or as high at the half-life involved in the light Majorana neutrino exchange.

In the light neutrino exchange, based on the above See-Saw type I, the decay rate is expressed as a product of three factors:

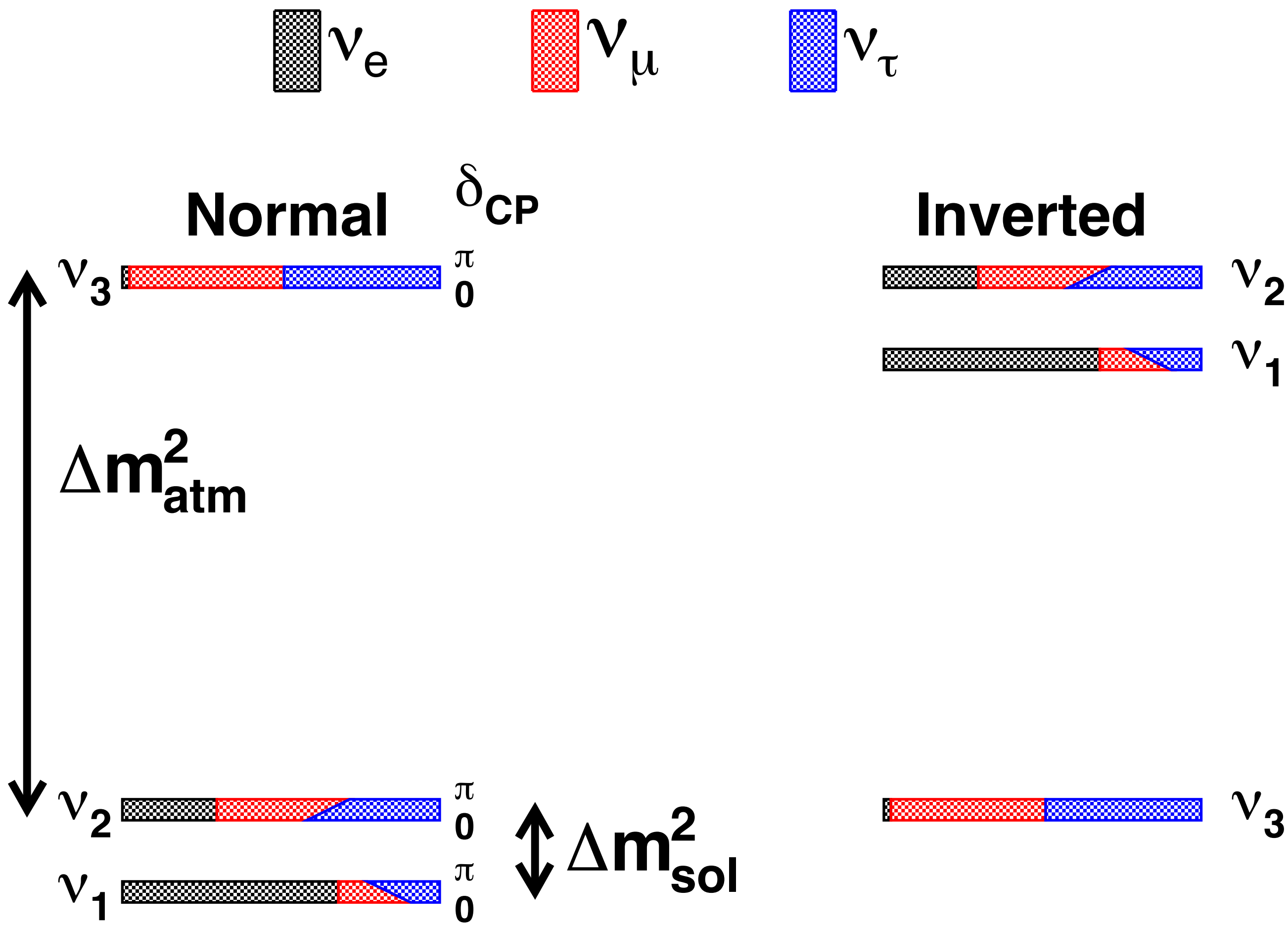
$$1/T_{1/2}^{0\nu} = G^{0\nu}(Q,Z)\, |M^{0\nu}|^2\, |\langle m_{\beta\beta}\rangle|^2, \quad \langle m_{\beta\beta}\rangle = |\Sigma_i U_{ei}^2\, m_i| \;,$$

which represents a simple relation between the decay rate and the parameters of the neutrino mass matrix.
The matrix elements of the first row of the PMNS matrix are, in general, complex numbers, thus U_{ei}^2 are also complex.

In the standard representation
$U_{e1} = \cos\theta_{13}\cos\theta_{12}$, $U_{e2} = \cos\theta_{13}\sin\theta_{12}\, e^{i\alpha}$, $U_{e3} = \sin\theta_{13}\, e^{i\beta}$,
where α and β are unknown Majorana phases.

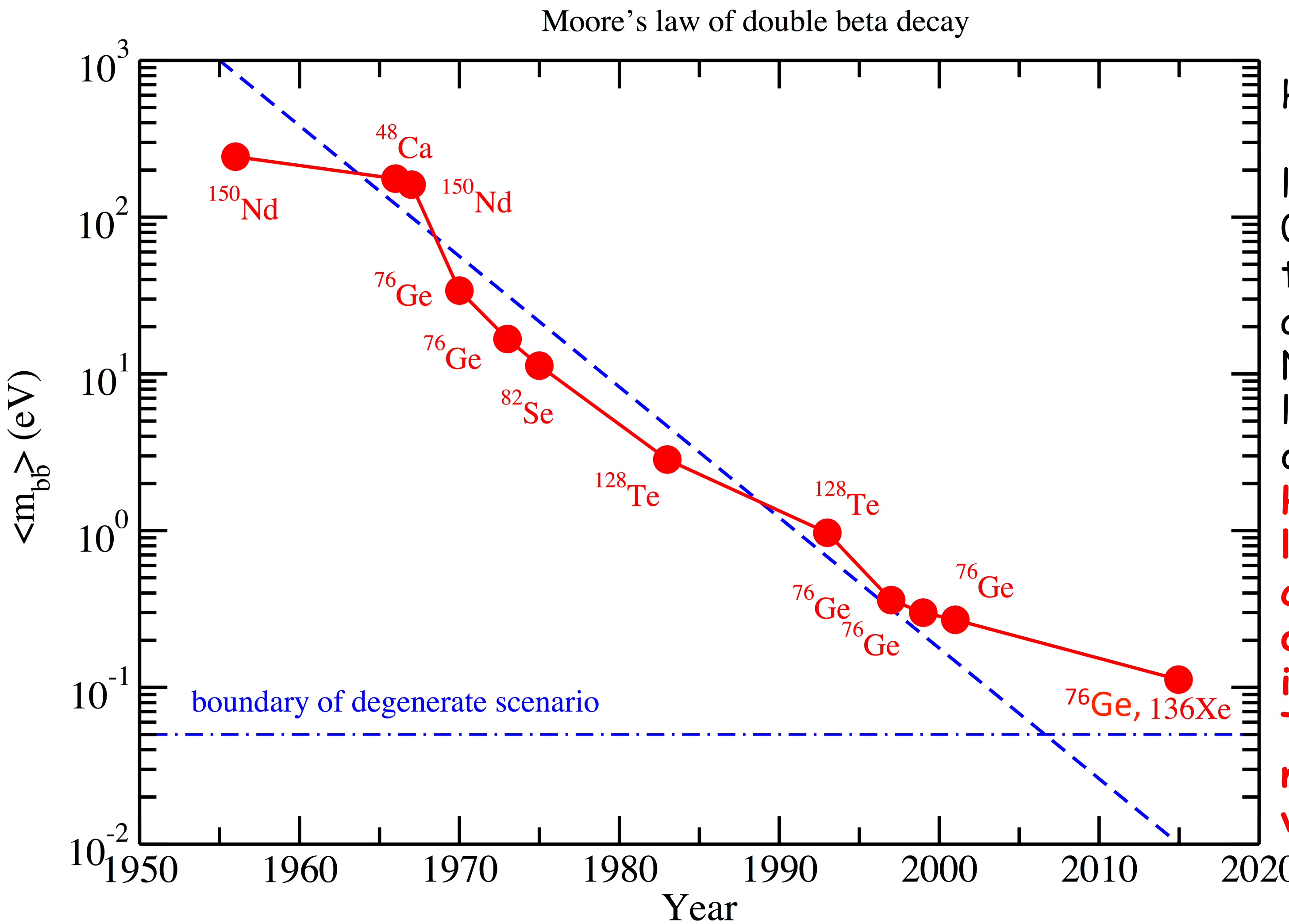
The mass squared differences Δm^2_{21} and Δm^2_{32} have been measured quite accurately, and the three mixing angles are known as well. However, we do not know the actual absolute neutrino mass, and the mass ordering (or hierarchy).

Neutrino Mass Hierarchy

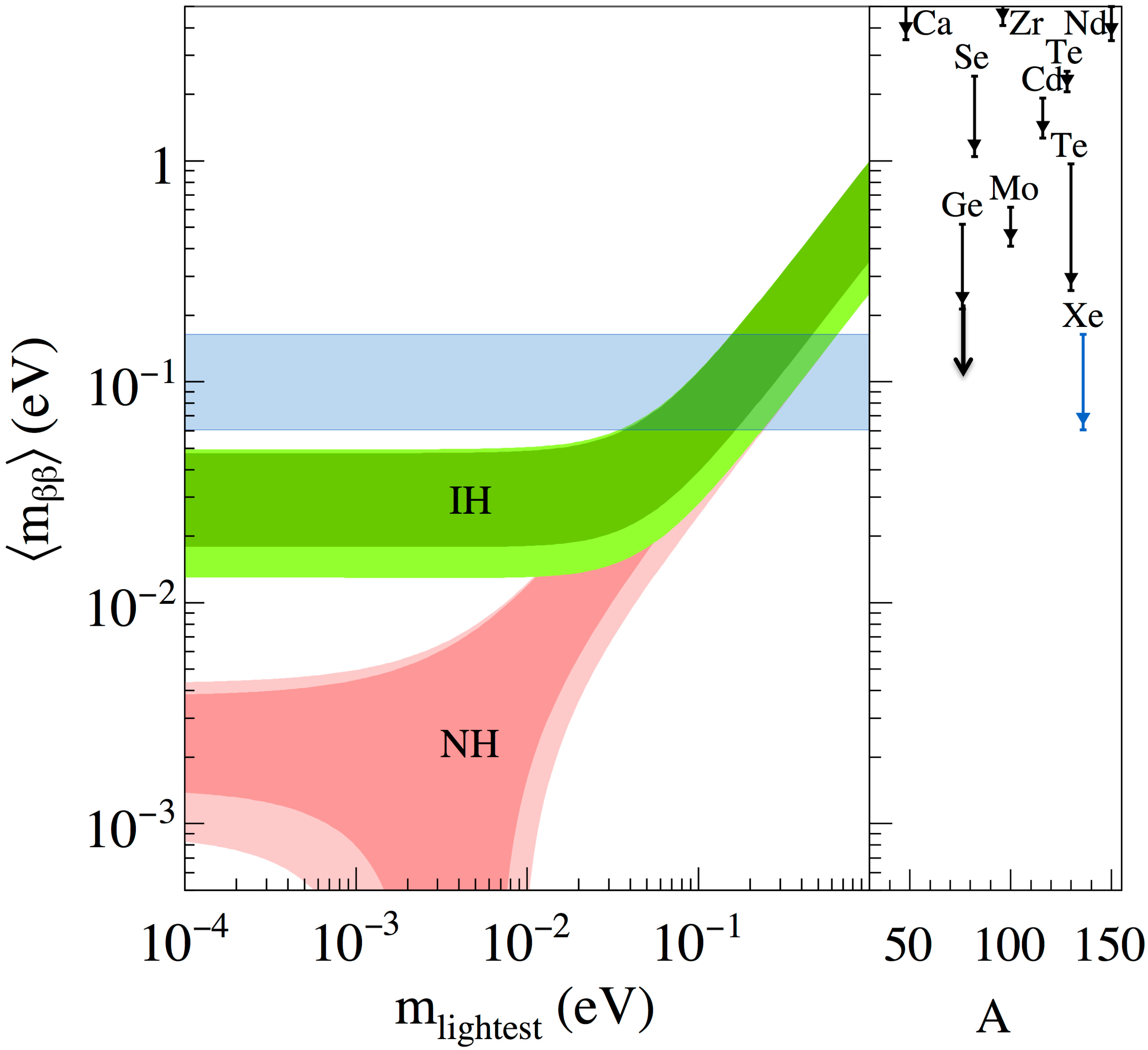


We do not know which of these two actually exists in nature.

History of $0\nu\beta\beta$ decay



Historically, there are > 100 experimental limits on $T_{1/2}$ of the $0\nu\beta\beta$ decay. Here are the records expressed as limits on $\langle m_{\beta\beta} \rangle$. Note the approximate linear slope vs time on such semilog plot.
However, during the last decade the complexity and cost of such experiments increased dramatically The constant slope is no longer obviously visible.



<$m_{\beta\beta}$> as a function of the mass of the lightest meutrino. Normal hierarchy in red, inverted hierarchy in green. The reach of the best experiments is indicated by the blue band. The sensitivity of the different tests is indicated in the right panel by the corresponding nuclei.

From Engel and Menendez

Note as a curiosity:
$\langle m_{\beta\beta} \rangle$ may vanish even though all m_i are nonvanishing and all ν_i are Majorana neutrinos.

What can we do in that case?

In principle, although probably not in practice, we can look for the lepton number violation involving muons.

Numerical example: take $\theta_{13} = 0$, and Majorana phase $\alpha_2 - \alpha_1 = \pi$ (only for this choice of phases can $\langle m_{\beta\beta} \rangle$ vanish when $\theta_{13} = 0$).
$\langle m_{\beta\beta} \rangle = 0$ if $m_1/m_2 = \tan^2\theta_{12}$, with $m_2 = (m_1^2 + \Delta m_{sol}^2)^{1/2}$.
That happens for $m_1 = 4.58$ meV and $m_2 = 10$ meV
(this is, therefore, fine tuning).
But then $\langle m_{\mu e} \rangle = \sin 2\theta_{12} \cos\theta_{23}/2 \times (m_1 + m_2) = 4.78$ meV,
Which is, at least in principle, observable using
$\mu^- + (Z,A) \rightarrow e^+ + (Z-2,A)$.

What are, in general, methods to determine m_ν?

Neutrino oscillations: $\Delta m^2_{21} = m^2_2 - m^2_1$, etc.

observed $\sim 10^{-5}$ eV^2 (only mass square differences, independent of Dirac vs. Majorana)

Single beta decay:
0.2 eV (independent of Dirac vs. Majorana)

$$\langle m_\beta \rangle^2 = \Sigma\, m_i^2\, |U_{ei}|^2$$

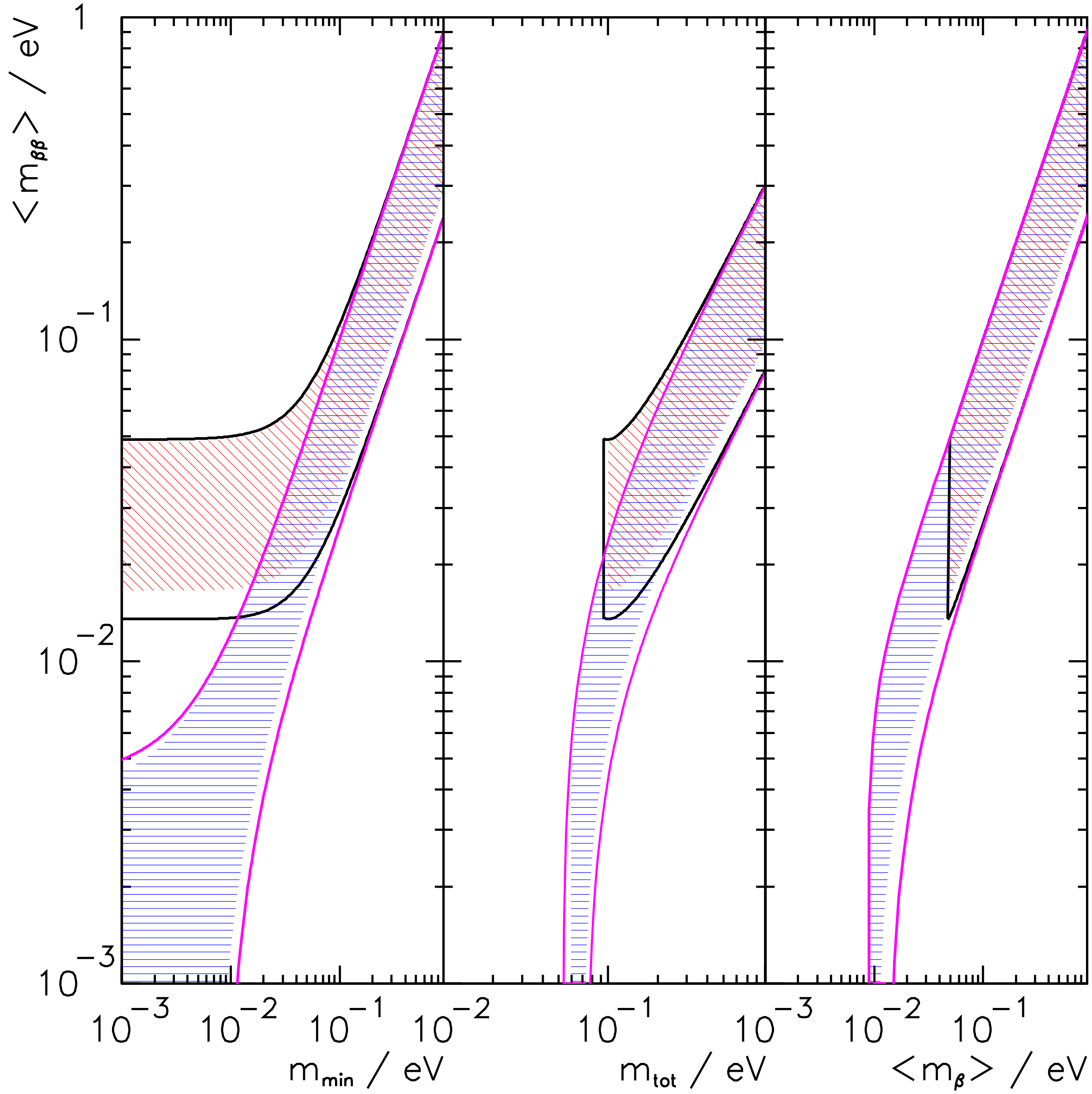
Double beta decay:
0.01 eV (only for Majorana)

$$\langle m_{\beta\beta} \rangle = |\Sigma\, m_i\, |U_{ei}|^2\, \varepsilon_i|$$

(Majorana phases)

Observational cosmology: $M = \Sigma\, m_i$
~0.01 eV (independent of Dirac vs. Majorana)

Relation of $m_{\beta\beta}$ and other ways of neutrino mass determination.



Blue shading ....NH

Pink shading ....IH

Is there anything else?

The two-body decays, like $\pi^+ \rightarrow \mu^+ + \nu_\mu$ are very simple conceptually: Consider pion decay in its rest frame, there

$$\mathbf{m_\nu^2 = m_\pi^2 + m_\mu^2 - 2m_\pi E_\mu} \ ,$$

but the sensitivity is only to $m_\nu \sim 170$ keV with little hope of a substantial improvement. That is so because the neutrino mass (squared) is a difference of two very large numbers ($\sim 3 \times 10^{16}$ eV^2).

Another conceptually simple methods of neutrino mass determination, like TOF from astronomical objects, are not sensitive enough either.

The time delay, with respect to massless particle, is

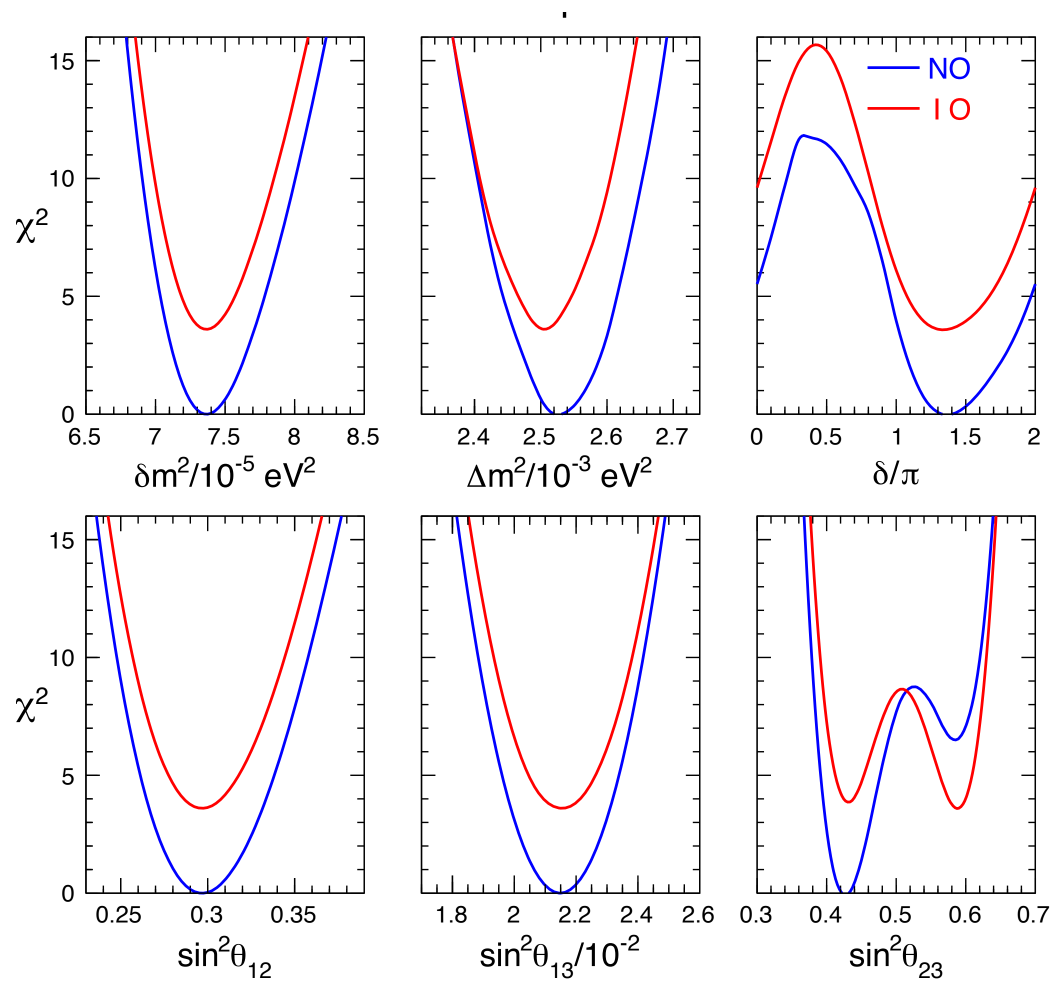
$\Delta t(E) = 0.514\ (m_\nu/E_\nu)^2 D$,

where m is in eV, E in MeV, D in 10 kpc, and Δt in sec.

But there are no massless particles emitted by SN at the same time as neutrinos (except, perhaps gravity waves). Alternatively, we might look for a time delay between the charged current, signal (i.e. ν_e) and the neutral current signal (dominated by ν_x). In addition , one might look for a broadening of the signal, or rearrangement according to the neutrino energy. None of that, realistically, is sensitive to the sub eV neutrino mass region.

What do we know about hierarchies now?

Capozzi et al. (1703.04471) obtain from global fit $\Delta\chi^2_{IH\text{-}NH} = 3.6$,
i.e. about 2σ preference for NH.



They use the term ``ordering" instead of hierarchy. Thus NO means NH and IO means IH

Similar preference for NH $\Delta\chi^2_{IH\text{-}NH} = 2.7$ is found in the analogous global oscillation analysis by Salas et al. 1708.01186. See also E. Esteban et al. JHEP 01(2017)087.

What about cosmology and astrophysics?

From oscillation results we know that the sum of the three neutrino masses, **$\Sigma = m_1 + m_2 + m_3$**, must be larger than **~0.06 eV** for NH and **~0.10 eV** for IH.

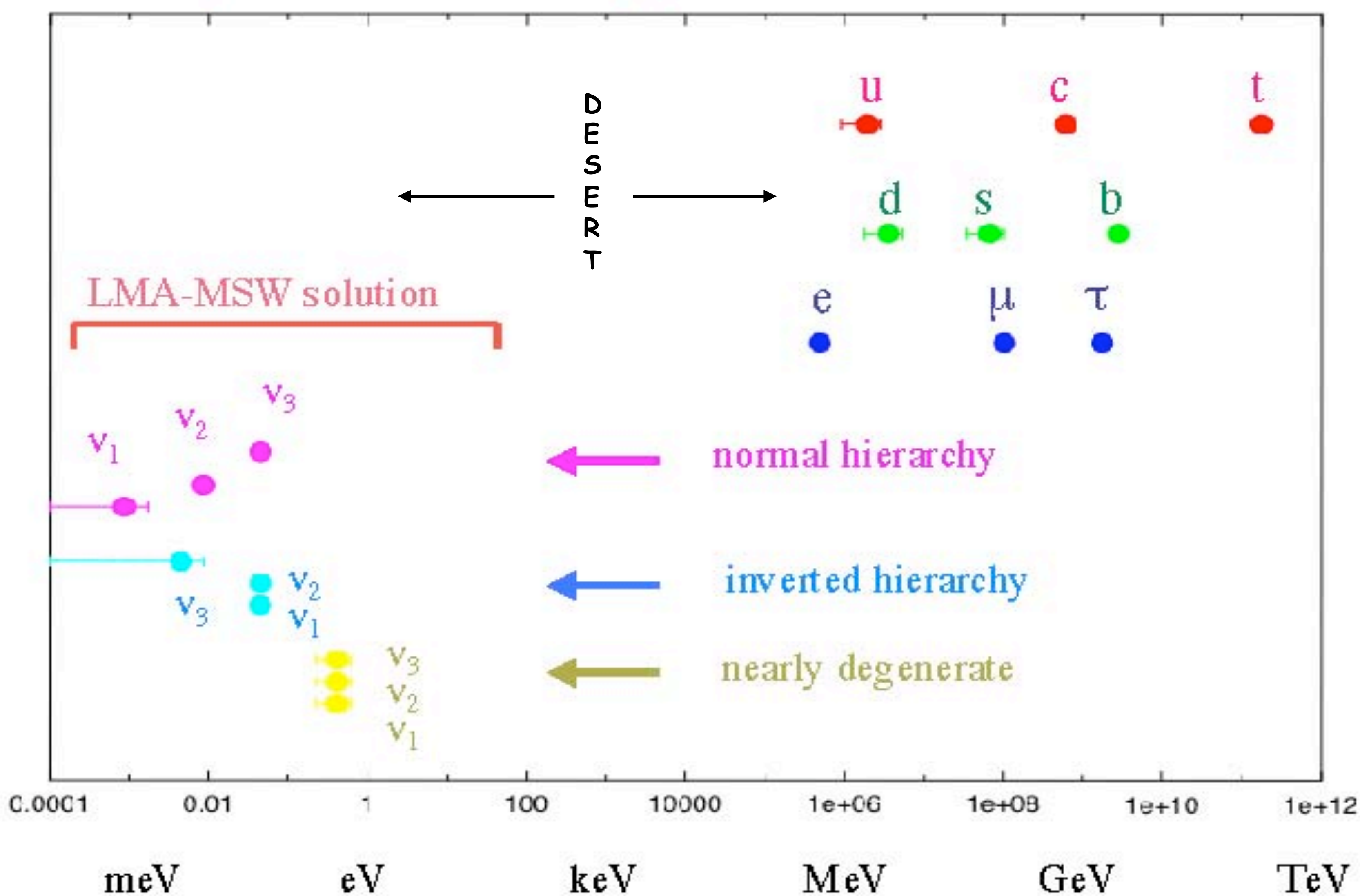
Σ can be constrained and, perhaps, eventually determined, by cosmology in combination with various astrophysics data. Recent Σ limits, at 95% CL, reach small values of 0.13 eV (Cuesta et al. 1511.05983) and 0.12 eV (Palanque-Delabrouille et al. 1506.05976). Based on that, Simpson et al. (1703.03425), use Bayesian analysis and claim a strong preference for NH (odds 42:1). This claim is based on using the logarithmic prior based on the so-called ``Bedford law" and is disputed (see Schwetz et al. 1703.04585).

Nevertheless, if Σ could be reliably restricted to values $\Sigma < 0.1$ eV, but still $\Sigma > 0.06$ eV, the NH would be obviously the only possibility.

Note, however, that the determination of Σ involves various model and systematic uncertainties (see e.g. talk by Maria Archdiacono in Erice 2017)

Why do we expect that the familiar light neutrinos are Majorana fermions?

Masses of neutrinos are much much smaller than the masses of other fermions

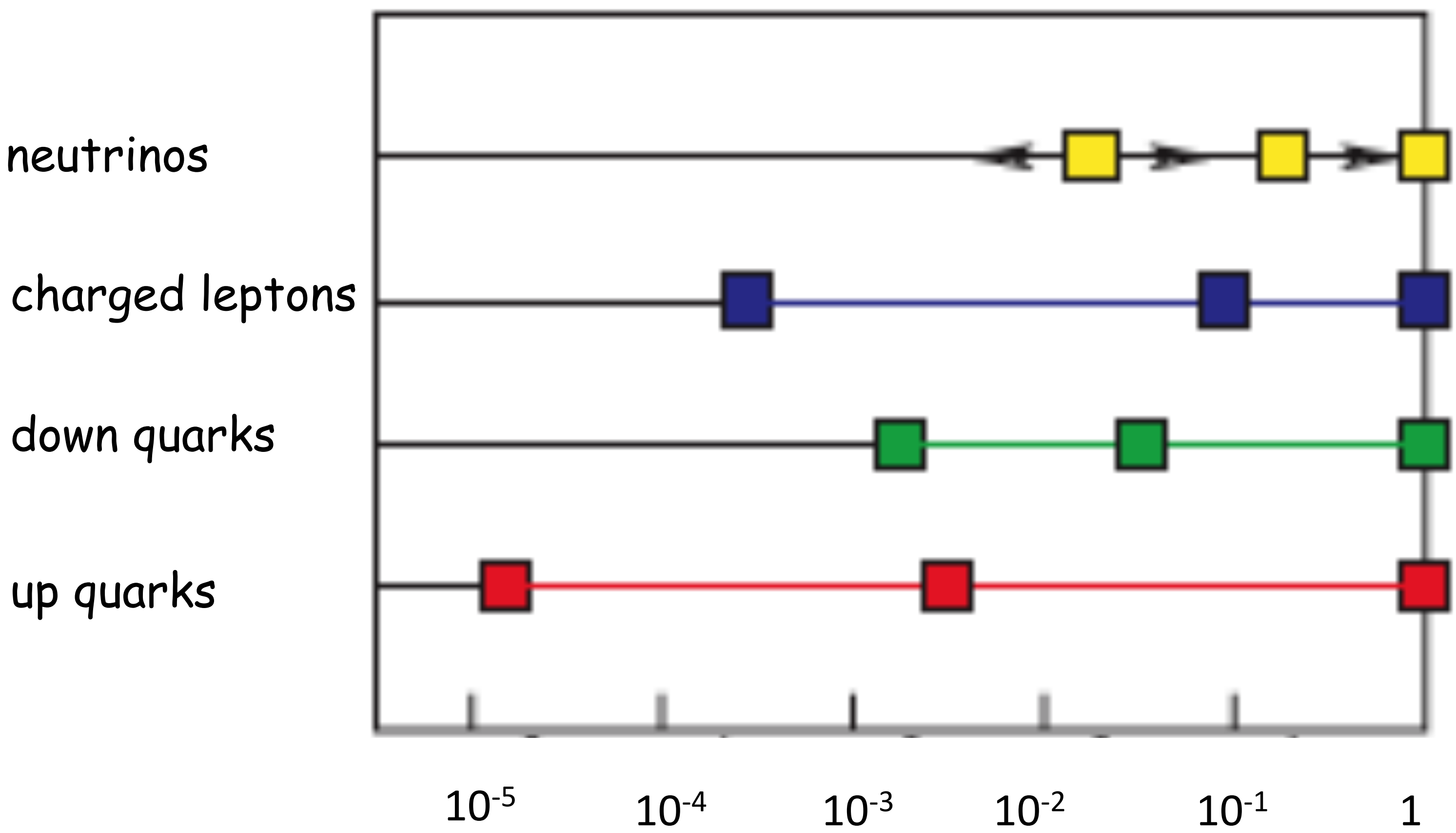


Is that a "Hint of" a new mass-generating mechanism?

Mass hierarchies of quarks and leptons

In this plot the mass of the heaviest particle is taken as unity

While the patterns of up quarks, down quarks, and charged leptons are not really identical, the neutrino masses are noticeably more squeezed together.



Physics beyond the Standard Model can be described by a tower of operators $\mathcal{O}^d$ of dimension d>4. These operators are made of the SM fields, are invariant under the SM gauge group and are *non-renormalizable*. They are weighed by the inverse powers of energy scale new physics Λ_{NP}

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm eff}^{d=5} + \mathscr{L}_{\rm eff}^{d=6} + \cdots \,, \quad \text{with} \quad \mathscr{L}_{\rm eff}^{d} \propto \frac{1}{\Lambda_{\rm NP}^{d-4}}\,\mathcal{O}^d \,.$$

If the Λ_{NP} is sufficiently large, the operators with the minimum possible dimension d would be the ones that have the easiest consequences to observe. Thus, d=5, is special in that sense.

Weinberg already in 1979 (PLR **43**, 1566) showed that there is **only one** dimension d=5 gauge-invariant operator given the particle content of the standard model:

$$L^{(5)} = C^{(5)}/\Lambda\ (\overline{L^c}\varepsilon H)(H^T\varepsilon L) + h.c.$$

Here $\overline{L^c} = L^T C$, where C is charge conjugation and $\varepsilon = -i\tau_2$. This operator clearly violates the lepton number by two units and represents neutrino Majorana mass

$$L^{(M)} = C^{(5)}/\Lambda\ v^2/2\ (\overline{\nu_L{}^c}\,\nu_L) + h.c.$$

(Here the Higgs operator H was replaced by its vacuum expectation value v = 245 GeV.)

If Λ is larger than v, the Higgs vacuum expectation value, the neutrinos will be `naturally' lighter than the charged fermions.

All other possible effective operators will be suppressed by higher powers of the energy scale Λ, i.e. Λ^{-d} with d > 1.

Neutrino mass is described by the Lorentz invariant and hermition mass term in the Lagrangian.

Possible mass terms are either $\overline{\psi}\psi$ and $\overline{\psi^c}\psi^c$ or the other possibility are terms $\overline{\psi}\psi^c$ and $\overline{\psi^c}\psi$.

Under the global phase transformation $\psi \longrightarrow e^{i\alpha}\psi$, $\psi^c \longrightarrow e^{-i\alpha}\psi^c$ the first group is invariant, but the second one is not.

The Dirac mass term is $\overline{\psi}m_D\psi$ and the Majorana mass term $\overline{\psi}m_M\psi^c$

The most general mass term depends on three real paramaters, m_D and the complex $m_M = m_1 + im_2$.

Lets rewrite the most general Lagrangian in the matrix form

$$\text{-2L}_M = \frac{1}{2}\left[\bar{\psi}, \bar{\psi^c}\right]\begin{bmatrix} m_D & m_M \\ m_M^* & m_D \end{bmatrix}\begin{bmatrix} \psi \\ \psi^c \end{bmatrix}$$

The eigenvalues of L_M are $m_D \pm |m_M|$, both real.
(Masses must be positive, if $|m_M| > m_D$, one of them is negative. This can be remedied by using $\gamma_5\psi$ instead of ψ. That obeys the same Dirac equation but with $-m$)

The eigenvectors of L_M are

$$\begin{bmatrix} \phi_+ \\ \phi_- \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{-i\theta}\psi + e^{i\theta}\psi^c \\ -e^{-i\theta}\psi + e^{i\theta}\psi^c \end{bmatrix}$$

Here $\tan 2\theta = m_2/m_1$

Both are eigenstates of charge conjugation; they are Majorana fields. When $m_M = 0$, ψ and ψ^c are eigenstates. They are then Dirac fields and no eigenstates of charge conjugation.

In practice, we should work with the chiral projections:

$$\psi_L = \frac{1-\gamma_5}{2}\psi : \quad (\psi_L)^c = \frac{1+\gamma_5}{2}\psi^c = (\psi^c)_R$$
$$\psi_R = \frac{1+\gamma_5}{2}\psi : \quad (\psi_R)^c = \frac{1-\gamma_5}{2}\psi^c = (\psi^c)_L$$

Note that the chiral projection and charge conjugation do not commute.

Mass terms using the chiral projections are
$\overline{\psi_L}\psi_R$ and $\overline{\psi_R}\psi_L$ for Dirac, and $\overline{\psi_L}(\psi^c)_R$ and $\overline{\psi_R}(\psi^c)_L$ for Majorana.

Terms like $\overline{\psi_L}\varphi_L$ or $\overline{\psi_R}\varphi_R$ vanish since $(1+\gamma_5)\,(1-\gamma_5) = 0$.

The mass terms of both kinds ``violate chirality", i.e. connect L and R, mixes them.

With the chiral projections the mass term eigenvalues depend, again, on the same three parameters,
$\lambda\pm = \frac{1}{2} \{ (m_R + m_L) \pm [(m_R - m_L)^2 + 4m_D{}^2]^{1/2}\}$
Where $m_L = m_1 - |m_2|$ and $m_R = m_1 + |m_2|$.

The general mass term is

$$m_D[\bar{\psi}_L\psi_R + h.c.] + m_L/2[(\bar{\psi}^c)_R\psi_L + h.c.] + m_R/2[(\bar{\psi}^c)_L\psi_R + h.c.]$$

It can be rewritten in terms of the charge conjugation eigenstates

$$f = [\psi_L + (\psi^c)_R]/\sqrt{2}$$
$$F = [\psi_R + (\psi^c)_L]/\sqrt{2}$$

In the form $m_D(\bar{f}F + \bar{F}f) + m_L\bar{f}f + m_R\bar{F}F$

Thus, as before in the matrix form with vectors (f,F)

$$M = \begin{bmatrix} m_R & m_D \\ m_D & m_L \end{bmatrix}$$

Lets consider some special cases:

1) $m_L = m_R = 0$. The eigenvalues are $+m_D$ and $-m_{D.}$ The negative value can be removed with the γ_5 trick. As expected, we recover the Dirac case.

2) m_L and m_R both $\ll m_D$. This is so-called ``quasi-Dirac" case. Pair of two almost degenerate Majorana states with the opposite CP eigenvalues.

3) Finally the most interesting case when $m_L \sim 0$, $m_R \gg m_D$

Now we can consider the underlying physics.

$$: \begin{bmatrix} m_R & m_D \\ m_D & m_L \end{bmatrix}$$

1) Let m_L be the parameter associated with the known light neutrinos
2) Let m_D be the characteristic mass of the charged fermions, leptons or quarks
3) And let m_R be the mass of a so far unknown very heavy, weak singlet, neutrino.

Thus $m_L \ll m_D \ll m_R$

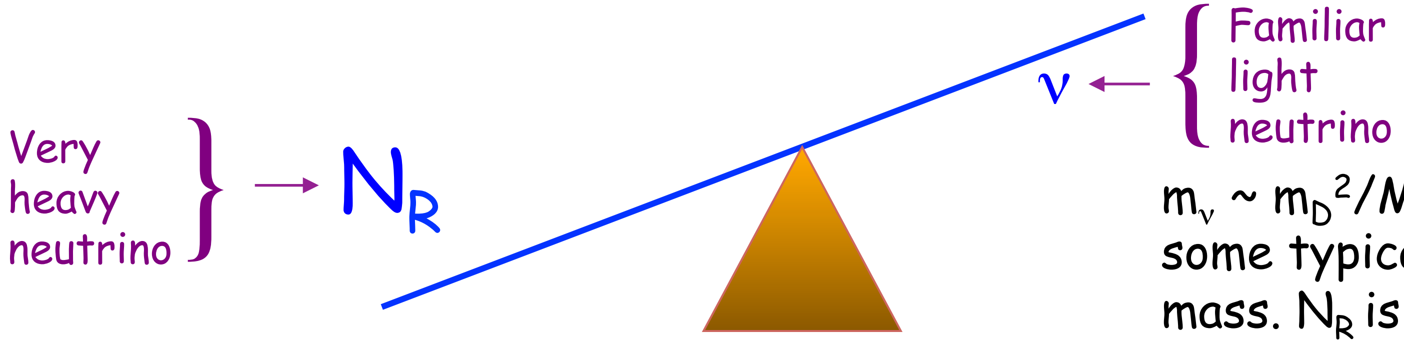
The eigenvalues are $\lambda_{1,2} = m_R /2 \pm\sqrt{(m_R^2/4 - m_D^2)}$

Now we can expand and find easily

$\lambda_1 = m_R$, $\lambda_2 = m_D^2/m_R \ll m_D$

And these are the main ingredient of the see-saw type I

The See-Saw (type I) Mechanism was suggested already in ~1980 by Minkowski (1977), Gell-Mann, Ramond, and Slansky(1979), Yanagida(1979), Mohapatra and Senjanovic (1980). It is related to the finding of Weinberg (1979) that there is only one operator of dimension 5 (with only one power of the scale Λ_{LNV} in the denominator). It represents a neutrino Majorana mass realized in the see-saw model.



$m_\nu \sim m_D^2/M_R$ with m_D some typical Dirac fermion mass. N_R is so heavy that it is unobservable

In the light neutrino exchange, based on the above See-Saw type I, the decay rate is expressed in the familiar form as a product of three factors:

$$1/T_{1/2}^{0\nu} = G^{0\nu}(Q,Z)\ |M^{0\nu}|^2\ |\langle m_{\beta\beta}\rangle|^2, \quad \langle m_{\beta\beta}\rangle = |\Sigma_i U_{ei}^2\ m_i|\ ,$$

This is thus a simple relation between the decay rate and the parameters of the neutrino mass matrix.

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Three regions of $\langle m_{\beta\beta}\rangle$ of interest:

i) Degenerate mass region where all $m_i \gg \Delta m_{31}^2$. There $\langle m_{\beta\beta}\rangle > 0.05$ eV. $T_{1/2}$ for $0\nu\beta\beta$ decay < 10^{26-27} y in this region. This region is explored now, at least in part, with $0\nu\beta\beta$ decay experiments using ~100 kg sources . Moreover, part of this mass region will be explored also by the study of ordinary β decay and most of it is being explored right now by the `observational cosmology'. These latter techniques are independent of whether neutrinos are Majorana or Dirac fermions.

ii) Inverted hierarchy region where m_3 could be < $|\Delta m^2_{31}|$. However, quasidenegerate normal hierarchy is also possible for $\langle m_{\beta\beta}\rangle$ ~ 20-100 meV. $T_{1/2}$ for $0\nu\beta\beta$ decay is 10^{27-28} years here, and could be explored with ~ton size experiments. Such experiments, with timeline ~10 years, will likely happen.

iii) Normal mass hierarchy, $\langle m_{\beta\beta}\rangle$ < 15 meV. It would be necessary to use ~100 ton experiments. There are no realistic ideas how to do it.