CP-Violating pion-nucleon couplings from quark Chromo-EDMs

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

Hadronic Matrix Elements for Probes of CP Violation
22-24 January, 2015

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Fundamental Symmetries and Low-Energy Nuclear Physics

- The Universe is matter dominated at roughly 1 ppb:

$$\eta \equiv \frac{X_{p+n}}{X_\gamma} = 6.19(15) \times 10^{-10}$$

- Sources of CP-violation beyond the Standard Model (SM) are needed to generate this observed asymmetry

- Assuming nature is CPT symmetric, this implies T-violation which implies fermions will have permanent electric dipole moments (EDMs)

- This has motivated significant experimental efforts to search (or plan to search) for permanent EDMs in a variety of systems
  e, n, p, deuteron, triton, ^{3}He, ..., ^{199}Hg, ^{225}Ra, ^{229}Pa,...

Fundamental Symmetries and Low-Energy Nuclear Physics

- In order to interpret a measurement/constraint of an EDM in a nucleon or nuclei as a value/bound of couplings to BSM physics, we must have a solution to QCD in the IR
- Our tools of choice are lattice QCD (LQCD) and Effective Field Theory (EFT)
- We desire to compute completely a nucleon EDM resulting from CP violating operators, however, yesterday, we heard a bit about how challenging this problem is
- In the meantime, we can exploit symmetries (tricks) to determine the long-range CP-violating π-N couplings from simple spectroscopic LQCD calculations which are expected to dominate the EDMs of certain nuclei (eg ^{225}Ra)

- In a large nucleus, the long-range pion exchange will dominate the nuclear EDM

$$\mathcal{L}_{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\vec{\pi}\cdot\vec{\tau}N - \frac{\bar{g}_1}{2F_\pi}\bar{N}\pi_3 N - \frac{\bar{g}_2}{2F_\pi}\pi_3\bar{N}\left(\tau_3 - \frac{\pi_3}{F_\pi}\vec{\pi}\cdot\vec{\tau}\right)N$$

- For the QCD theta term

$$\{\bar{g}_1, \bar{g}_2\} \sim \bar{g}_0\frac{m_\pi^2}{\Lambda_\chi^2}$$

- For more generic CP Violating operators

$$\bar{g}_2 \sim \{\bar{g}_0, \bar{g}_1\}\frac{m_\pi^2}{\Lambda_\chi^2} \qquad \bar{g}_1 \sim \bar{g}_0$$

- The nuclear EDM is proportional to the Schiff moment

$$S = \sum_{i \neq 0} \frac{\langle \Phi_0 | S_z | \Phi_i \rangle \langle \Phi_i | H_{CPV} | \Phi_0 \rangle}{E_0 - E_i} + c.c.$$

$$S = \frac{2 M_N g_A}{F_\pi} \left(a_0 \bar{g}_0 + a_1 \bar{g}_1 + a_2 \bar{g}_2 \right)$$

- The Schiff parameters $\{a_0, a_1, a_2\}$ are computed with nuclear models under the assumption the CPV operator does not significantly distort the nuclear wave-function

- For a QCD theta term only $\bar{g}_1 \sim \bar{g}_2 \sim 0$ and thus a constraint on $\bar{\theta}$ can be made through the relation

$$\bar{g}_0 = \frac{\delta M_{n-p}^{m_d - m_u}}{m_d - m_u} \frac{2 m_d m_u}{m_d + m_u} \bar{\theta} = \alpha \frac{2 m_d m_u}{m_d + m_u} \bar{\theta}$$

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$$S = \sum_{i \neq 0} \frac{\langle \Phi_0 | S_z | \Phi_i \rangle \langle \Phi_i | H_{CPV} | \Phi_0 \rangle}{E_0 - E_i} + c.c.$$

$$S = \frac{2 M_N g_A}{F_\pi} \left(a_0 \bar{g}_0 + a_1 \bar{g}_1 + a_2 \bar{g}_2 \right)$$

- ^{225}Ra is interesting nucleus as it is octupole deformed
  - "stiff" core making nuclear model calculations more reliable
  - nearly degenerate parity partner state

$$E^{-}_{1/2} - E^{+}_{1/2} = 55 \text{ KeV}$$

  - $10^2 - 10^3$ enhancement of $\{a_0, a_1, a_2\}$

- Sources of CP-Violation in quark sector:

Operator	[Operator]	No. Operators
$\bar{\theta}$	4	1
quark EDM	6	2
quark Chromo-EDM	6	2
Weinberg (GGG)	6	1
4-quark	6	2
4-quark induced	6	1

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$$\mathcal{L}_{CPV} = -\frac{g_s^2\bar{\theta}}{32\pi^2}\tilde{G}_{\mu\nu}G^{\mu\nu} - \frac{i}{2}\bar{q}\sigma^{\mu\nu}\gamma_5\left(\tilde{d}_0 + \tilde{d}_3\tau_3\right)G_{\mu\nu}q$$

$$\Downarrow$$

$$\mathcal{L}_{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\vec{\pi}\cdot\vec{\tau}N - \frac{\bar{g}_1}{2F_\pi}\bar{N}\pi_3 N - \frac{\bar{g}_2}{2F_\pi}\pi_3\bar{N}\left(\tau_3 - \frac{\pi_3}{F_\pi}\vec{\pi}\cdot\vec{\tau}\right)N$$

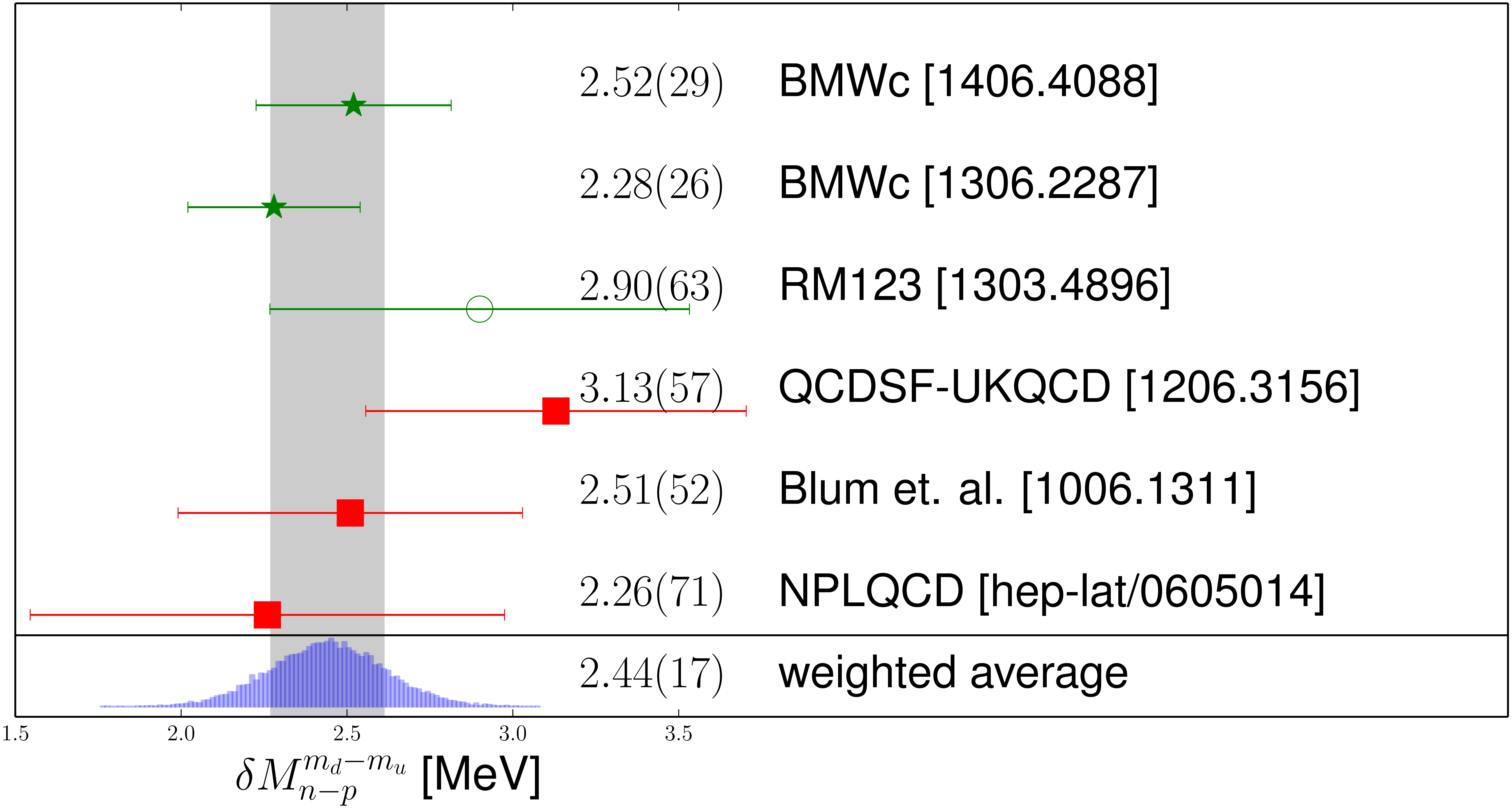
QCD Isospin Violation and CP-violating π-N

- A precise determination of the strong isospin breaking contribution to Mn-Mp teaches us about CP-violation

$$\bar{g}_0 = \frac{\delta M_{n-p}^{m_d - m_u}}{m_d - m_u} \frac{2 m_d m_u}{m_d + m_u} \bar{\theta} = \alpha \frac{2 m_d m_u}{m_d + m_u} \bar{\theta}$$

Isospin Violation and Lattice QCD

$$\delta M_{n-p}^{m_d-m_u} = 2.44(17)\ \mathrm{MeV}$$



Isospin Violation and Lattice QCD

strong isospin breaking correction

$$\delta M_{n-p}^{m_d-m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d-m_u} = 2.44(17)\ \mathrm{MeV}$$

lattice average

B. Tiburzi, AWL	Nucl. Phys. A764 (2006)
Beane, Orginos, Savage	Nucl. Phys. B768 (2007)
AWL	arXiv:0904.2404
Blum, Izubuchi, etal	Phys. Rev. D82 (2010)
AWL	PoS Lattice2010 (2010)
de Divitiis etal	JHEP 1204 (2012)
Horsley etal	Phys. Rev. D86 (2012)
de Divitiis etal	Phys. Rev. D87 (2013)
Borsanyi etal	arXiv:1306.2287
Borsanyi etal	arXiv:1406.4088

But in (most) lattice calculations $m_u = m_d = m_l$?
(except latest)

Isospin Violation and Lattice QCD

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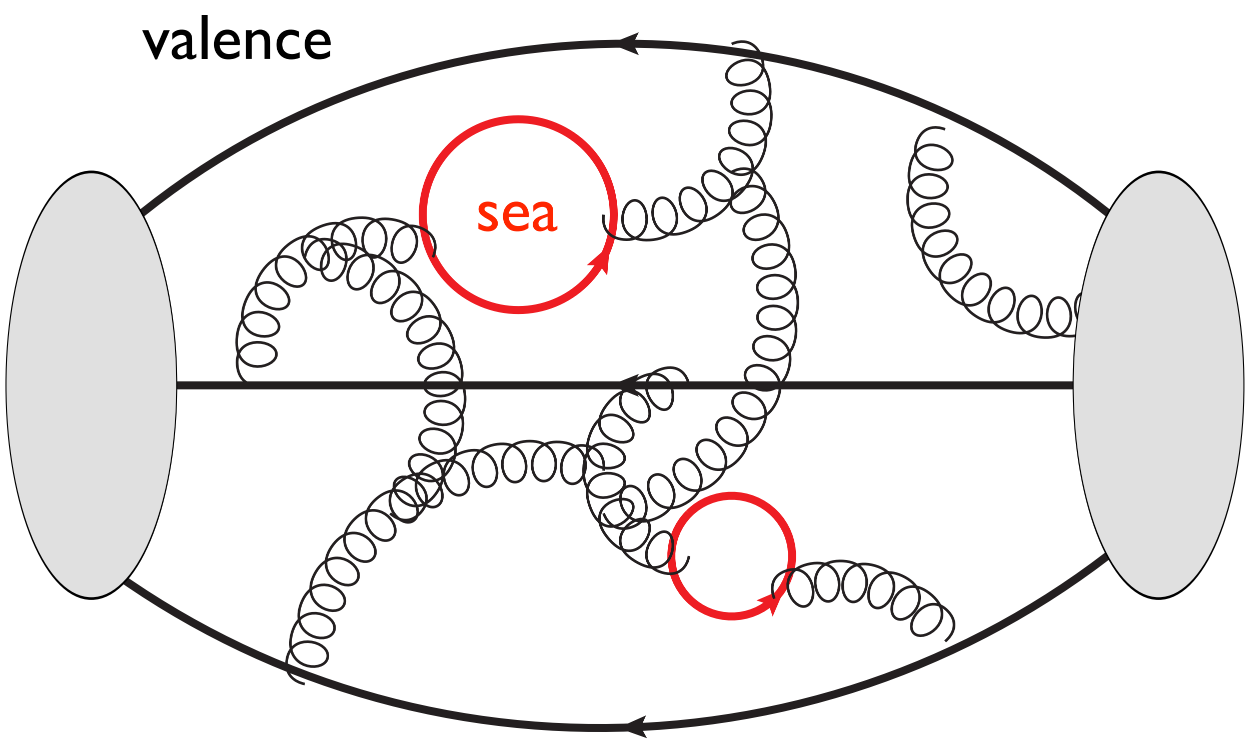

$$m_{u,d}^{valence} \neq m_l^{sea}$$

"partially quenched" lattice QCD trick that works on the computer but introduces error which must be corrected

Isospin Violation and Lattice QCD

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

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can we improve this method?

of course!

"Symmetric breaking of isospin symmetry" AWL arXiv:0904.2404

Isospin Violation and Lattice QCD

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

$$\mathcal{Z}_{u,d} = \int DU_\mu \ \mathrm{Det}(D + m_l - \delta\tau_3)\ e^{-S[U_\mu]}$$

$$= \int DU_\mu \ \mathrm{Det}(D + m_l)\ \det\left(1 - \frac{\delta^2}{(D+m_l)^2}\right)\ e^{-S[U_\mu]}$$

Isospin Violation and Lattice QCD

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

$$\mathcal{Z}_{u,d} = \int DU_\mu \ \mathrm{Det}(D + m_l - \delta\tau_3) \ e^{-S[U_\mu]}$$

$$= \int DU_\mu \ \mathrm{Det}(D + m_l) \ \det\left(1 - \cancel{\frac{\delta^2}{(D+m_l)^2}}\right) \ e^{-S[U_\mu]}$$

Isospin symmetric quantities: error $\mathcal{O}(\delta^2)$

Isospin violating quantities: error $\mathcal{O}(\delta^3)$

see also

de Divitiis etal JHEP 1204 (2012)

de Divitiis etal Phys. Rev. D87 (2013)

Isospin Violation and Lattice QCD

$$m^{val} = m^{sea} - \delta\tau_3$$

Partially Quenched Pion Lagrangian

$$\begin{aligned}
\mathcal{L}^{(4|2)} =& \frac{f^2}{8}\mathrm{str}\left(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right) + \frac{2\mathrm{Bf}^2}{8}\mathrm{str}\left(\mathrm{m}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger\mathrm{m}_\mathrm{Q}\right) \\
&+ L_1^{(PQ)}[\mathrm{str}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger)]^2 + \mathrm{L}_2^{(\mathrm{PQ})}\mathrm{str}(\partial_\mu\Sigma\partial_\nu\Sigma^\dagger)\mathrm{str}(\partial^\mu\Sigma\partial^\nu\Sigma^\dagger) + \mathrm{L}_3^{(\mathrm{PQ})}\mathrm{str}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\partial_\nu\Sigma\partial^\nu\Sigma^\dagger) \\
&+ L_4^{(PQ)}\mathrm{str}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger)\mathrm{str}(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q}) + \mathrm{L}_5^{(\mathrm{PQ})}\mathrm{str}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q})) \\
&+ L_6^{(PQ)}[\mathrm{str}(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q})]^2 + \mathrm{L}_7^{(\mathrm{PQ})}[\mathrm{str}(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma - \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q})]^2 \\
&+ L_8^{(PQ)}\mathrm{str}(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma 2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q}\Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q})
\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}
\mathcal{L}^{(2)} =& \frac{f^2}{8}\mathrm{tr}\left(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right) + \frac{2\mathrm{Bf}^2}{8}\mathrm{tr}\left(\mathrm{m}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger\mathrm{m}_\mathrm{Q}\right) \\
&+ \frac{l_1}{4}[\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger)]^2 + \frac{\mathrm{l}_2}{4}\mathrm{tr}(\partial_\mu\Sigma\partial_\nu\Sigma^\dagger)\mathrm{tr}(\partial^\mu\Sigma\partial^\nu\Sigma^\dagger) + \frac{\mathrm{l}_3 + \mathrm{l}_4}{16}[\mathrm{tr}(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q})]^2 \\
&+ \frac{l_4}{8}\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma + \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q})) + \frac{\mathrm{l}_7}{16}[\mathrm{tr}(2\mathrm{Bm}_\mathrm{Q}^\dagger\Sigma - \Sigma^\dagger 2\mathrm{Bm}_\mathrm{Q})]^2
\end{aligned}$$

$$\Rightarrow$$

$$l_1 = 4L_1^{(PQ)} + 2L_3^{(PQ)} \qquad l_2 = 4L_2^{(PQ)}$$

$$l_3 + l_4 = 16L_6^{(PQ)} + 8L_8^{(PQ)} \qquad l_4 = 8L_4^{(PQ)} + 4L_5^{(PQ)}$$

$$l_7 = 16L_7^{(PQ)} + 8L_8^{(PQ)}$$

Isospin Violation and Lattice QCD

AWL arXiv:0904.2404

$$m^{val} = m^{sea} - \delta\tau_3$$

Partially Quenched Hairpin Interactions

Partially Quenched (mixed-action) theories exhibit unitarity violating sicknesses - eg. double pole structure of flavor neutral mesons. Generally

$$\mathcal{G}_{\eta_u\eta_u}(p^2) = \frac{i}{p^2 - m_{\eta_u}^2 + i\epsilon} - \frac{i}{2}\frac{p^2 - m_{jj}^2}{(p^2 - m_\pi^2)^2} \qquad \mathcal{G}_{\eta_u\eta_d}(p^2) = -\frac{i}{2}\frac{p^2 - m_{jj}^2}{(p^2 - m_\pi^2)^2}$$

$$m_{jj} = \text{sea pion mass}$$

It is useful to re-write Lagrangian in terms of the fields JW Chen, D O'Connell, AWL hep-lat/0611003

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(|\eta_u\rangle - |\eta_d\rangle) \qquad |\bar{\eta}\rangle = \frac{1}{\sqrt{2}}(|\eta_u\rangle + |\eta_d\rangle)$$

$$\mathcal{G}_{\pi^0} = \frac{i}{p^2 - m_\pi^2 + i\epsilon} \qquad \mathcal{G}_{\bar{\eta}} = \frac{i\Delta_{PQ}^2}{(p^2 - m_\pi^2 + i\epsilon)^2} \qquad \Delta_{PQ}^2 = m_{jj}^2 - m_\pi^2 \simeq 2B(m_q^{sea} - m_q^{val})$$

For symmetric isospin breaking:

$$\mathcal{G}_{\pi^0} = \frac{i}{p^2 - m_\pi^2 + i\epsilon} \qquad \mathcal{G}_{\bar{\eta}} = \frac{i\Delta_{PQ}^4}{(p^2 - m_\pi^2 + i\epsilon)^3} \qquad \Delta_{PQ}^2 = \hat{\delta} = B(m_d - m_u) = 2B\delta$$

Isospin Violation and Lattice QCD

AWL arXiv:0904.2404

$$m^{val} = m^{sea} - \delta\tau_3$$

Partially Quenched Hairpin Interactions

In general, the pion mass at NLO is

$$m_\pi^2 = 2Bm_l \left\{ 1 + \underbrace{\frac{m_\pi^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right) + 4l_3^r(\mu)\frac{2Bm_l}{f_\pi^2}}_{\text{QCD}} \underbrace{- \frac{\Delta_{PQ}^2}{(4\pi f_\pi)^2}\left[1 + \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \frac{\Delta_{PQ}^2}{f_\pi^2} l_{PQ}(\mu)}_{\text{Partially Quenched}} \right\}$$

For symmetric isospin breaking:

$$m_{\pi^\pm}^2 = 2Bm_l \left\{ 1 + \frac{2Bm_l}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right) + 4l_3(\mu)\frac{2Bm_l}{f_\pi^2} \right\} + \frac{\Delta_{PQ}^4}{2(4\pi f_\pi)^2}$$

Suppressed Partially Quenched effects

$$m_{\pi^0}^2 = m_{\pi^\pm}^2 + 4l_7 \frac{(2B\delta)^2}{f_\pi^2}$$

pion mass splitting free of "error" at this order, as expected: splitting exactly as in QCD at NLO

Isospin Violation and Lattice QCD

AWL arXiv:0904.2404

$$m^{val} = m^{sea} - \delta\tau_3$$

Partially Quenched Nucleon Lagrangian

$$\mathcal{L}^{(PQ)} = \left(\overline{\mathcal{B}} v \cdot D \mathcal{B}\right) + \frac{\alpha_M^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{B}}\mathcal{B}\mathcal{M}_+\right) + \frac{\beta_M^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{B}}\mathcal{M}_+\mathcal{B}\right) + \frac{\sigma_M^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{B}}\mathcal{B}\right)\operatorname{tr}(\mathcal{M}_+)$$
$$- \left(\overline{\mathcal{T}}_\mu v \cdot D\, \mathcal{T}_\mu\right) - \Delta\left(\overline{\mathcal{T}}_\mu \mathcal{T}_\mu\right) + \frac{\gamma_M^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{T}}_\mu \mathcal{M}_+ \mathcal{T}_\mu\right) - \frac{\overline{\sigma}_M^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{T}}_\mu \mathcal{T}_\mu\right)\operatorname{tr}(\mathcal{M}_+)$$
$$+ 2\alpha^{(PQ)}\left(\overline{\mathcal{B}} S^\mu \mathcal{B} A_\mu\right) + 2\beta^{(PQ)}\left(\overline{\mathcal{B}} S^\mu A_\mu \mathcal{B}\right) + 2\mathcal{H}^{(PQ)}\left(\overline{\mathcal{T}}^\nu S^\mu A_\mu \mathcal{T}_\nu\right) + \sqrt{\frac{3}{2}}\mathcal{C}\left[\left(\overline{\mathcal{T}}^\nu A_\nu \mathcal{B}\right) + \left(\overline{\mathcal{B}} A_\nu \mathcal{T}^\nu\right)\right]$$

$$\mathcal{L} = \overline{N} v \cdot D N + \frac{\alpha_M}{(4\pi f)}\overline{N}\mathcal{M}_+ N + \frac{\sigma_M}{(4\pi f)}\overline{N} N \operatorname{tr}(\mathcal{M}_+)$$
$$+ \left(\overline{T}_\mu v \cdot D\, T_\mu\right) + \Delta\left(\overline{T}_\mu T_\mu\right) + \frac{\gamma_M}{(4\pi f)}\left(\overline{T}_\mu \mathcal{M}_+ T_\mu\right) + \frac{\overline{\sigma}_M}{(4\pi f)}\left(\overline{T}_\mu T_\mu\right)\operatorname{tr}(\mathcal{M}_+)$$
$$+ 2\, g_A\, \overline{N} S \cdot A\, N - 2\, g_{\Delta\Delta}\, \overline{T}_\mu S \cdot A\, T_\mu + g_{\Delta N}\left[\overline{T}^{kji}_\mu A_i^{\mu,i'} \epsilon_{ji'} N_k + h.c.\right].$$

$$\alpha_M = \frac{2}{3}\alpha_M^{(PQ)} - \frac{1}{3}\beta_M^{(PQ)},$$
$$\sigma_M = \sigma_M^{(PQ)} + \frac{1}{6}\alpha_M^{(PQ)} + \frac{2}{3}\beta_M^{(PQ)},$$
$$\gamma_M = \gamma_M^{(PQ)}\ ,\ \bar{\sigma}_M = \bar{\sigma}_M^{(PQ)},$$

$$g_A = \frac{2}{3}\alpha^{(PQ)} - \frac{1}{3}\beta^{(PQ)}\ ,\ g_1 = \frac{1}{3}\alpha^{(PQ)} + \frac{4}{3}\beta^{PQ)},$$
$$g_{\Delta\Delta} = \mathcal{H}\ ,\ g_{\Delta N} = -\mathcal{C},$$

Isospin Violation and Lattice QCD

AWL arXiv:0904.2404

$$m^{val} = m^{sea} - \delta\tau_3$$

Nucleon Masses

$$M_n = M_0 + \frac{2B\delta}{4\pi f_\pi}\frac{\alpha_N}{2} + \frac{m_\pi^2}{4\pi f_\pi}\left(\frac{\alpha_N}{2} + \sigma_N(\mu)\right) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2}m_\pi^3 - \frac{8g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2}\mathcal{F}(m_\pi,\Delta,\mu) + \frac{3\pi\Delta_{PQ}^4(g_A+g_1)^2}{8m_\pi(4\pi f_\pi)^2}$$

$$M_p = M_0 - \frac{2B\delta}{4\pi f_\pi}\frac{\alpha_N}{2} + \frac{m_\pi^2}{4\pi f_\pi}\left(\frac{\alpha_N}{2} + \sigma_N(\mu)\right) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2}m_\pi^3 - \frac{8g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2}\mathcal{F}(m_\pi,\Delta,\mu) + \frac{3\pi\Delta_{PQ}^4(g_A+g_1)^2}{8m_\pi(4\pi f_\pi)^2}$$

Notice in the isospin splitting, not only the isospin violation appears as expected, but the non-analytic pion loop corrections exactly cancel, and the PQ effects exactly cancel! (This is only with "symmetric isospin breaking")

$$M_n - M_p = 2\alpha_N\delta\frac{B}{4\pi f_\pi} + \mathcal{O}(\delta^2, \delta m_\pi)$$

The expansion for M_n-M_p becomes similar to that of the pions (only even powers of the pion mass)

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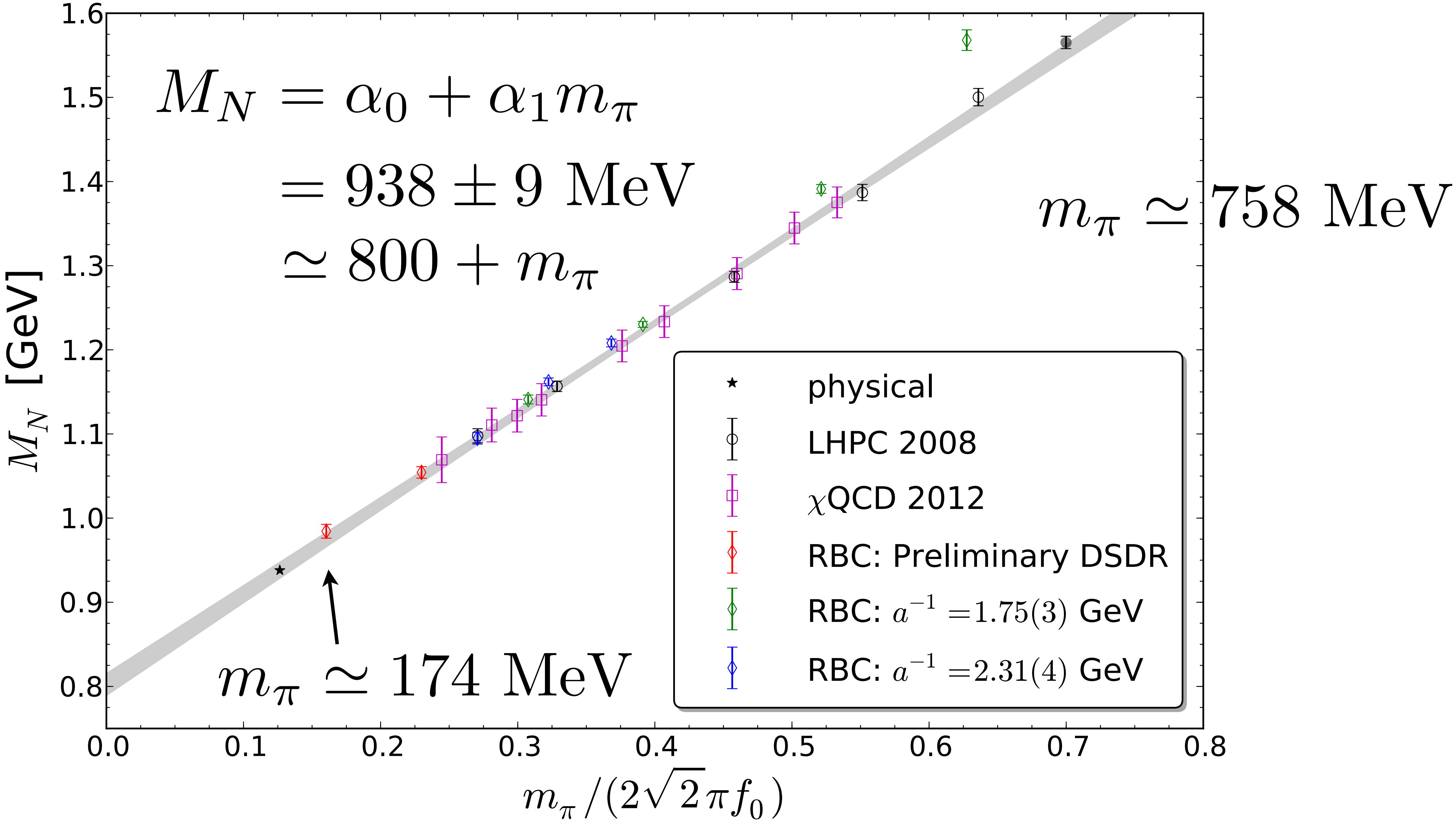
"Ruler Plot" (blame Brian Tiburzi)

(Lattice 2008, Chiral Dynamics 2012 AWL)



Trying to fit nucleon mass results to baryon chiral perturbation theory, with g_A as a free parameter, leads to g_A~0. Serious challenges to convergence of SU(2) baryon chiral perturbation theory

Isospin Violation and Lattice QCD

AWL arXiv:0904.2404

$$m^{val} = m^{sea} - \delta\tau_3$$

NNLO operators (SU(2))

BCT & AWL hep-lat/0501018

$$\mathcal{L}_M = \frac{1}{(4\pi f)^3}\Big\{ b_1^M \bar{N}\,\mathcal{M}_+^2 N + b_5^M \bar{N}N\,\mathrm{tr}(\mathcal{M}_+^2) + b_6^M \bar{N}\,\mathcal{M}_+ N\,\mathrm{tr}(\mathcal{M}_+) + b_8^M \bar{N}N\,[\mathrm{tr}(\mathcal{M}_+)]^2$$
$$+ t_1^M\,\bar{T}_\mu^{kji}(\mathcal{M}_+\mathcal{M}_+)_i{}^{i'}T_{\mu,i'jk} + t_2^M\,\bar{T}_\mu^{kji}(\mathcal{M}_+)_i{}^{i'}(\mathcal{M}_+)_j{}^{j'}T_{\mu,i'j'k} + t_3^M\,\bar{T}_\mu T_\mu \mathrm{tr}(\mathcal{M}_+^2)$$
$$+ t_4^M\,\left(\bar{T}_\mu\mathcal{M}_+T_\mu\right)\mathrm{tr}(\mathcal{M}_+) + t_5^M\,\bar{T}_\mu T_\mu[\mathrm{tr}(\mathcal{M}_+)]^2\Big\}$$

In QCD

$$\delta m_N = b_5^M\,\frac{m_\pi^4 + (2B\delta)^2}{2(4\pi f_\pi)^3}$$

In symmetric isospin breaking PQQCD

$$\delta m_N = b_5^M\,\frac{m_\pi^4}{2(4\pi f_\pi)^3}$$

This is an error of this partially quenched calculation, which must be removed from the LQCD calculation, to compare with experiment.
But NOTE! In M_n-M_p, this error exactly cancels (as we expect)

Full NNLO Nucleon mass splitting:

$$M_n - M_p = \frac{2B\delta}{4\pi f_\pi}\Bigg\{\alpha_N + \frac{m_\pi^2}{(4\pi f_\pi)^2}(b_1^M + b_6^M) + \frac{\mathcal{J}(m_\pi,\Delta,\mu)}{(4\pi f_\pi)^2} 4g_{\pi N\Delta}^2\left(\frac{5}{9}\gamma_M - \alpha_N\right)$$

$$\frac{m_\pi^2}{(4\pi f_\pi)^2}\left[\frac{20}{9}\gamma_M g_{\pi N\Delta}^2 - 4\alpha_N(g_A^2 + g_{\pi N\Delta}^2) - \alpha_N(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right]$$

$$+ \frac{\alpha_N \Delta_{PQ}^4}{m_\pi^2(4\pi f_\pi)^2}\left(2 - \frac{3}{2}(g_A + g_1)^2\right)\Bigg\}$$

LQCD Calculation

PRELIMINARY

lattice QCD calculation performed using the Spectrum Collaboration anisotropic clover-Wilson gauge ensembles (developed @JLAB)

ensemble				$a_t m_\pi$	$a_t m_K$	$a_t\delta$ $[N_{cfg} \times N_{src}]$			
L	T	$a_t m_l$	$a_t m_s$			0.0002	0.0004	0.0010	0.0020
16	128	-0.0830	-0.0743	0.0800	0.1033	207×16	207×16	207×16	207×16
16	128	-0.0840	-0.0743	–	–	166×25	166×25	166×25	166×50
20	128	-0.0840	-0.0743	–	–	120×25	–	–	–
24	128	-0.0840	-0.0743	–	–	97×25	–	193×25	–
32	256	-0.0840	-0.0743	0.0689	0.0968	291×10	291×10	291×10	–
24	128	-0.0860	-0.0743	–	–	118×26	–	–	–
32	256	-0.0860	-0.0743	0.0393	0.0833	842×11	–	–	–

e m c

C.Aubin, W.Detmold, E Mereghetti, K.Orginos, S.Syritsyn, B.Tiburzi, AWL

LQCD Calculation

PRELIMINARY

Scale Setting: $l_\Omega = \dfrac{m_\pi^2}{m_\Omega^2}$ $\quad s_\Omega = \dfrac{2m_K^2 - m_\pi^2}{m_\Omega^2}$

Compute these masses, then extrapolate

$$m_\Omega(l_\Omega, s_\Omega) = m_0 + c_l^{(1)} l_\Omega + c_s^{(1)} s_\Omega + \cdots$$

the scale is determined at the physical point

$$a_t m_\Omega^* = a_t m_\Omega(l_\Omega^*, s_\Omega^*)$$

$$a_t^* \equiv \frac{a_t m_\Omega^*}{m_\Omega^{\mathrm{phy}}}$$

This is a "quark mass independent" scale setting scheme

$$m_h[\mathrm{MeV}] = \frac{a_t m_h}{a_t m_\Omega}\frac{a_t m_\Omega}{a_t m_\Omega^*} m_\Omega^{\mathrm{phy}} = a_t m_h \frac{m_\Omega^{\mathrm{phy}}}{a_t m_\Omega^*} = a_t m_h \frac{1}{a_t^*}$$

But recall the lattices generated were generated with fixed strange quark mass this makes it challenging to extrapolate in s_Ω

Also, input strange quark mass was about 10% too light

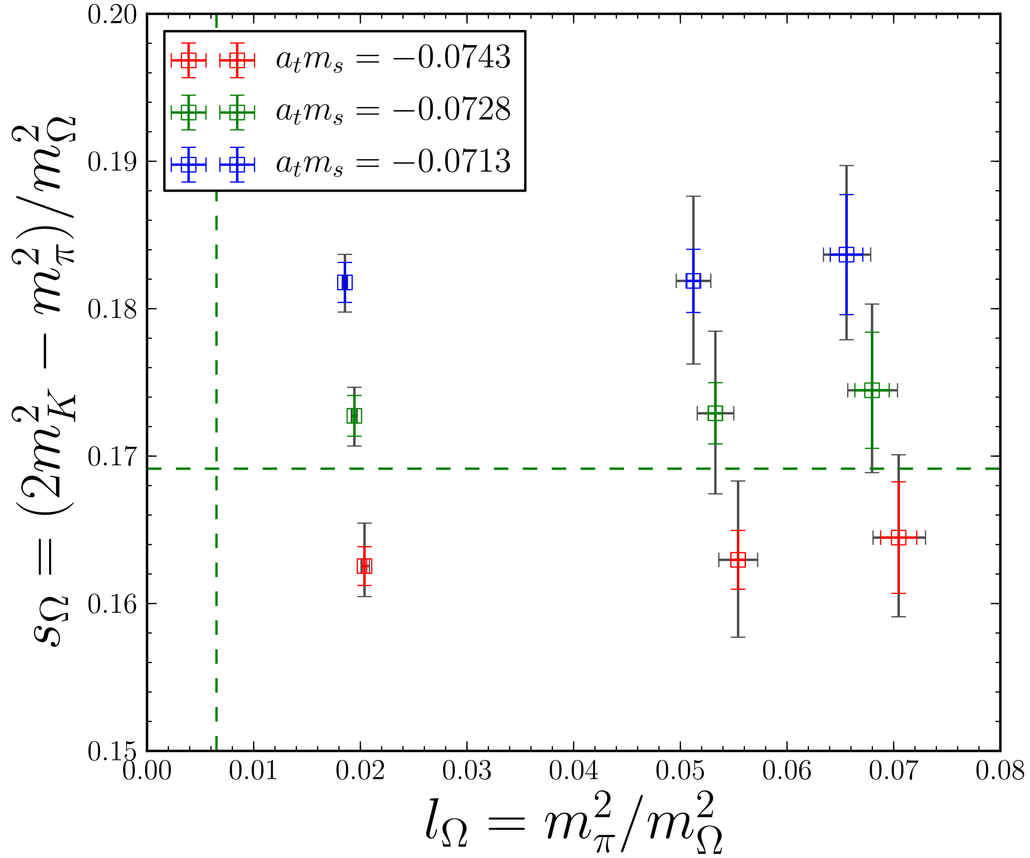
$a_t m_s^{sea} = -0.0743$ $\quad a_t m_s^{val} = \{-0.0743, -0.0728, -0.0713\}$

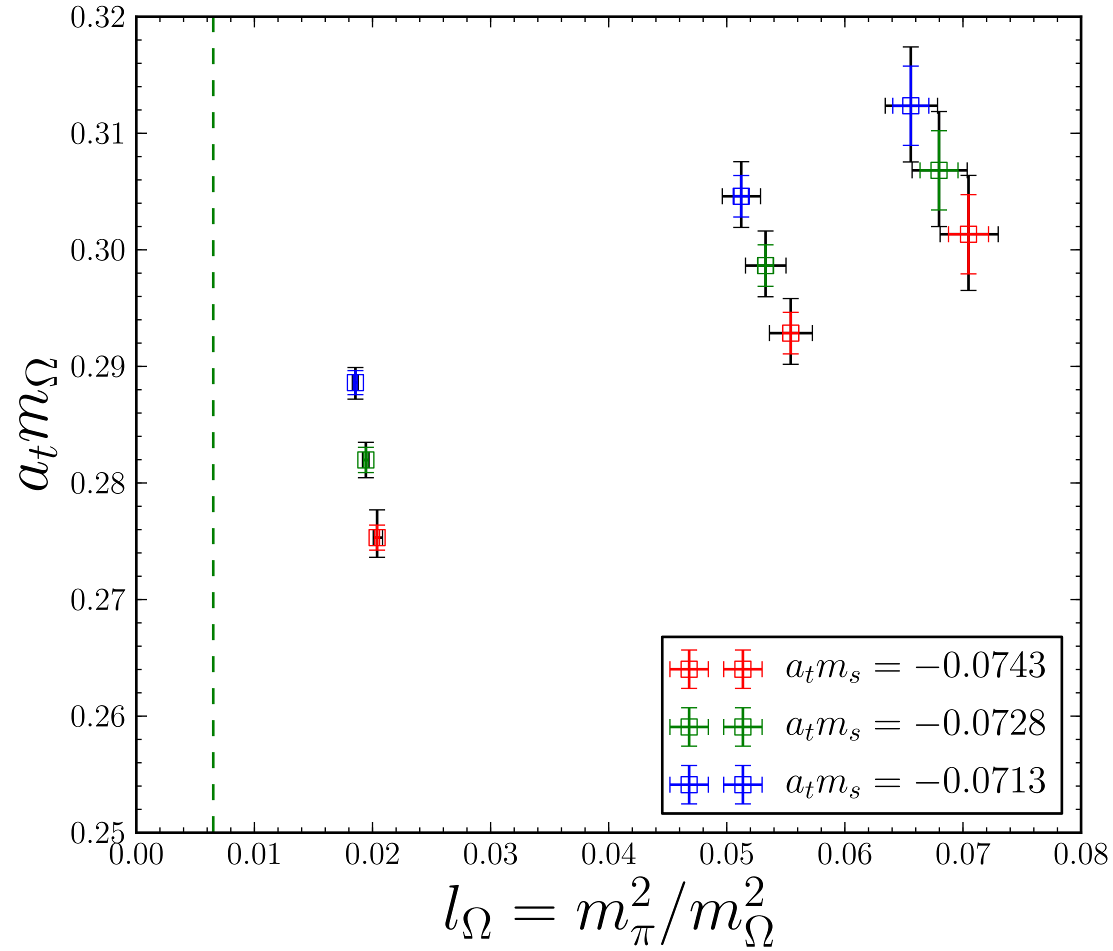
lightest heaviest

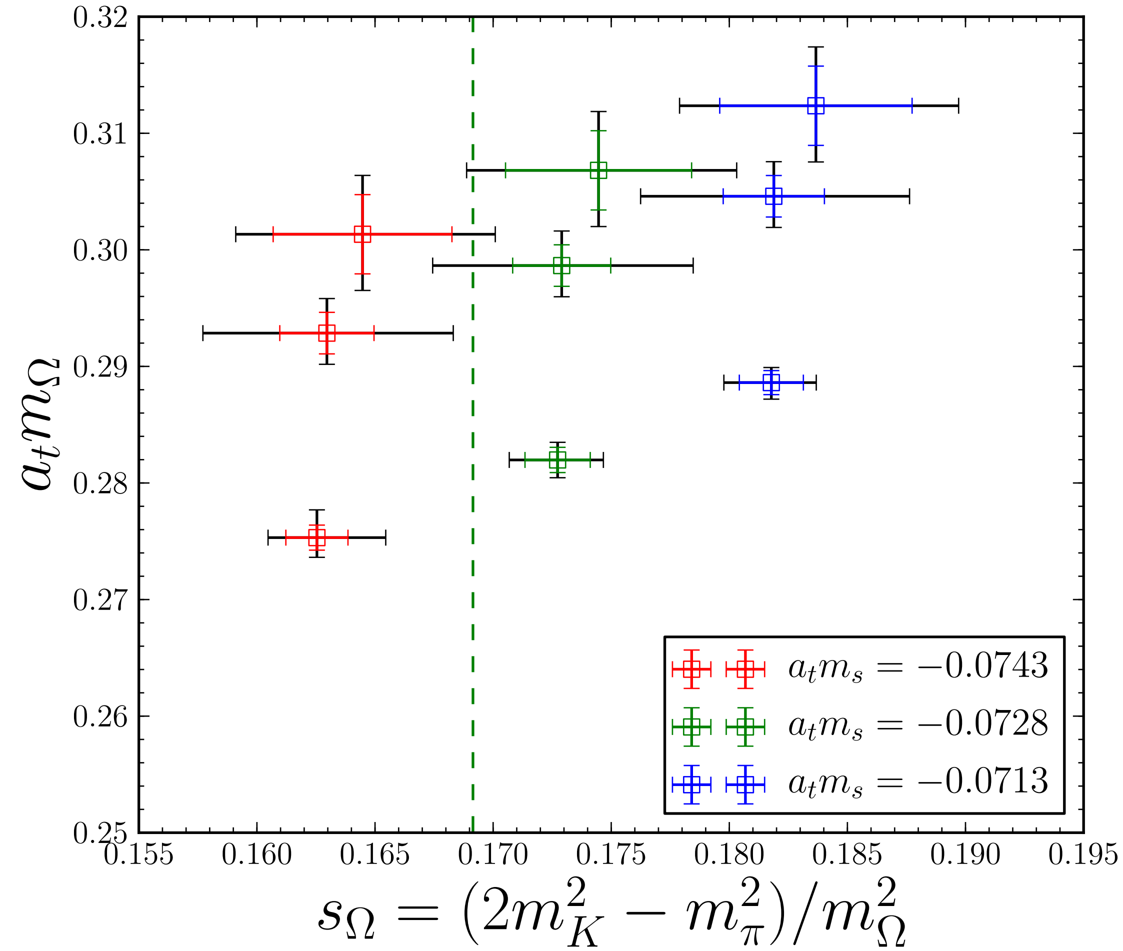
LQCD Calculation

Scale Setting

PRELIMINARY







Extrapolation and scale setting

$$m_\Omega(l_\Omega, s_\Omega) = m_0 + c_l^{(1)} l_\Omega + c_s^{(1)} s_\Omega + \cdots$$

but we also have the partially quenched results using SU(3) symmetry to motivate the formula:

$$m_\Omega(l_\Omega, s_\Omega^{sea}, s_\Omega^{val}) = m_0 + \tilde{c}_l^{(1)} \left(l_\Omega + \frac{1}{2} s_\Omega^{sea} \right) + \tilde{c}_s^{(1)} s_\Omega^{val} + \cdots$$

TABLE II: Correlated extrapolation

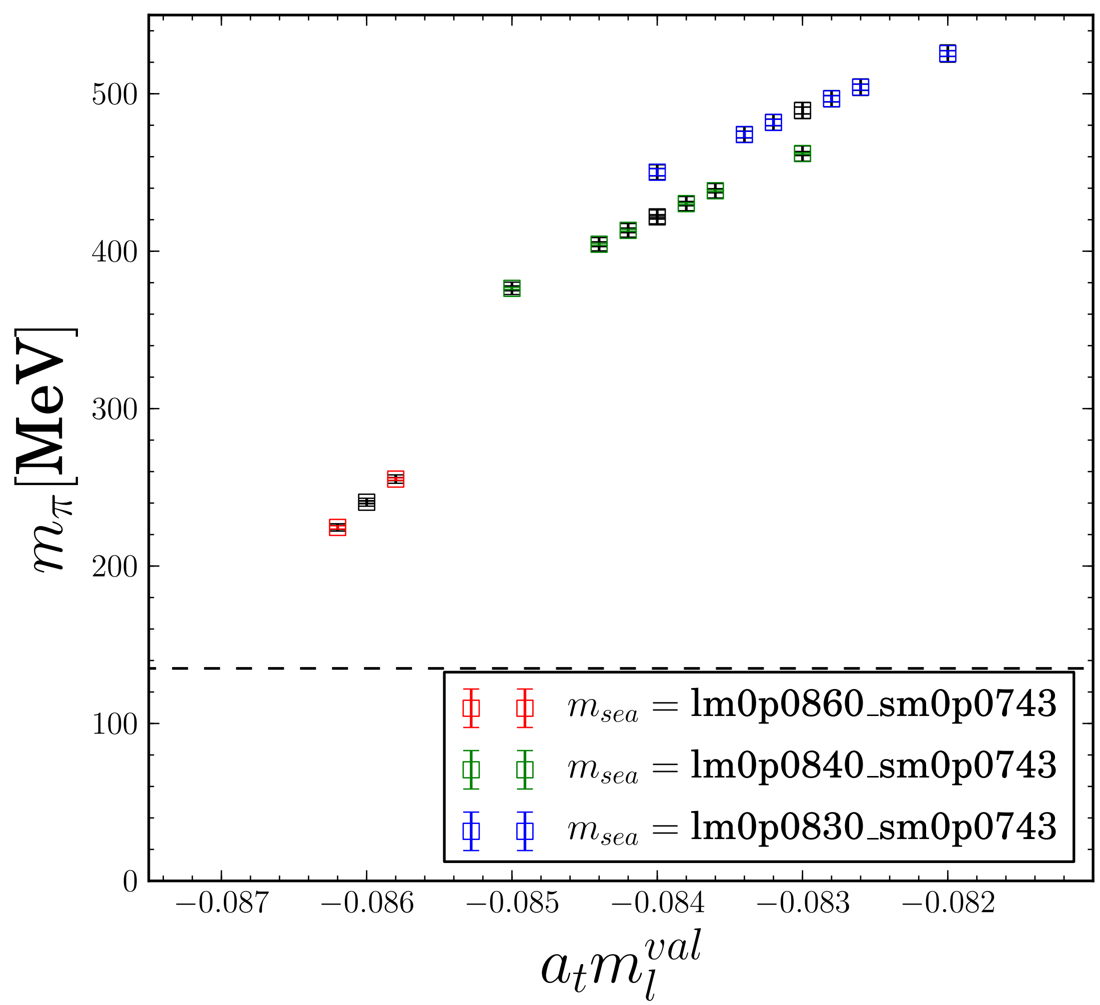
PQ	$a_t m_0$	$a_t c_l^{(1)}$	$a_t c_s^{(1)}$	χ^2	dof	Q	$a_t m_\Omega^{phys}$	a_t[fm]	a_t^{-1}[MeV]
no	0.146(13)(20)	0.49(6)(7)	0.73(7)(11)	6.70	6	0.35	0.2741(18)(22)	0.0322(2)(3)	6101(40)(49)
yes	0.108(14)(23)	0.48(6)(7)	0.73(7)(11)	6.94	6	0.33	0.2750(62)(78)	0.0324(7)(9)	6082(137)(173)

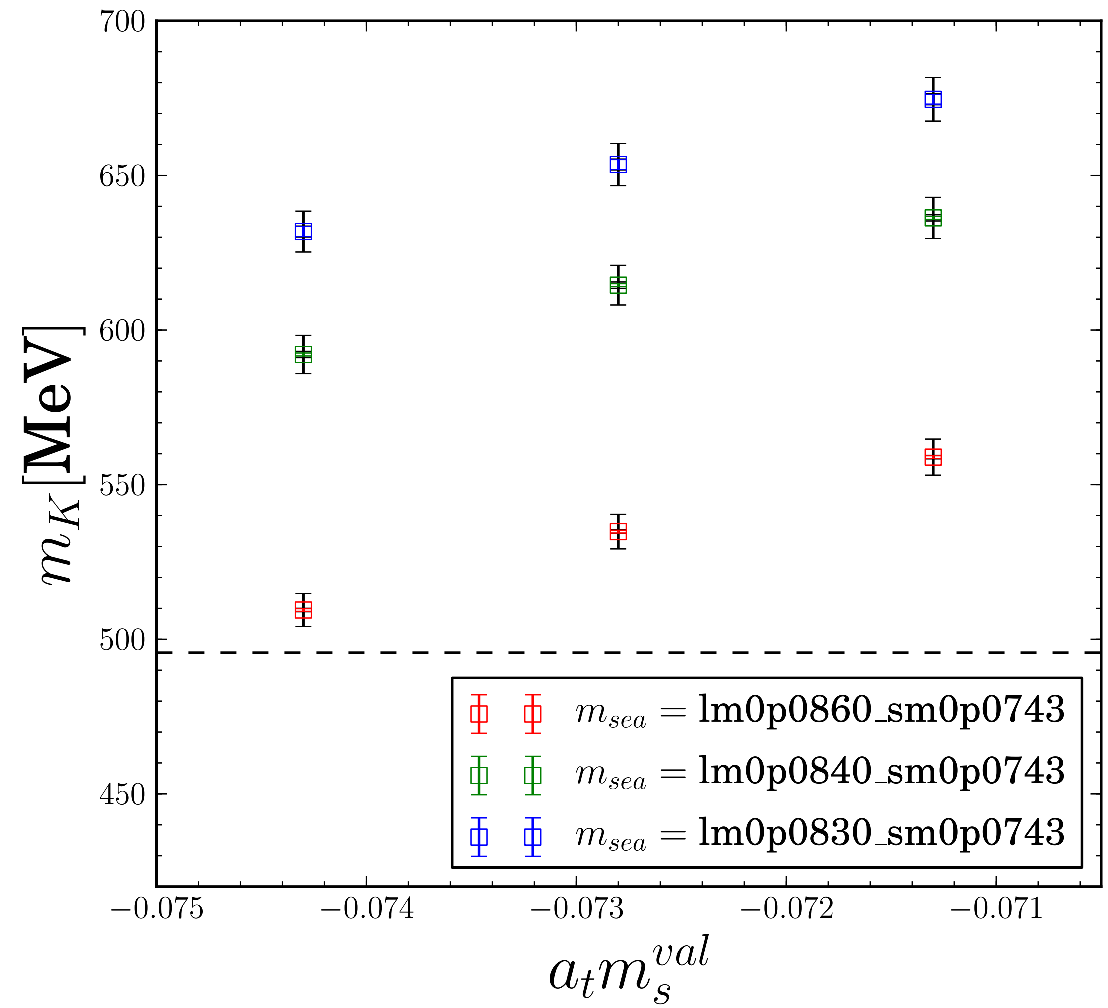
partial quenching has no discernible effect on scale or $c_{l,s}^{(1)}$

LQCD Calculation

Scale Setting

PRELIMINARY





ensemble				m_π	m_K	$a_t\delta$ $[N_{cfg} \times N_{src}]$			
L	T	$a_t m_l$	$a_t m_s$	[MeV]	[MeV]	0.0002	0.0004	0.0010	0.0020
16	128	-0.0830	-0.0743	488	620	207×16	207×16	207×16	207×16
16	128	-0.0840	-0.0743	420	591	166×25	166×25	166×25	166×50
20	128	-0.0840	-0.0743	420	591	120×25	–	–	–
24	128	-0.0840	-0.0743	420	591	97×25	–	193×25	–
32	256	-0.0840	-0.0743	420	591	291×10	291×10	291×10	–
24	128	-0.0860	-0.0743	240	508	118×26	–	–	–
32	256	-0.0860	-0.0743	240	508	842×11	–	–	–

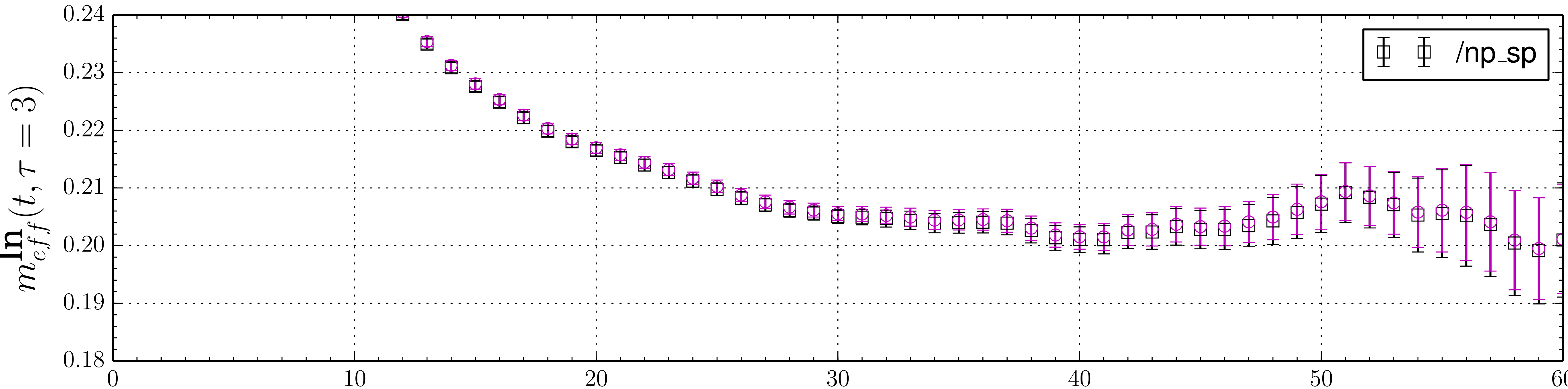
$$\delta^{latt}[\text{MeV}] = \begin{cases} 1.22 \\ 2.44 \\ 6.10 \\ 12.2 \end{cases}$$

not renormalized

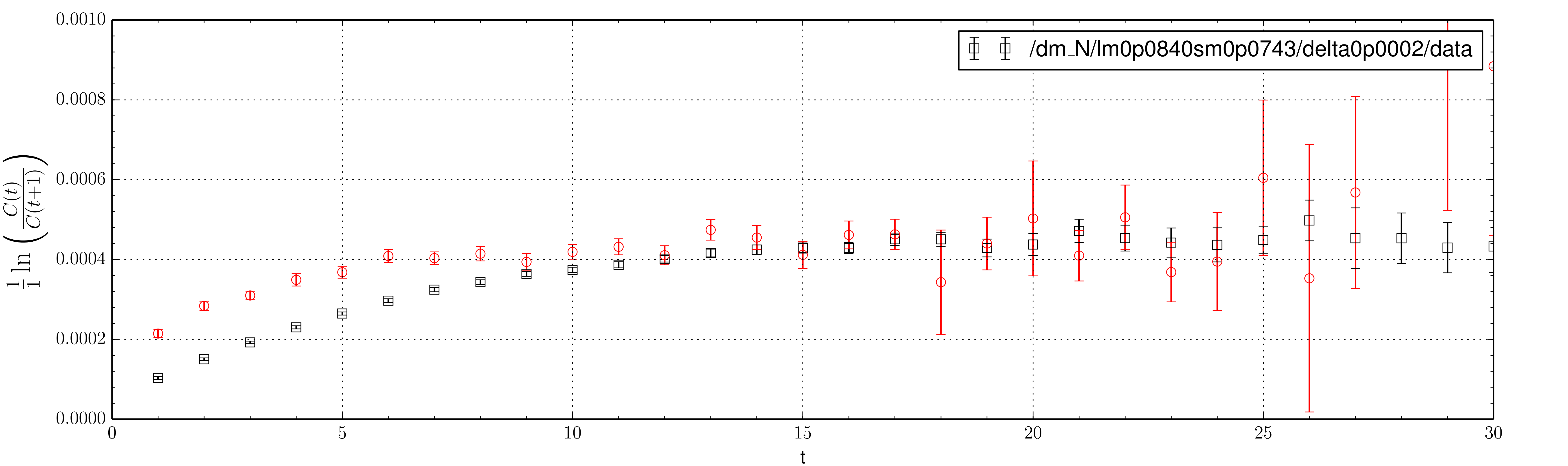
LQCD Calculation $M_n - M_p$

PRELIMINARY

Nucleon Mass Splitting



Ratio $C_n(t)$ / $C_p(t)$



LQCD Calculation $M_n - M_p$

PRELIMINARY

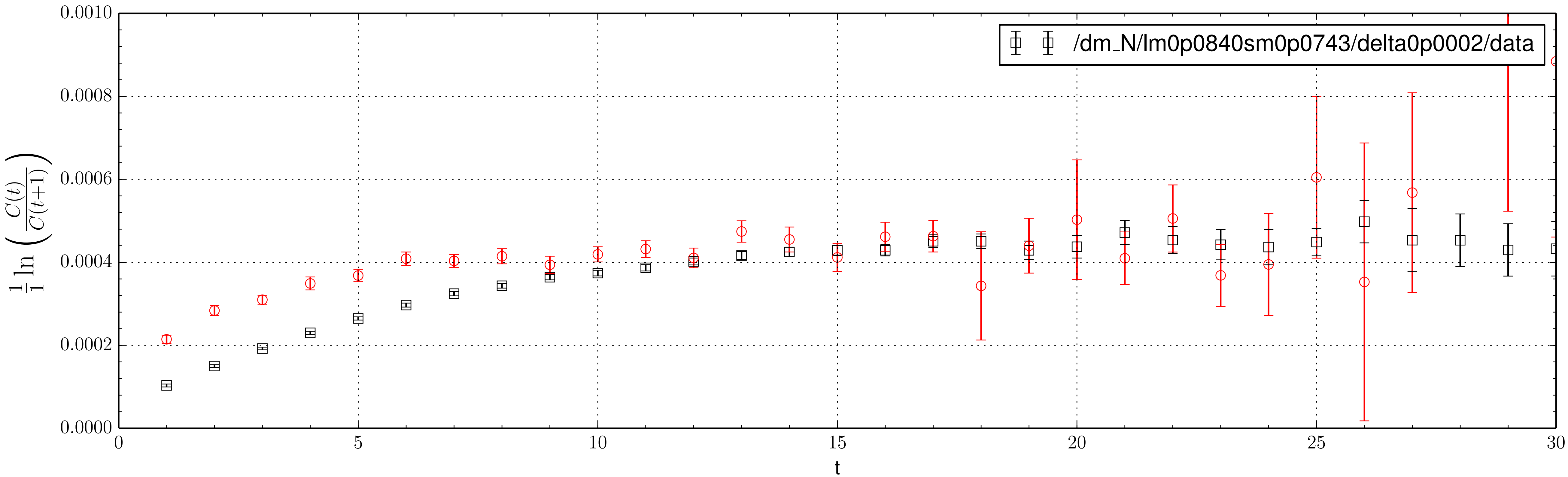
Nucleon Mass Splitting

$$\frac{C_n(t)}{C_p(t)} = e^{-\delta M_N t} \frac{A_0 + \delta_0^n + (A_1 + \delta_1^n) e^{-(\Delta + \delta\Delta^n)t} + \cdots}{A_0 + \delta_0^p + (A_1 + \delta_1^p) e^{-(\Delta+\delta\Delta^p)t}}$$

$$= e^{-\delta M_N t} \left\{ 1 + (\delta_0^n - \delta_0^p) + [\delta_1^n - \delta_1^p - A_1(\delta\Delta^n - \delta\Delta^p)t] \, e^{-\Delta t} \right\}$$

In ratio, excited state mass gap is the nucleon excited state, $\Delta >> M_n - M_p$

Ratio $C_n(t)$ / $C_p(t)$



LQCD Calculation $M_n - M_p$

PRELIMINARY

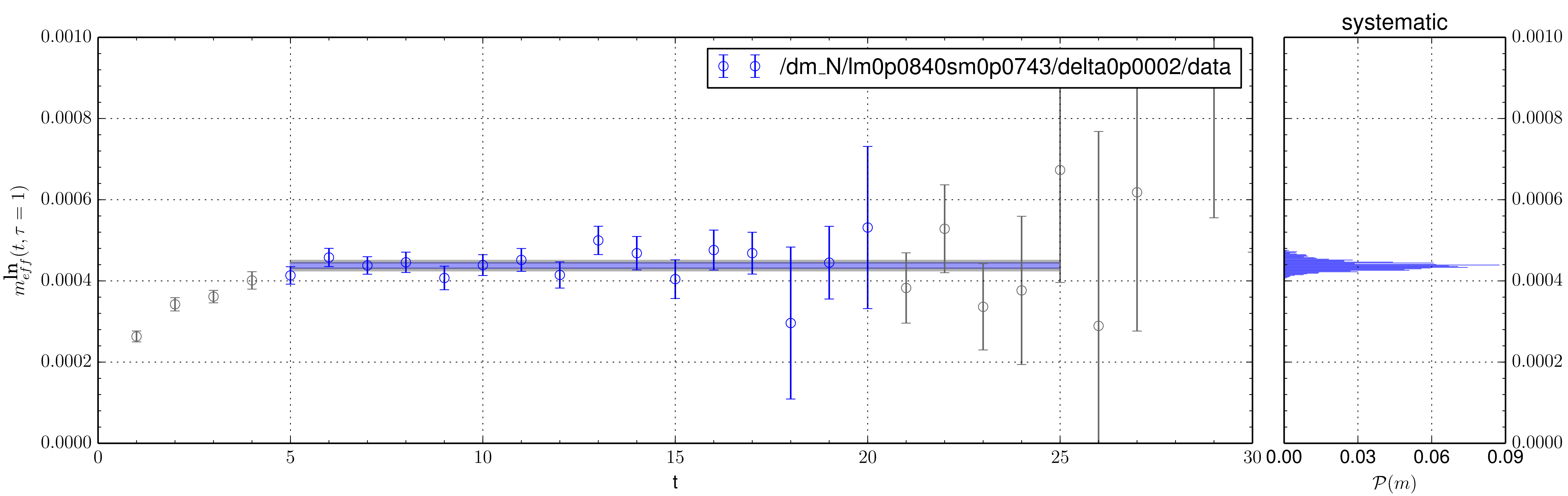
Nucleon Mass Splitting

Ratio $C_n(t)$ / $C_p(t)$

$$\frac{C_n(t)}{C_p(t)} = e^{-\delta M_N t} \frac{A_0 + \delta_0^n + (A_1 + \delta_1^n) e^{-(\Delta + \delta\Delta^n)t} + \cdots}{A_0 + \delta_0^p + (A_1 + \delta_1^p) e^{-(\Delta + \delta\Delta^p)t}}$$

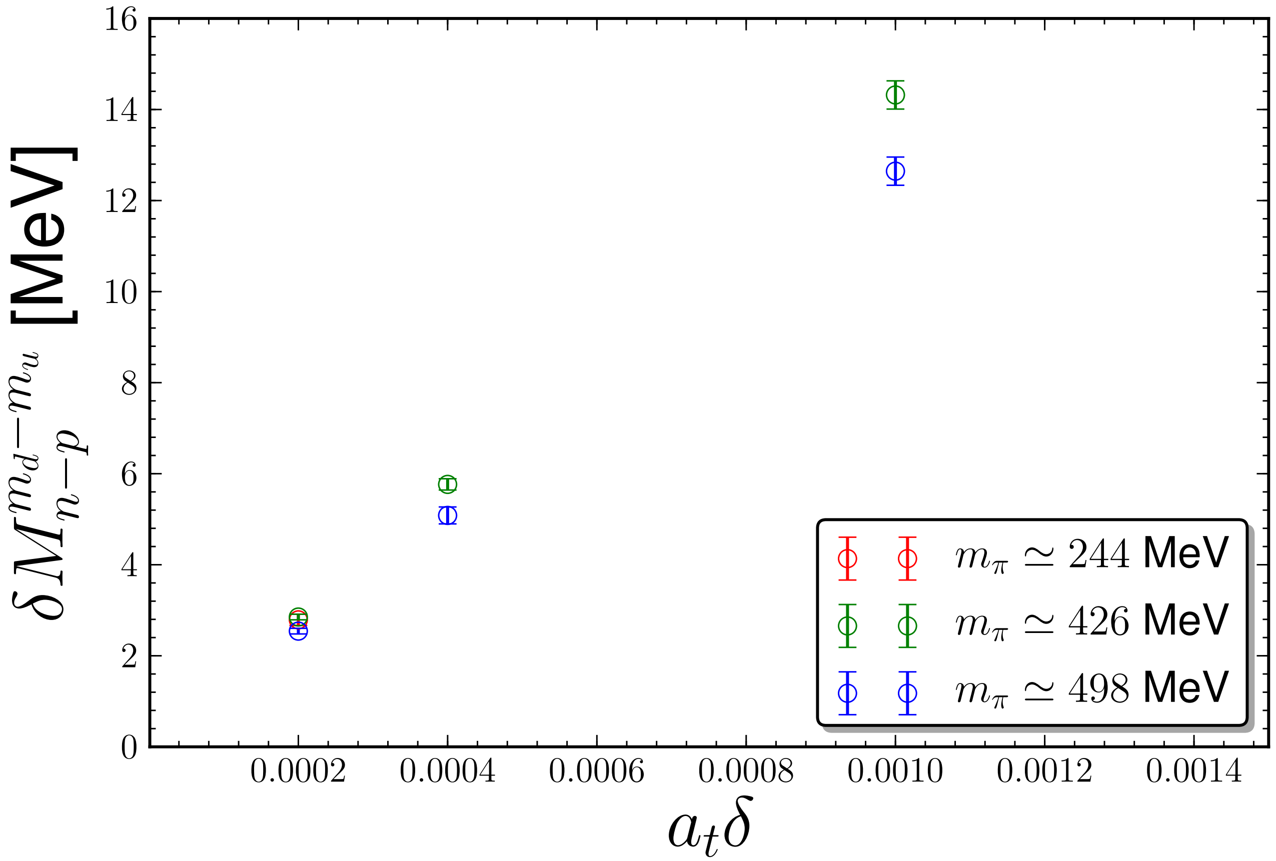
$$= e^{-\delta M_N t} \left\{ 1 + (\delta_0^n - \delta_0^p) + \left[\delta_1^n - \delta_1^p - A_1(\delta\Delta^n - \delta\Delta^p)t\right] e^{-\Delta t} \right\}$$

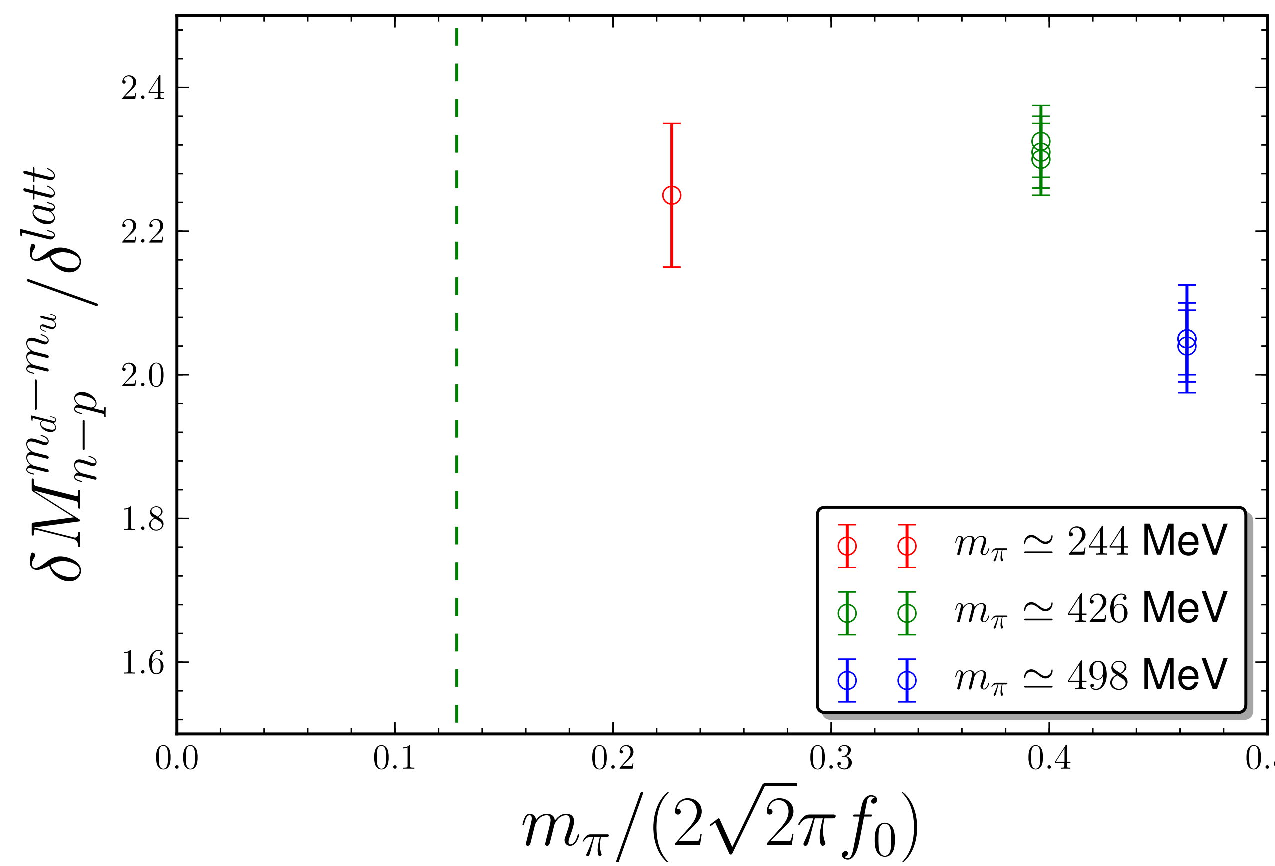
In ratio, excited state mass gap is the nucleon excited state, $\Delta >> M_n - M_p$



LQCD Calculation

PRELIMINARY



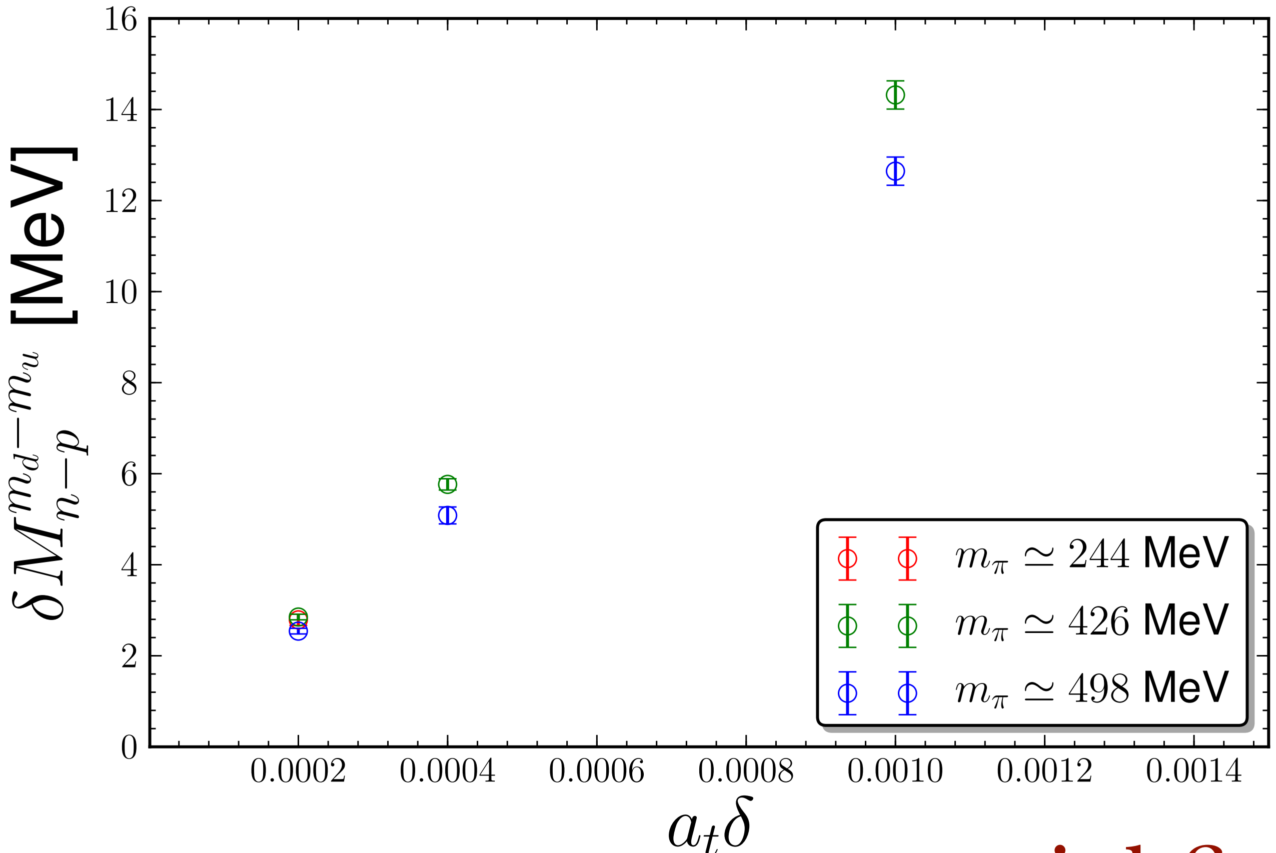


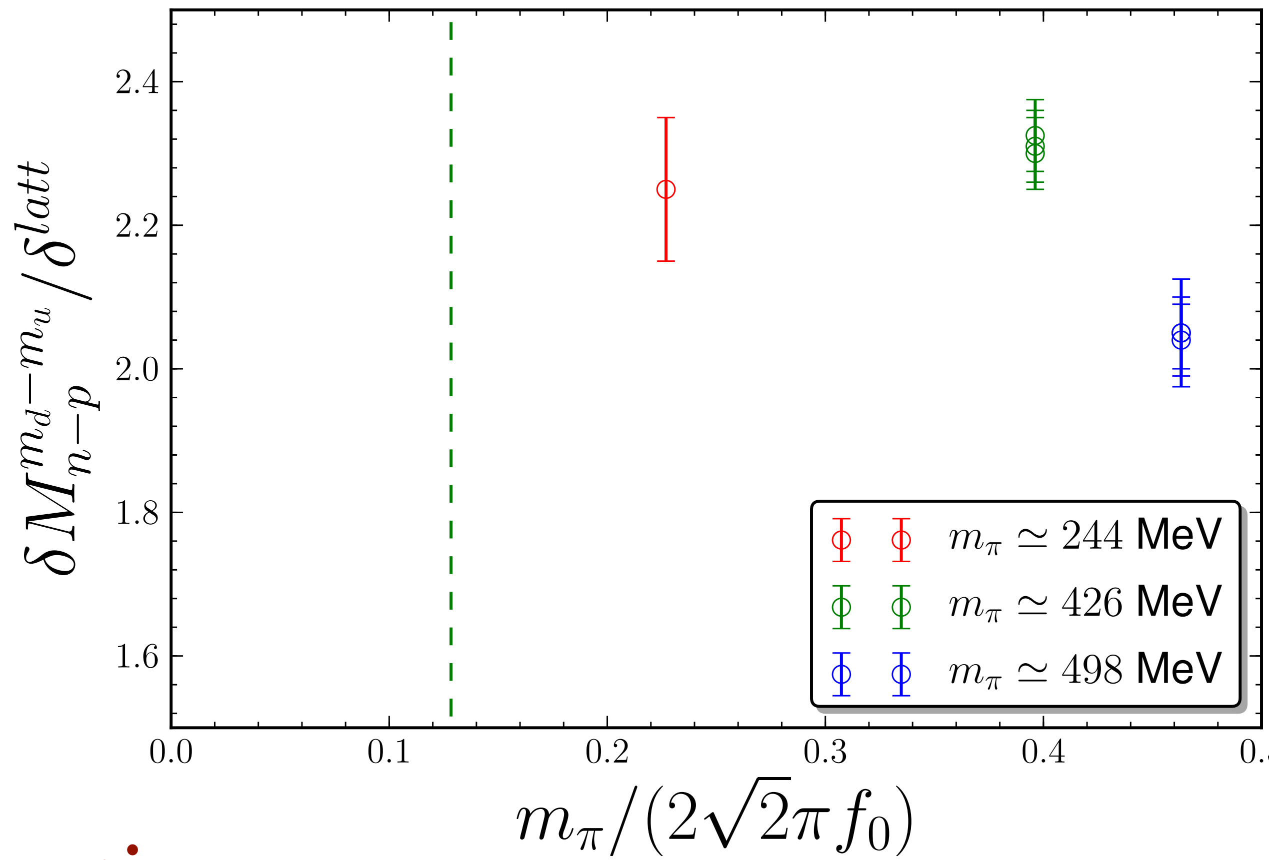
slope depends slightly on pion mass

no evidence for deviations from linear δ dependence

LQCD Calculation

PRELIMINARY





trial fit functions

polynomial in m_π^2

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha + \beta\frac{m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\chi^2/dof = 13/5 = 2.6$$

NNLO χPT

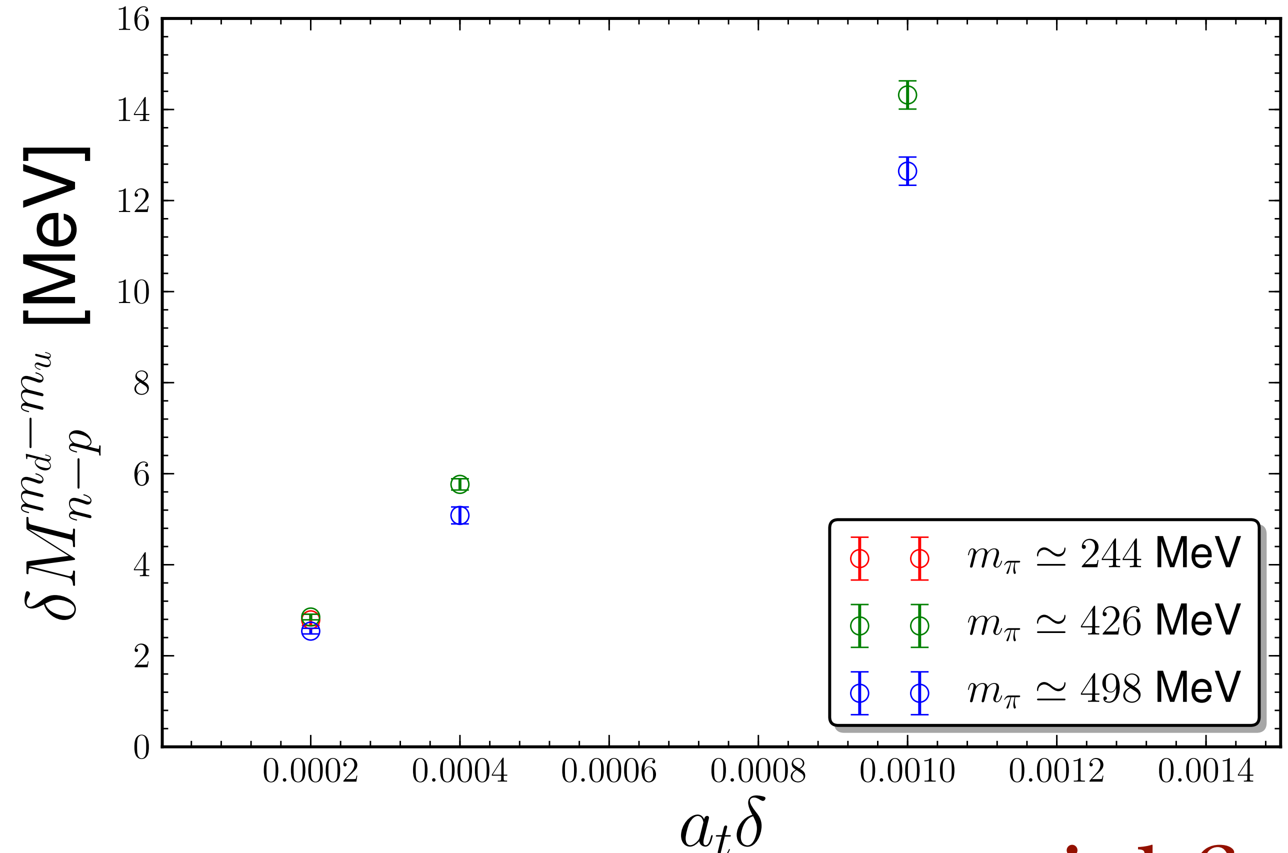
$$\delta M_{n-p}^{m_d-m_u} = \delta\Bigg\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\Bigg\}$$

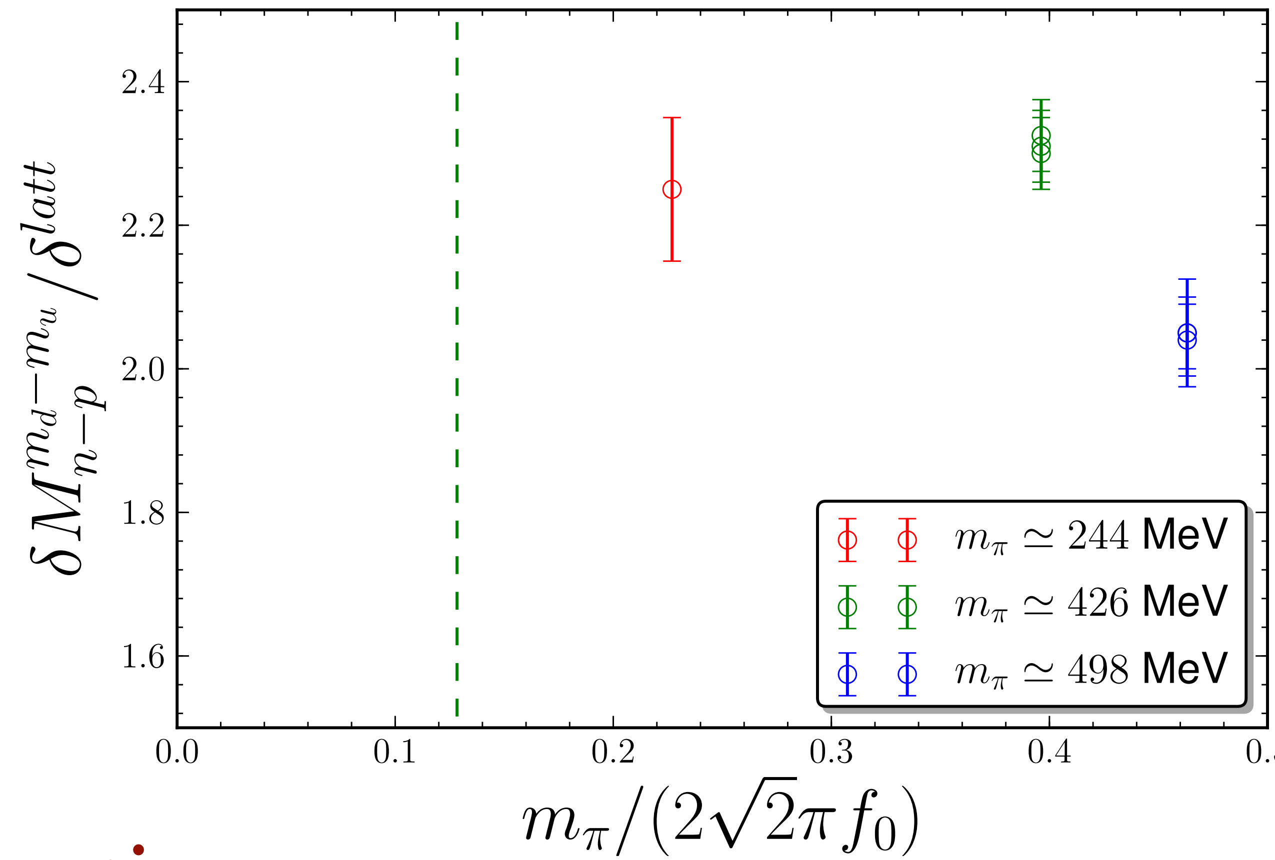
$(g_A = 1.27, f_\pi = 130 \text{ MeV})$

$$\chi^2/dof = 1.66/5 = 0.33$$

LQCD Calculation

PRELIMINARY





trial fit functions

polynomial in m_π^2

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha + \beta\frac{m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\chi^2/dof = 13/5 = 2.6$$

NNLO χPT

$$\delta M_{n-p}^{m_d-m_u} = \delta\Bigg\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\Bigg\}$$

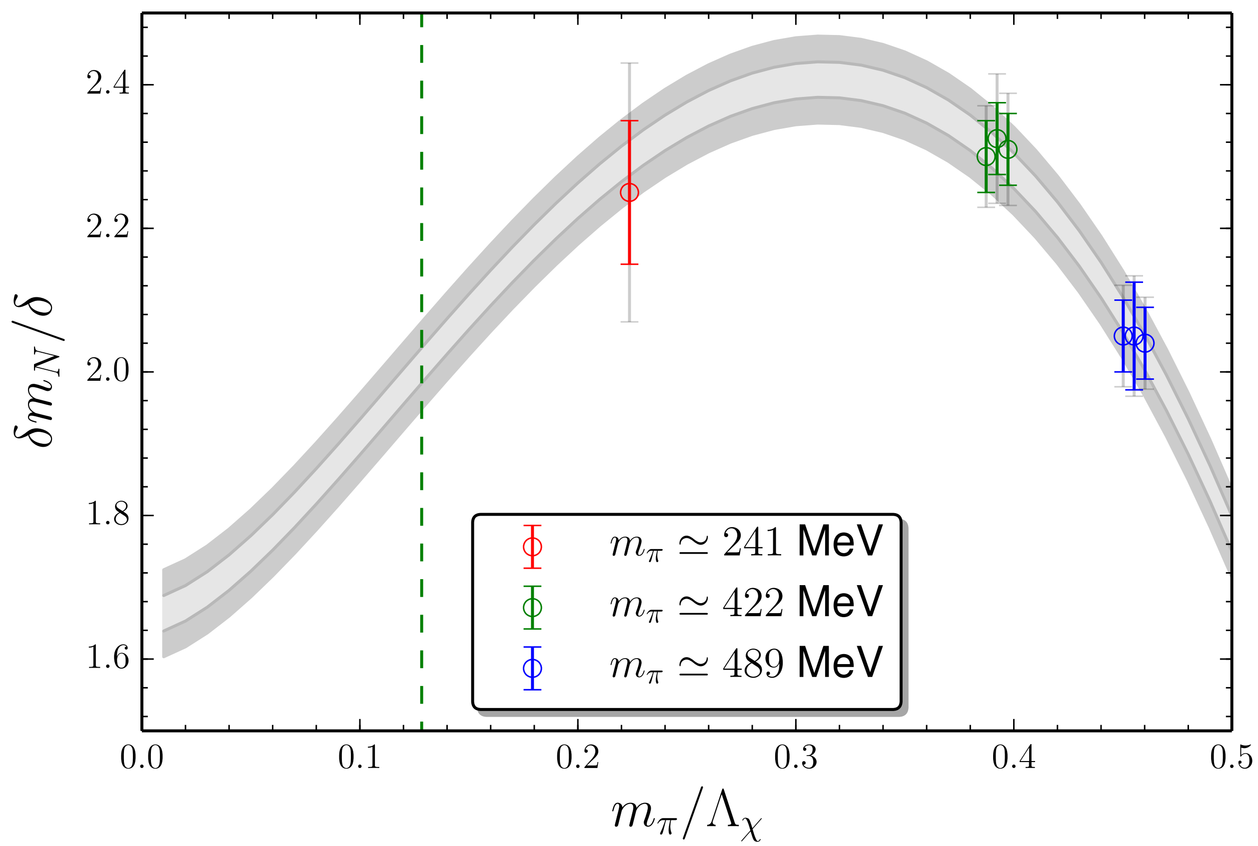
$(f_\pi = 130 \text{ MeV})$

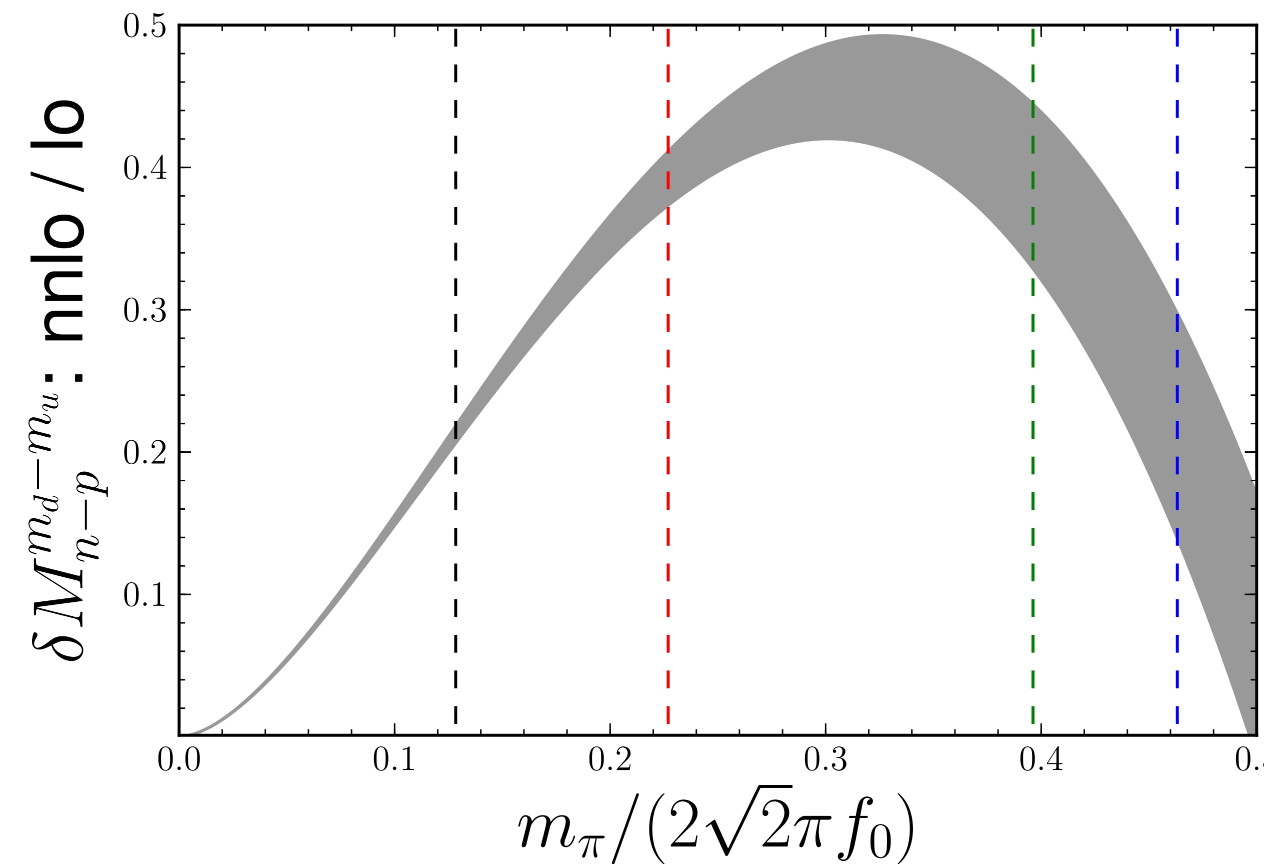
$$\chi^2/dof = 1.34/4 = 0.33$$

$$\Longrightarrow \quad g_A = 1.50(.29)$$

LQCD Calculation

PRELIMINARY





NNLO χPT

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$(g_A = 1.27, f_\pi = 130\ \mathrm{MeV})$

$$\chi^2/dof = 1.66/5 = 0.33$$

ratio of NNLO to LO correction

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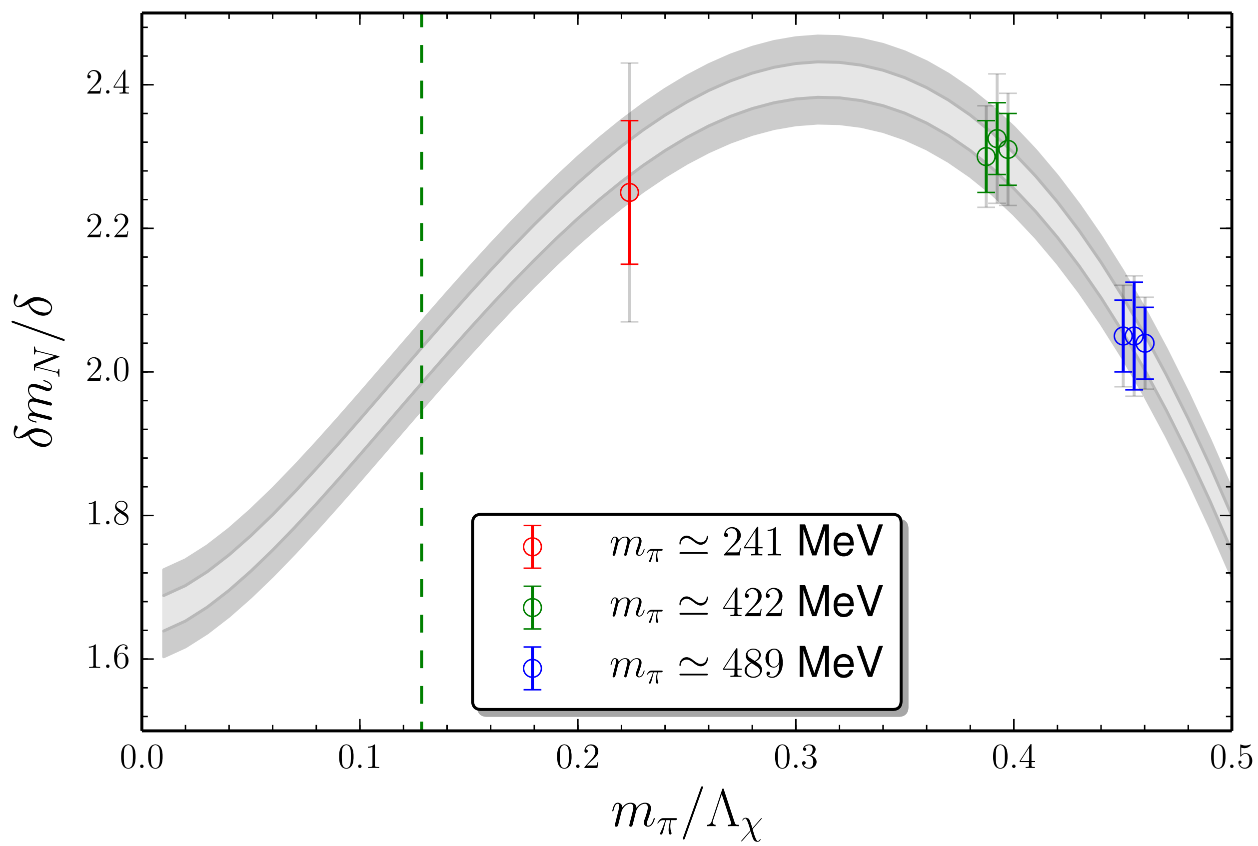
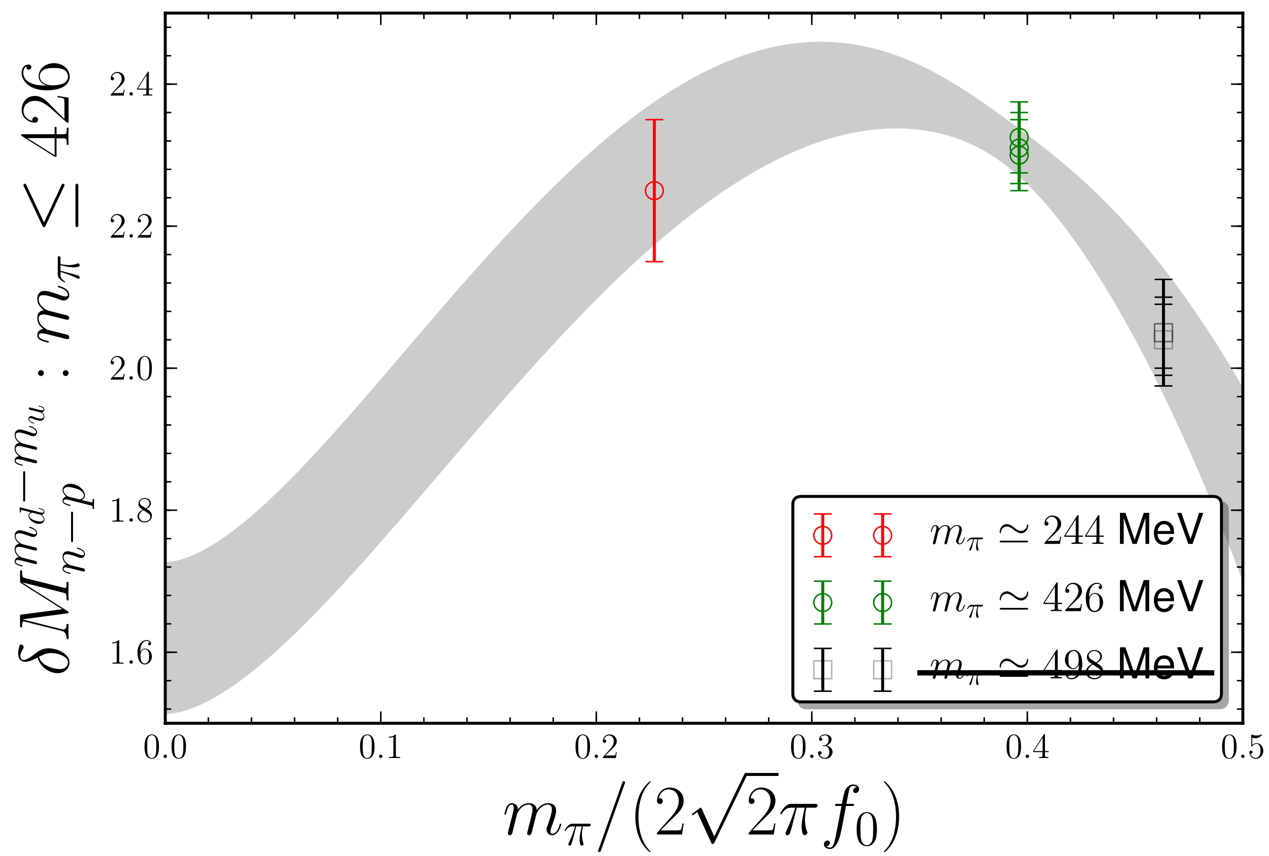
AWL

LQCD Calculation

PRELIMINARY





NNLO χPT

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$(g_A = 1.27, f_\pi = 130 \text{ MeV})$

$$\chi^2/dof = 1.66/5 = 0.33$$

exclude heavy mass point

this is striking evidence of a chiral logarithm

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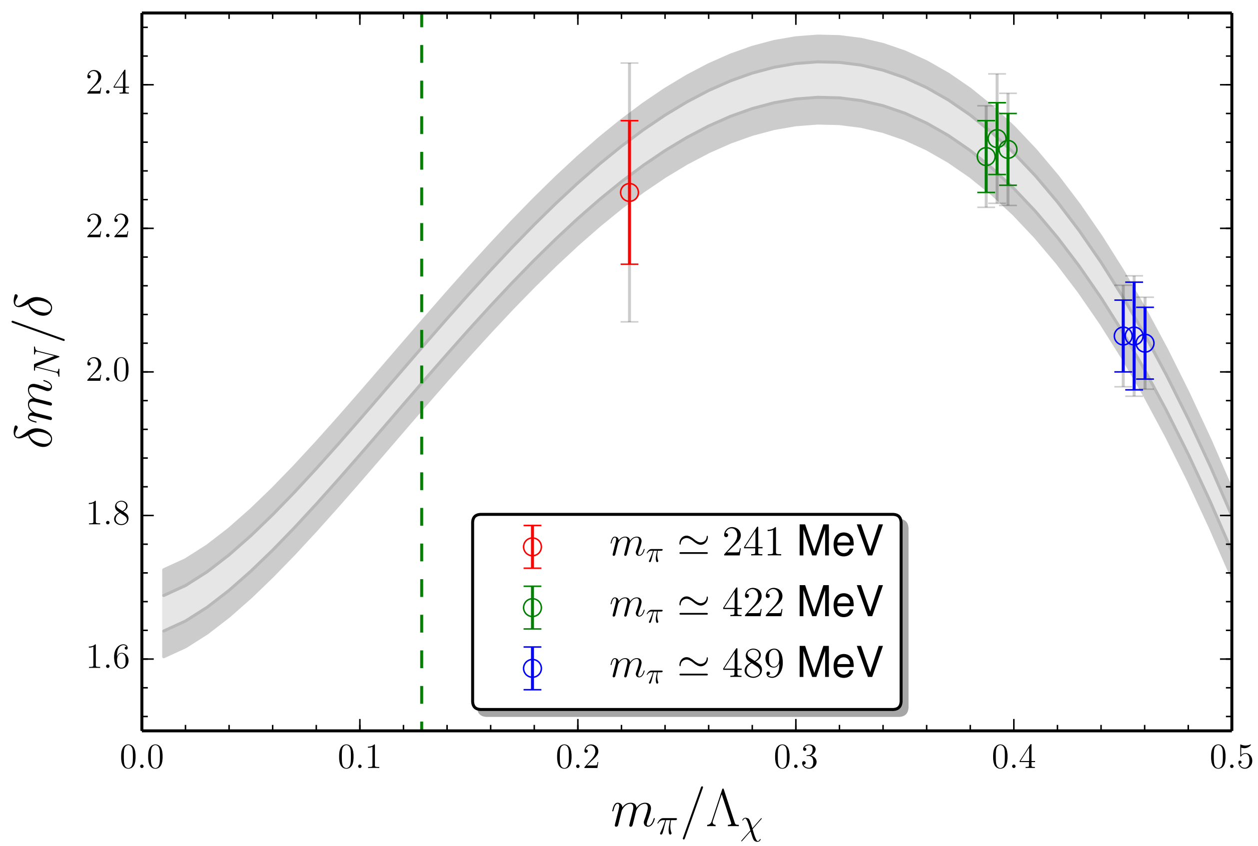
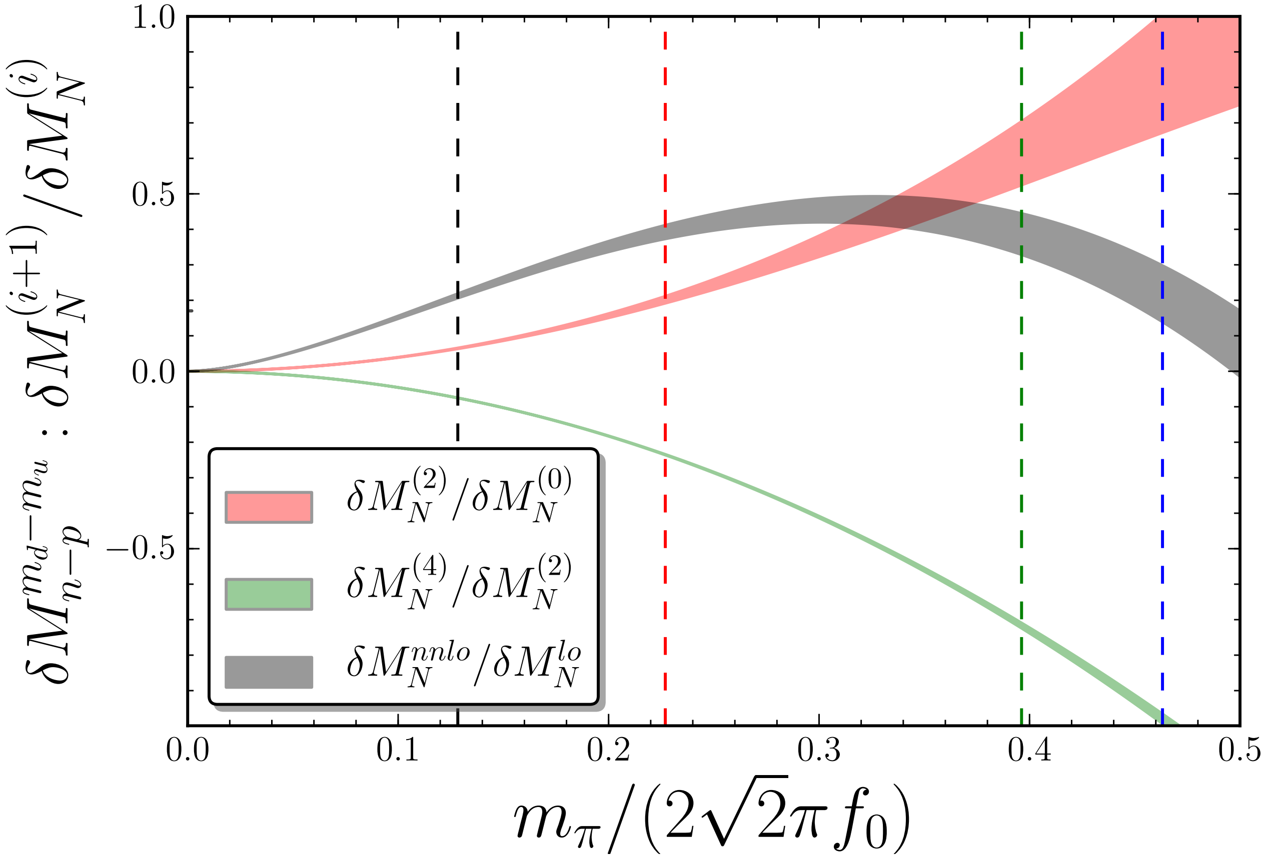
Emanuele Mereghetti,
K.Orginos, S.Syritsyn,
B.Tiburzi,
AWL

LQCD Calculation

PRELIMINARY





$$\delta M_{n-p}^{\delta} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\delta M_{n-p}^{\delta} = \delta\left\{\alpha + \beta\frac{m_\pi^2}{8\pi^2 f_\pi^2} + \gamma\frac{m_\pi^4}{(8\pi^2 f_\pi^2)^2}\right\}$$

adding $\gamma\frac{m_\pi^4}{(8\pi^2 f_\pi^2)^2}$ counterterm does not improve fit: γ consistent with zero

higher order polynomial gives good fit but poorer convergence

LQCD Calculation $M_{\Xi^-} - M_{\Xi^0}$ PRELIMINARY

If this story is true, we should be able to predict the behavior of the Ξ isospin splitting. The Ξ is also an iso-doublet, so the chiral Lagrangian will be identical in form, with only the LECs being different.

$$\delta M_N^{m_d-m_u} = \delta\left\{\alpha_N\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_N(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\delta M_\Xi^{m_d-m_u} = \delta\left\{\alpha_\Xi\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_\Xi^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_\Xi(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$g_A = 1.27 \qquad\qquad g_\Xi \simeq 0.24$$

Contribution from log should be ~10 times smaller, or, rather insignificant

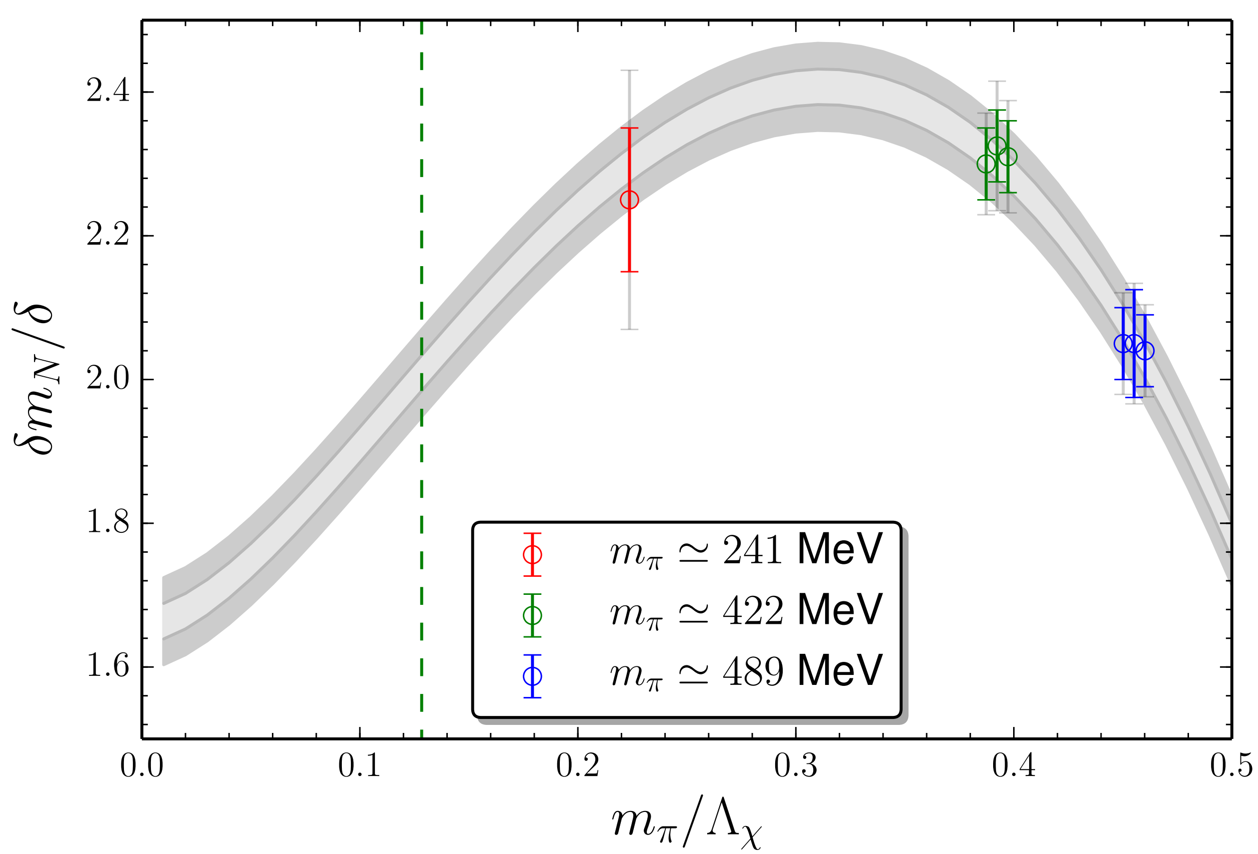
LQCD Calculation $M_{\Xi^-} - M_{\Xi^0}$

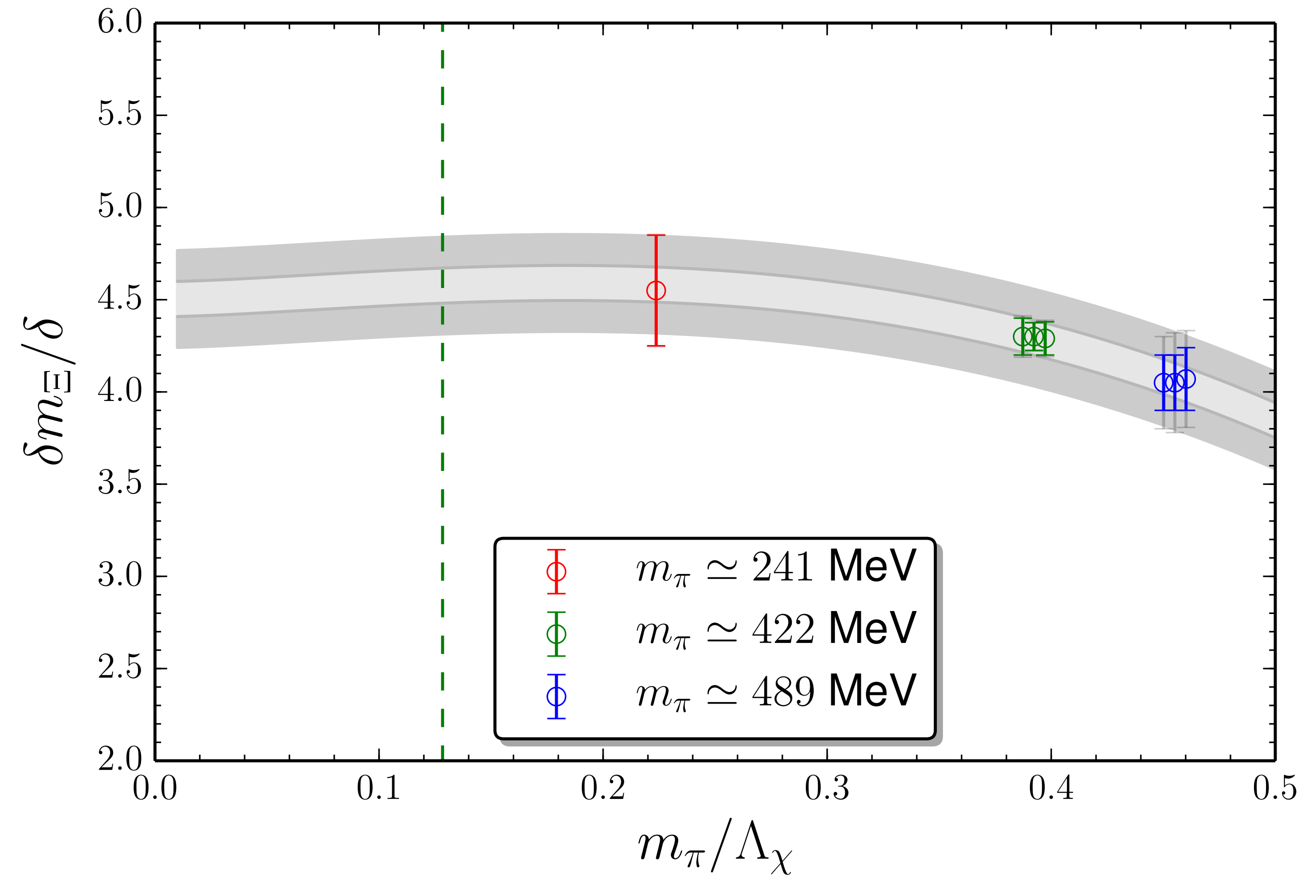
PRELIMINARY

$$\delta M_N^{m_d-m_u} = \delta\left\{\alpha_N\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_N(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\delta M_\Xi^{m_d-m_u} = \delta\left\{\alpha_\Xi\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_\Xi^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_\Xi(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$g_A = 1.27 \qquad\qquad g_\Xi \simeq 0.24$$





$$\alpha_N = 1.66 \pm 0.06 \pm 0.09 \qquad\qquad \alpha_\Xi = 4.50 \pm 0.24 \pm 0.42$$

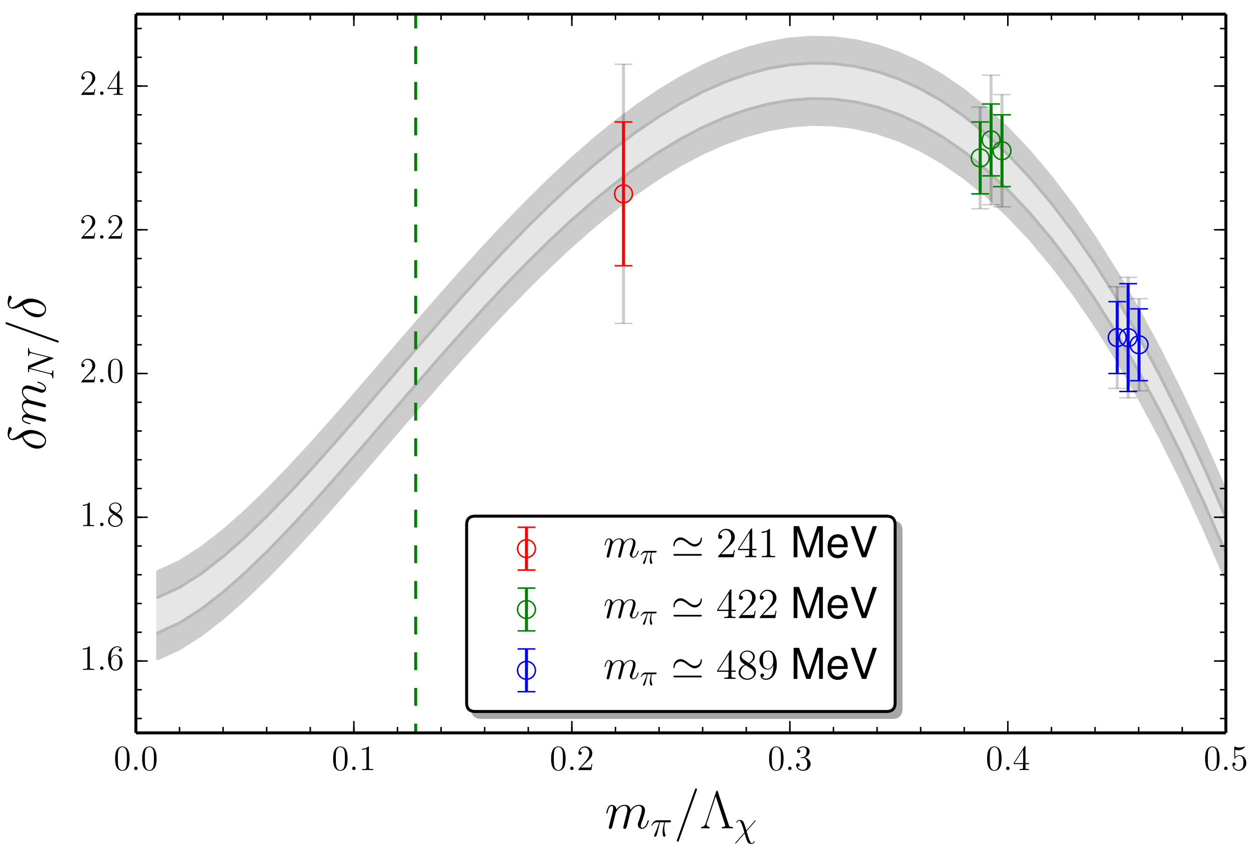
LQCD Calculation $\quad M_{\Xi^-} - M_{\Xi^0}$

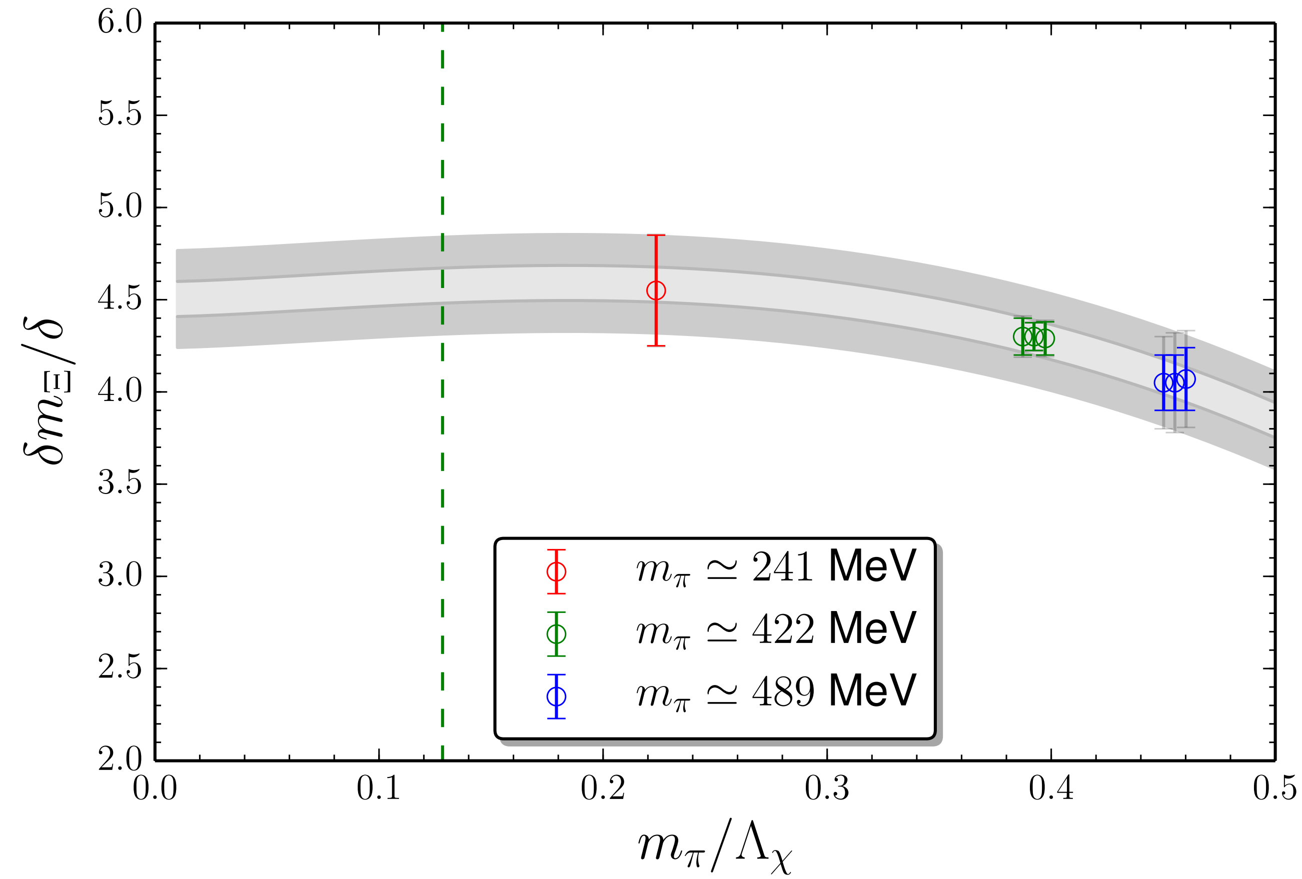
PRELIMINARY

$$\delta M_N^{m_d-m_u} = \delta\left\{\alpha_N\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_N(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\delta M_\Xi^{m_d-m_u} = \delta\left\{\alpha_\Xi\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_\Xi^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_\Xi(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$g_A = 1.27 \qquad\qquad g_\Xi \simeq 0.24$$





$$\alpha_N = 1.66 \pm 0.06 \pm 0.09 \qquad\qquad \alpha_\Xi = 4.50 \pm 0.24 \pm 0.42$$

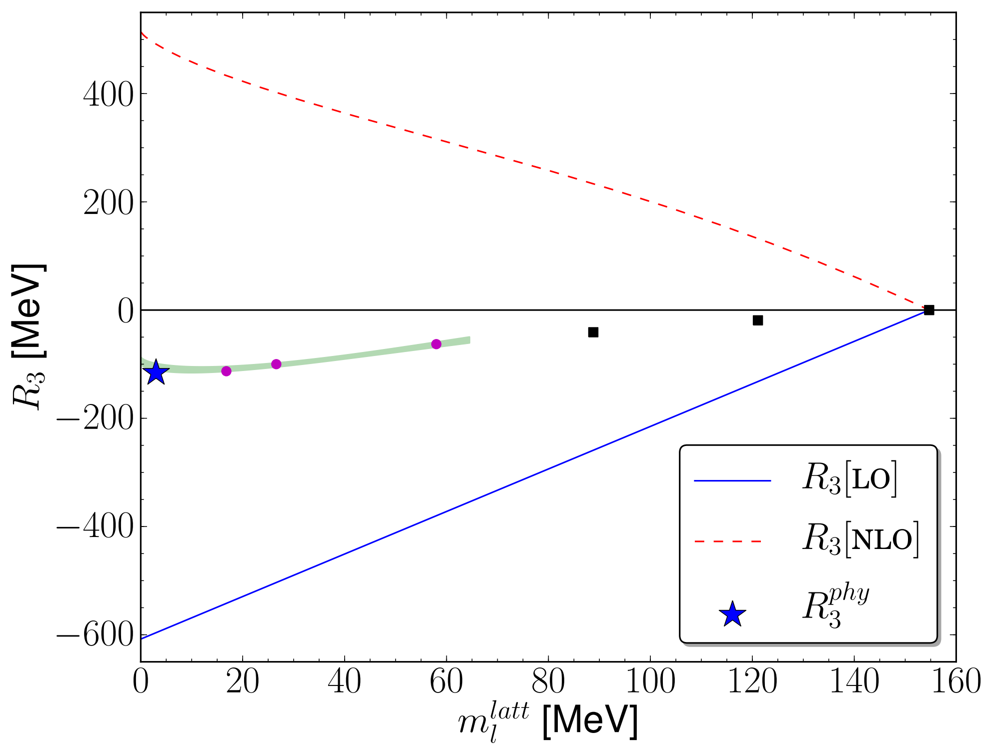
Expectations from SU(3):

$$\alpha_N = -(b_F + b_D) \qquad\qquad \alpha_\Xi = -(b_F - b_D)$$

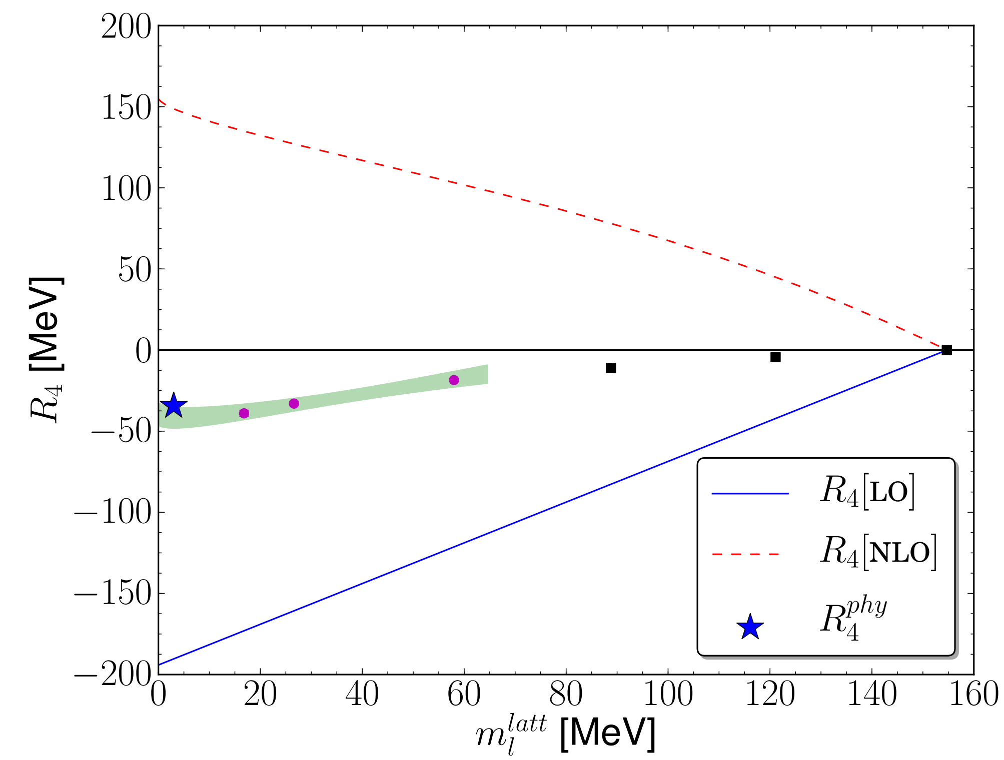
LQCD Calculation $M_{\Xi^-} - M_{\Xi^0}$ arXiv:1112.2658

Now for something really crazy: combine SU(3) and large-N$_c$ expansions

$$R_3 \propto m_s - m_l$$



$$R_4 \propto (m_s - m_l)/N_c$$



$$R_3(m_l, m_s) = \frac{20}{39}\, b_1\,(m_s - m_l) - \frac{20a_1^2 - 5a_2^2}{117}\frac{3\mathcal{F}_\pi^0 - 2\mathcal{F}_K^0 - \mathcal{F}_\eta^0}{(4\pi f)^2} - \frac{a_1^2}{117}\left[35\frac{3\mathcal{F}_\pi^\Delta - 2\mathcal{F}_K^\Delta - \mathcal{F}_\eta^\Delta}{(4\pi f)^2} - \frac{3\mathcal{F}_\pi^{-\Delta} - 2\mathcal{F}_K^{-\Delta} - \mathcal{F}_\eta^{-\Delta}}{(4\pi f)^2}\right],$$

$$b_D = \frac{1}{4}b_2$$

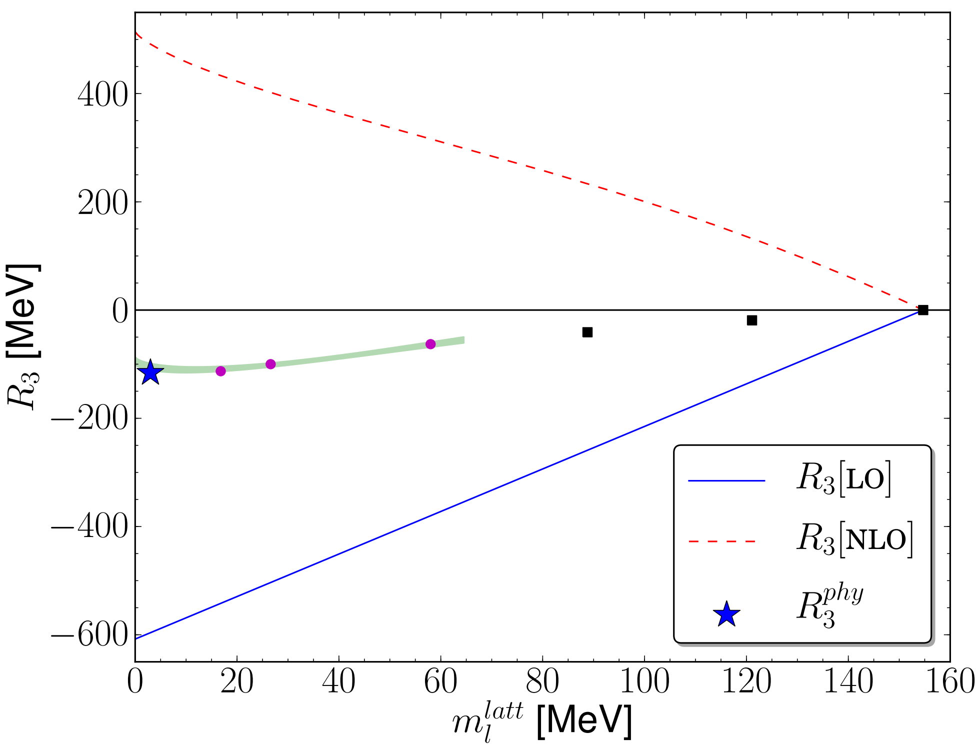
$$b_F = \frac{1}{2}b_1 + \frac{1}{6}b_2$$

$$R_4(m_l, m_s) = -\frac{5}{18}\, b_2\,(m_s - m_l) + \frac{a_1^2 + 4a_1a_2 + a_2^2}{36}\frac{3\mathcal{F}_\pi^0 - 2\mathcal{F}_K^0 - \mathcal{F}_\eta^0}{(4\pi f)^2} - \frac{2a_1^2}{9}\frac{3\mathcal{F}_\pi^\Delta - 2\mathcal{F}_K^\Delta - \mathcal{F}_\eta^\Delta}{(4\pi f)^2}$$

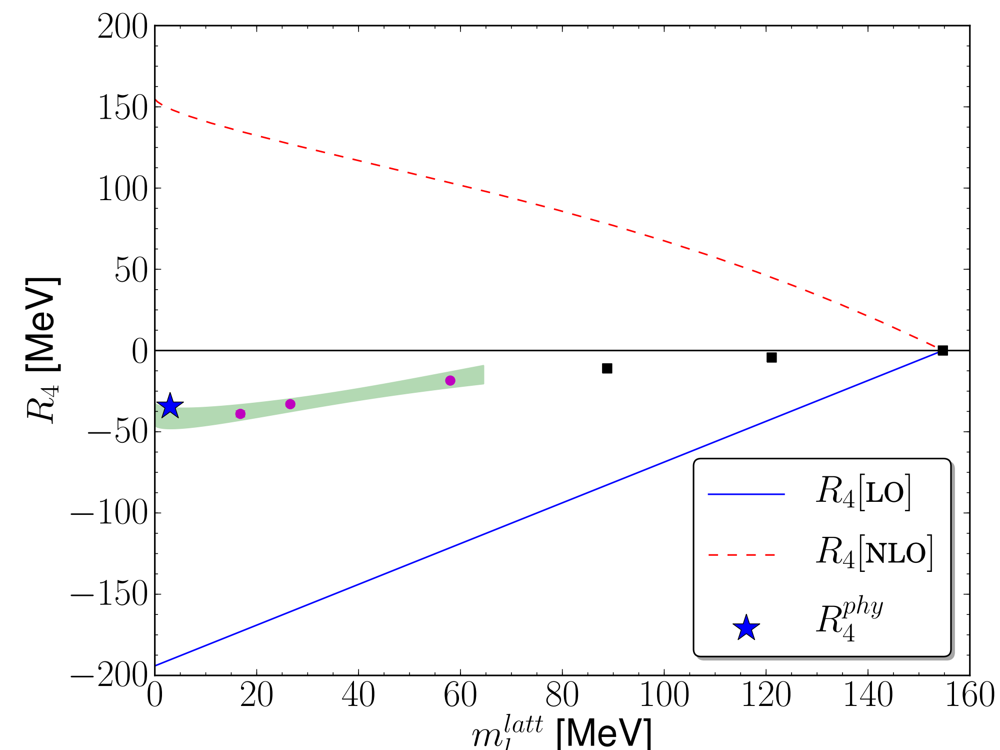
LQCD Calculation $M_{\Xi^-} - M_{\Xi^0}$ arXiv:1112.2658

Now for something really crazy: combine SU(3) and large-N$_c$ expansions

$$R_3 \propto m_s - m_l$$



$$R_4 \propto (m_s - m_l)/N_c$$



Fit yields

$$b_1[\mathrm{NLO}] = -6.6(5)\,, \qquad b_2[\mathrm{NLO}] = 4.3(4)\,, \qquad a_1[\mathrm{NLO}] = 1.4(1)\,.$$

$$D = 0.70(5)\,, \qquad F = 0.47(3)\,, \qquad C = -1.4(1)\,, \qquad H = -2.1(2)$$

$$b_D \simeq 1.07 \qquad b_F \simeq -2.58$$

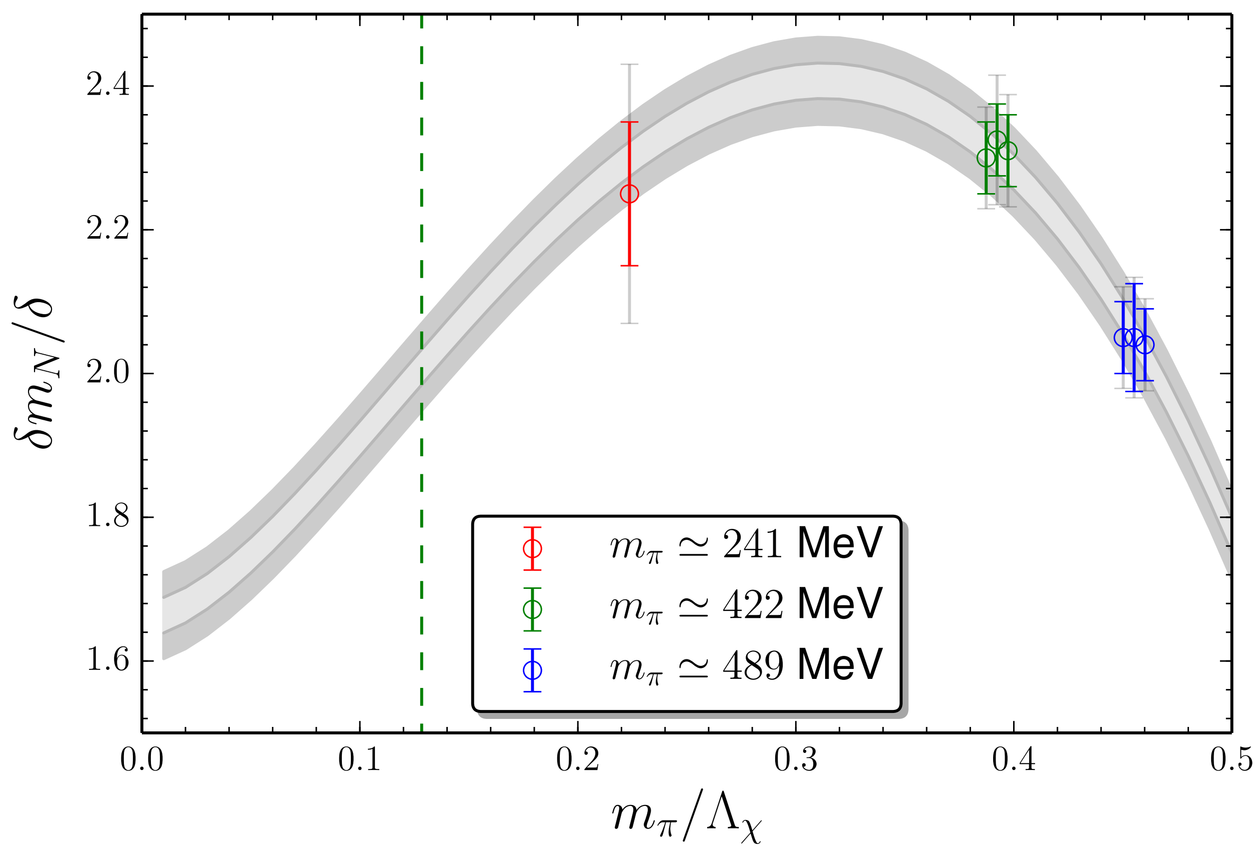
LQCD Calculation $M_{\Xi^-} - M_{\Xi^0}$

PRELIMINARY

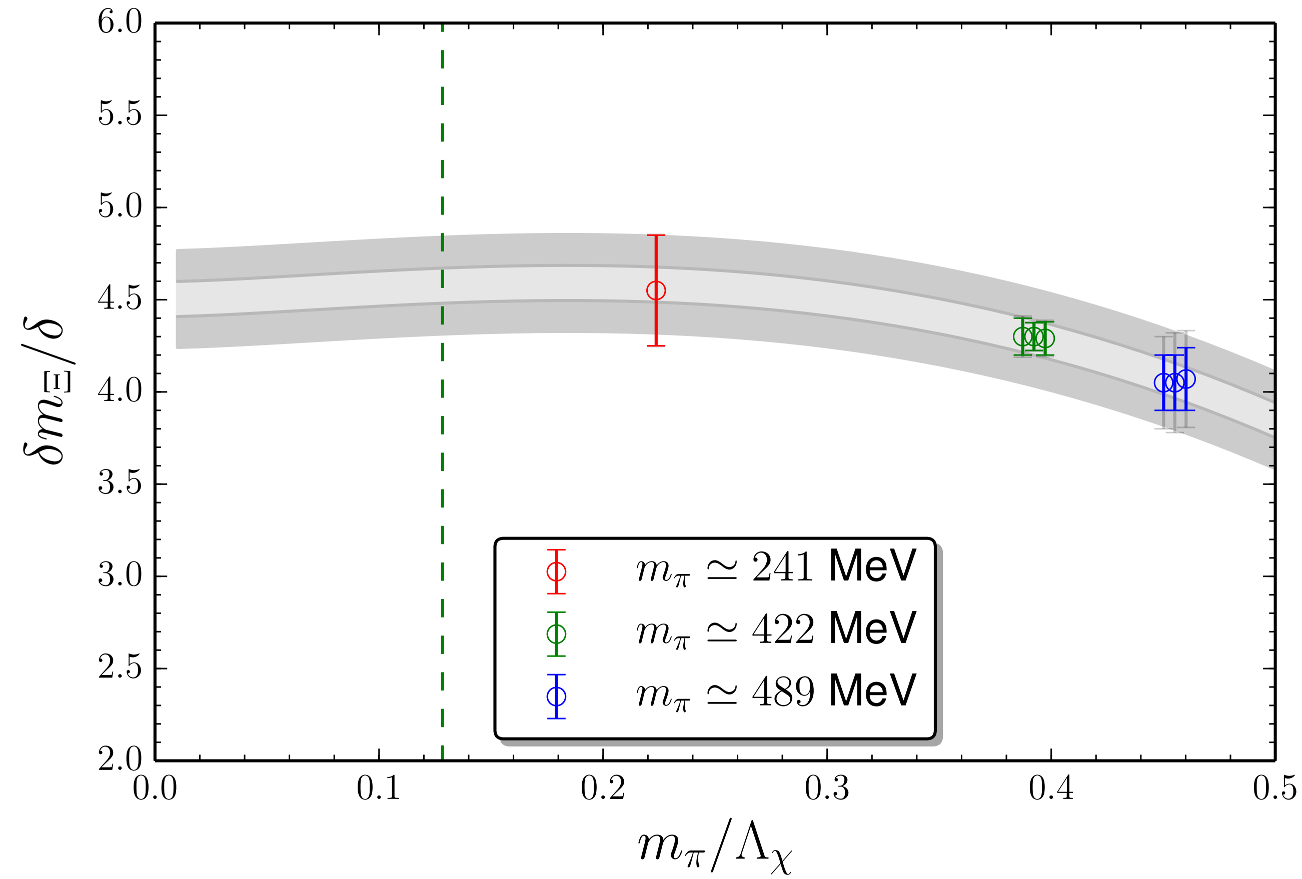
$$\delta M_N^{m_d-m_u} = \delta\left\{\alpha_N\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_N(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\delta M_\Xi^{m_d-m_u} = \delta\left\{\alpha_\Xi\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_\Xi^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_\Xi(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$g_A = 1.27$



$g_\Xi \simeq 0.24$



$\alpha_N = 1.66 \pm 0.06 \pm 0.09$

$\alpha_\Xi = 4.50 \pm 0.24 \pm 0.42$

$$\frac{\alpha_\Xi}{\alpha_N} = \frac{4.50}{1.66} \simeq 2.71$$

Expectations from SU(3):

$b_D \simeq 1.07 \qquad b_F \simeq -2.58$

$$\frac{\alpha_\Xi}{\alpha_N} = \frac{b_F - b_D}{b_F + b_D} \simeq 2.43$$

agreement within uncertainties and 10%!

Computational Strategy

- QCD Theta term

$$\mathcal{L}_{CPV} = -\frac{g_s^2\bar{\theta}}{32\pi^2}\tilde{G}_{\mu\nu}G^{\mu\nu} \Rightarrow \mathcal{L}^{\chi}_{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\vec{\pi}\cdot\vec{\tau}N$$

Symmetries $\Rightarrow$ $\bar{g}_0 = \frac{\delta M_{n-p}^{m_d-m_u}}{m_d-m_u}\frac{2m_d m_u}{m_d+m_u}\bar{\theta}$

$$\delta M_{n-p}^{m_d-m_u} = \alpha(m_d-m_u)$$

Simple spectroscopic calculation allows us to determine this long-range CP-Violating pion-nucleon coupling

This strategy was developed in conversations with Emanuele Mereghetti while we were both at LBNL

Computational Strategy

- Quark Chromo-EDM Operators

$$\mathcal{L}^{6}_{\bar{q}q} = -\frac{i}{2}\bar{q}\sigma^{\mu\nu}\gamma_5(\tilde{d}_0 + \tilde{d}_3\tau_3)G_{\mu\nu}q - \frac{1}{2}\bar{q}\sigma^{\mu\nu}(\tilde{c}_3\tau_3 + \tilde{c}_0)G_{\mu\nu}q$$

Computational Strategy

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Symmetries ⇒

$$\bar{g}_0 = \delta_q M_N \frac{\tilde{d}_0}{\tilde{c}_3} + \delta M_N \frac{\Delta_q m_\pi^2}{m_\pi^2}\frac{\tilde{d}_3}{\tilde{c}_0}$$

$$\bar{g}_3 = -2\sigma_{\pi N}\left(\frac{\Delta_q M_N}{\sigma_{\pi N}} - \frac{\Delta_q m_\pi^2}{m_\pi^2}\right)\frac{\tilde{d}_3}{\tilde{c}_0}$$

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Again, all that is needed are simple spectroscopic quantities

$$\begin{aligned}
\delta M_N &= \text{nucleon mass splitting induced by } \mathcal{O} = \delta\,\bar{q}\,\tau_3\,q\,,\\
\sigma_{\pi N} &= \text{nucleon sigma-term induced by } \mathcal{O} = -\bar{m}\bar{q}q\,,\\
\delta_q M_N &= \text{nucleon mass splitting induced by } \mathcal{O} = -(\tilde{c}_3/2)\,\bar{q}\sigma^{\mu\nu}\tau_3 G_{\mu\nu}q\,,\\
\Delta_q M_N &= \text{nucleon sigma-term induced by } \mathcal{O} = -(\tilde{c}_0/2)\,\bar{q}\sigma^{\mu\nu}G_{\mu\nu}q\,,\\
\Delta_q m_\pi^2 &= \text{pion sigma-term induced by } \mathcal{O} = -(\tilde{c}_0/2)\,\bar{q}\sigma^{\mu\nu}G_{\mu\nu}q\,,
\end{aligned}$$

Computational Strategy

- Quark Chromo-EDM Operators

$$\mathcal{O}_0 = -\frac{1}{2}\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \qquad \mathcal{O}_3 = -\frac{1}{2}\bar{q}\tau_3\sigma_{\mu\nu}G^{\mu\nu}q$$

You may recognize these operators...

Computational Strategy

- Quark Chromo-EDM Operators

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You may recognize these operators...

The quantities of interest can be determined by making use of the Feynman-Hellman Theorem and simple spectroscopic LQCD calculations

$$\Delta_q M_N = \tilde{c}_0 \frac{\partial M_N[\tilde{c}_0\mathcal{O}_0]}{\partial \tilde{c}_0} \qquad \delta_q M_N = \tilde{c}_3 \frac{\partial M_N[\tilde{c}_c\mathcal{O}_3]}{\partial \tilde{c}_3}$$

Computational Strategy

- Quark Chromo-EDM Operators

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We also need to determine

$$\sigma_{\pi N} = m_l \frac{\partial M_N}{\partial m_l} \qquad\qquad \delta M_N = \delta \frac{\partial M_N}{\partial \delta}$$

$$m_l = \frac{m_d + m_u}{2} \qquad\qquad \delta = \frac{m_d - m_u}{2}$$

Computational Strategy

- Quark Chromo-EDM Operators

$$\mathcal{O}_0 = -\frac{1}{2}\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \qquad \mathcal{O}_3 = -\frac{1}{2}\bar{q}\tau_3\sigma_{\mu\nu}G^{\mu\nu}q$$

Simple spectroscopic LQCD calculations can be used to determine these important long-range CP-Violating pion-nucleon couplings

Spectroscopic calculations are what we are best at

Computational Strategy

- Quark Chromo-EDM Operators

$$\mathcal{O}_0 = -\frac{1}{2}\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \qquad\qquad \mathcal{O}_3 = -\frac{1}{2}\bar{q}\tau_3\sigma_{\mu\nu}G^{\mu\nu}q$$

  The leading contribution will come from the valence quarks (experience with valence/sea quark mass contribution to nucleon mass) - begin with this contribution

- Invert the valence quarks with a modified Dirac operator

$$D_\lambda = D + \lambda\{\mathcal{O}_0, \mathcal{O}_3\}$$

- Construct nucleon correlation function with these quarks and determine the resulting nucleon mass

- Vary λ and determine slope of mass correction to get derivative

$$\Delta_q M_N = \tilde{c}_0 \frac{\partial M_N[\tilde{c}_0\mathcal{O}_0]}{\partial\tilde{c}_0} \qquad\qquad \delta_q M_N = \tilde{c}_3 \frac{\partial M_N[\tilde{c}_c\mathcal{O}_3]}{\partial\tilde{c}_3}$$

Choice of Action

- Our choice of action for these calculations is

  Domain-Wall Valence fermions on dynamical HISQ ensembles

  There are several reasons for this choice

  it is not my fond familiarity with Mixed-Action calculations :)

Choice of Action

- Our choice of action for these calculations is

  Domain-Wall Valence fermions on dynamical HISQ ensembles

  - control of chiral symmetry is important - renormalization
  - calculations at/near physical pion mass
  - multiple volumes to perform volume study
  - multiple lattice spacings for continuum limit
  - publicly available configurations

Choice of Action

- The only choice of dynamical configurations which satisfy these criteria are the dynamical HISQ ensembles from MILC with 2+1+1 dynamical flavors

a [fm]	m_l/m_s	V	$m_\pi L$	m_π [MeV]
0.15	1/5	$16^3 \times 48$	3.78	306
0.15	1/10	$24^3 \times 48$	3.99	217
0.15	1/27	$32^3 \times 48$	3.30	135
0.12	1/5	$24^3 \times 64$	4.54	309
0.12	1/10	$24^3 \times 64$	3.22	221
0.12	1/10	$32^3 \times 64$	4.29	221
0.12	1/10	$40^3 \times 64$	5.36	221
0.12	1/27	$48^3 \times 64$	3.88	135
0.09	1/5	$32^3 \times 96$	4.50	314
0.09	1/10	$48^3 \times 96$	4.71	221
0.09	1/27	$64^3 \times 96$	3.66	130

Choice of Action

- To control the chiral symmetry and greatly simplify the operator renormalization, Domain-Wall valence fermions are very suitable choice

Choice of Action

- To control the chiral symmetry and greatly simplify the operator renormalization, Domain-Wall valence fermions are very suitable choice
- Consider the contribution from the “clover operator”

$$\mathcal{O}_0 = -\frac{1}{2}\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q$$

  - one may first think clover-Wilson fermions are perfect, as a simple re-tuning of the clover-coefficient is all that is needed
  - changing the clover-coefficient will change the nucleon mass with an additive mix of both the physical shift of interest and a discretization correction to the quark mass
  - disentangling the physical shift to the nucleon mass from this operator would involve a complicated tuning problem involving non-perturbative renormalization for each choice of clover-coefficient to subtract unphysical discretization effects

Choice of Action

- To control the chiral symmetry and greatly simplify the operator renormalization, Domain-Wall valence fermions are very suitable choice
- There are other simplifications which occur with DWF involving both the chiral symmetry properties and renormalization

Outlook

- By exploiting symmetries, we can study the modification of the nucleon spectrum in the presence of CP conserving operators which will then determine the values of the CP-violating π-N couplings which arise from the quark chromo-EDM operators
- The long-range CPV pion-nucleon couplings could allow for a direct connection with fundamental coefficients in dimension-6 quark operators and real nuclear physics, ^{225}Ra
- We hope to have initial results this year

Chris Bouchard, Chris Monahan, Emanuele Mereghetti, Kostas Orginos, AWL, …

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Thank You!