Electroweak Radiative Corrections & CKM Unitarity

W. J. Marciano
UMASS Workshop
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with A. Czarnecki & A. Sirlin Update
Outline

1. Early History at a Glance
2. Muon vs Neutron Decay
3. CKM Unitarity
   *Seng, Gorchtein, Patel, Ramsey-Musolf (PRL2018)*
   \[ \Delta V_R = 0.02361(38) \Rightarrow 0.02467(22) \]
   Universal (Inner) Radiative Correction
5. CMS Update \[ \Delta V_R = 0.02428(32) \]
6. Implications for the Neutron Lifetime
EW Radiative Corrections
Some Pioneering Work

R. Behrends, R. Finkelstein & A. Sirlin (1956)
S. Berman (1958)
T. Kinoshita & A. Sirlin (1959) 60th Anniversary
S. Berman & A. Sirlin (1962)
J. D. Bjorken (1966)
E. Abers, R. Norton & D. Dicus (1967)
A. Sirlin (1967) Classic

1967 EW Unification – Renormalizable (1972)

A. Sirlin (1974) RC to Neutron Beta Decay

Beginning of a new era
Muon vs Neutron Decay

Muon Lifetime=2.1969803(22)x10^{-6}s

\[ G_\mu = 1.1663787(6) \times 10^{-5}\text{GeV}^{-2} \]

SU(2)\text{\textsubscript{L}}\times\text{U(1)}\text{\textsubscript{Y}} Standard Model Electroweak Radiative

 Corrections to \( \mu \rightarrow e\nu_e \nu_\mu \) and \( n \rightarrow p\nu_e \) both Infinite but
renormalized using \( (G_F^0 \rightarrow G_\mu) \)

Quark mixing divergences absorbed in \( V_{ud}^0 \rightarrow V_{ud} \) maintaining
Unitarity

The CKM Quark Mixing Matrix:

\[
V^{\text{CKM}} = \begin{vmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{vmatrix}
\]

3x3 Unitary Matrix

Unitarity \( \rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \)

Real “Apparent” Deviation from 1 Implies “New Physics”
Short-distance behavior of $\mu$ and $\beta$ decays differ due to weak hypercharges of $\mu_L(Y=-1)$ and $d_L(Y=+1/3)$

*Nevertheless, the ratio of $\tau_n$ and $\tau_\mu$ is finite!*

- **Muon Decay** \[ \Gamma_0(\mu \rightarrow e\nu\nu) = F(m_e^2/m_\mu^2)G_0^2m_\mu^5/192\pi^3 = 1/\tau_\mu^0 \]
- **Neutron Decay** \[ \Gamma_0(n \rightarrow p\nu\nu) = fG_0^2|V_{ud}|^2m_e^5(1+3g_A^2)/2\pi^3 = 1/\tau_n^0 \]

\[ F(x)=1-8x+8x^3-x^4-12x^2\ln x \quad \text{Phase Space Factor} \]

\[ f=1.6887(1) \quad \text{phase space factor Fermi function(\sim3.6\%),} \]

proton recoil, finite nucleon size… Uncertainty$\leq10^{-4}$

$g_A$ and $\tau_n$ important for Unitarity test, solar neutrino flux, primordial abundances, spin content of proton, Goldberger-Treiman/Muon Capture, Bjorken Sum Rule, lattice benchmark…

*Must be precisely determined!*
Loop and Tree Level Corrections to Muon Decay

- $\nu_\mu \rightarrow e$ + $Z^0$ + $W^\pm$ + $W^+$

- $H$ + $W$ + $W^*$

- $Z'$ Boson + SUSY + Technicolor

+ ...
Electroweak Radiative Corrections to Neutron Beta Decay

Include Virtual Corrections + Inclusive Bremsstrahlung
Normalize using $G_\mu$ from the muon lifetime
Absorbs Ultraviolet Divergences & some finite parts

$$\frac{1}{\tau_n} = f G_\mu^2 |V_{ud}|^2 m_e^5 \frac{(1+3g_A^2)(1+RC)}{2\pi^3}$$

$$f = 1.6887(1) \text{ (Includes Fermi Function)}$$

RC calculated for (Conserved) Vector Current since it is not renormalized by strong interaction at zero momentum transfer.

**Same** RC used to define $g_A$: $[A(g_A) = (1.001)A^{\text{exp}}]$  

$$RC = \frac{\alpha}{2\pi} [\langle g(E_m) \rangle + 3\ln(m_Z/m_p) + \ln(m_Z/m_A) + 2C + A_{\text{QCD}}]$$

$$\Delta^V_R = \frac{\alpha}{2\pi} [3\ln(m_Z/m_p) + \ln(m_Z/m_A) + 2C + A_{\text{QCD}}] \text{ “Inner”}$$

+ higher order

$$g(E_e) = \text{Universal Function (1967 Sirlin))}$$
\[ \frac{\alpha}{2\pi} g(E_m = 1.292581 \text{MeV}) = 0.015035 \] long distance loops and brem. averaged over the decay spectrum. Independent of Strong Int. up to \( O(E_e/m_P) \)

\( g(E_e) \) also applies to Nuclei \( A \). Sirlin (1967)

\[ 3 \frac{\alpha}{2\pi} \ln \left( \frac{m_Z}{m_p} \right) \] short-distance (Vector) log \textit{not} renormalized by strong int.

\[ \left[ \frac{\alpha}{2\pi} \ln \left( \frac{m_Z}{m_A} \right) + 2C + A_{QCD} \right] \] Induced by axial-current loop
Includes hadronic uncertainty

\( m_A = 1.2 \text{GeV} \) long/short distance matching scale (factor 2 unc.)

\( C = 0.8 g_A (\mu_N + \mu_P) = 0.891 \) (long distance \( \gamma W \) Box diagram) \( 1986 \)

\( A_{QCD} = -\frac{\alpha_s}{\pi} \left( \ln \left( \frac{m_Z}{m_A} \right) + \text{cons} \right) = -0.34 \) QCD Correction

\[ \left[ \frac{\alpha}{\pi} \ln \left( \frac{m_Z}{m} \right) \right]^n \] leading logs summed via renormalization group,

Next to leading short distance logs \(-0.0001\)

and \(-\alpha^2 \ln \left( \frac{m_p}{m_e} \right) = -0.00043\) estimated (for neutron decay)

Czarnecki, WJM, Sirlin (2004) \( 1 + RC = 1.0390(8) \) main unc. from \( m_A \)

matching short and long distance \( \gamma W \) (VA) Box

\textbf{DR Currently give I+RC = 1.0399(2)}
The Infamous $\gamma W$ Box Diagram

Weak Axial-Vector Induced Radiative Corrections

$AV$ Loop $\rightarrow V \rightarrow$ Superallowed $B$-decays

\[ RC = \frac{\alpha}{4\pi} \int_0^\infty dQ^2 \frac{m_W^2}{Q^2 + m_W^2} F(Q^2) \]

Large $Q^2$ $F(Q^2) = \frac{1}{Q^2} \left[ 1 - \frac{6\alpha_s(Q^2)}{\pi} + \ldots \right] + O(\frac{1}{Q^4})$

Small $Q^2$ $\rightarrow$ Nucleon Form Factors

$\frac{\alpha}{2\pi} \left\{ \frac{\lambda_\pi}{m_\pi} + \frac{\lambda_\rho}{m_\rho} + 2\lambda \right\}$ $m_r =$ matching

QCD $\rightarrow$ Long Distance
1.) Use large $N_{\text{QCD}}$ Interpolator to connect long-short distances

2.) Relate neutron beta decay to Bjorken Sum Rule ($N_F=3$)

$$1 - \frac{\alpha_s}{\pi} \rightarrow 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.583 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.212 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3$$

$$-175.7 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^4 \quad \text{(Baikov, Chetyrkin and Kuhn)}$$

The extra QCD corrections lead to a matching between short and long distance corrections at about $Q^2=1.08\text{GeV}^2$

$$1+RC= 1.0390(8) \rightarrow 1.03886(39) \quad \text{for Neutron Beta Decay}$$

Reduction by $1.4\times10^{-4}$ (Same for $0^+ \rightarrow 0^+$ beta decays)
RC Error Budget

1) Neglected Two Loop Effects: $\pm 0.0001$ conservative

2) Long Distance $\alpha/\pi C \sim \alpha/\pi (0.75g_A(\mu_N+\mu_P))=0.0020$
   
   Assumed Uncertainty $\pm 10\% \rightarrow \pm 0.0002$ reasonable?

3) Long-Short Distance Loop Matching: $0.8\text{GeV} < Q < 1.5\text{GeV}$
   
   $\pm 100\% \rightarrow \pm 0.0003$ conservative

Total RC Error $\pm 0.00038 \rightarrow \Delta V_{ud} = \pm 0.00019$

More Aggressive Analysis $\rightarrow \Delta V_{ud} = \pm 0.00013$

(1/2 conservative)
Superallowed \((0^+ \rightarrow 0^+)\) Beta Decays & \(V_{ud}\)

RC same as in Neutron Decay but with \(g(E_m)\) averaged Nuclear decay spectrum, C modified by Nucleon-Nucleon Interactions and
\(+Z \alpha^2 \ln(m_p/m_e)\) corrections (opposite sign from neutron)

\[
ft = |V_{ud}|^2(2984.5s)(1+ \Delta^V_R)(1+NP \text{ corr.})
\]

Nuclear Physics (NP) isospin breaking effects
(Hardy & Towner Calculations)

ft values + RC for 13 precisely measured nuclei found to be consistent with CVC: Average \(\rightarrow V_{ud}\)
Superallowed Nuclear Beta Decays

RC Uncertainty-Same as Neutron Decay

Nuclear Unc. - Significantly Reduced (2006-08)
Nuclear Coulomb Corrections Improved

\[ |V_{ud}| = 0.97425(11)_{\text{Nuc}}^{(19)}_{\text{RC}} \]

(2008 Hardy and Towner Update)

(0.97418((13)(14)(19) in PDG08)
(0.97377(11)(15)(19) in PDG06)
(0.97340(80) in 2004) Factor of 3 worse

2018 PDG \[ |V_{ud}| = 0.97420(10)_{\text{Nuc}}^{(18)}_{\text{RC}} \]
2019 DR \[ |V_{ud}| = 0.97370(10)_{\text{Nuc}}^{(11)}_{\text{RC}} \]
Superallowed Nuclear Beta Decays

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2018 PDG $|V_{ud}| = 0.97420(10)_{Nuc}(18)_{RC}$
2019 DR $|V_{ud}| = 0.97370(14)$
The Kaon Revolution of 2004-2005

(Starting with BNL E865) +FNAL, Frascati & CERN

$BR(K \rightarrow \pi e \nu)$ increased by $\approx 6\%$!

All Major $K_L$ BRs Changed! $\varepsilon_K$ changed by $3.7\sigma$!

Now Based on: $\Gamma(K \rightarrow \pi l \nu)_{\text{exp}}$ & $\Gamma(K \rightarrow \mu \nu)/\Gamma(\pi \rightarrow \mu \nu)_{\text{exp}}$

+ Lattice Matrix Elements $f_+(0)=0.960(5)$ & $f_K/f_\pi=1.193(6)$

2010 Flavianet Analysis:

$$|V_{us}|=0.2253(13) \quad \text{from } K \rightarrow \pi l \nu \quad \text{Vector}$$

$$|V_{us}|=0.2252(13) \quad \text{from } K \rightarrow \mu \nu \quad \text{Axial-Vector}$$

$$|V_{us}|=0.2253(9) \quad \text{Kaon Average} \quad (\text{was } \sim 0.220 \text{ pre 2004})$$

(Watch for lattice updates)
2018 STATUS of CKM Unitarity

\[ V_{ud} = 0.97420(10)_{\text{exp.,nucl.}}^{\exp.} (18)_{\text{RC}}^{\exp,\text{nucl.}} (\text{superallowed}), \]

\[ V_{us} = 0.2238(4)_{\text{exp+RC}}^{\exp,\text{RC}} (6)_{\text{lattice}}^{\text{lattice}} (K_{l3} \text{ decays}) \]

\[ V_{us} = 0.2253(7) (K_{\mu 2}/\pi_{\mu 2} \text{ decays}) \]

**Average** \[ V_{us} = 0.2243(9) \quad S=1.8 \]

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{V_{ud}(4)}^{V_{ud}(4)}_{V_{us}(4)} = 0.9994(6) \]

**Good Agreement With Unitarity**
“Reduced Hadronic Uncertainties in the determination of $V_{ud}$”

Radiative Corrections: Axial Induced part via Dispersion Relation.

$\Delta V_R = 0.02361(38) \rightarrow 0.02467(22)$

$|V_{ud}| = 0.97420(21) \rightarrow 0.97370(14)$

$|V_{ud}|^2 + |V_{us}|^2 = 0.9994(4)(4) \rightarrow 0.9984(3)(4)$

3 sigma deviation from unitarity

“New Physics”?
Comment on CKM Unitarity

- Most believable: $K_{\mu2}/\pi\mu_2$ & $F_K/F_\pi$ lattice
- $|V_{us}|/|V_{ud}|=0.2313(5)$ plus
- $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$
- $V_{ud}=0.97427(11)$, $V_{us}=0.2254(4)$

- $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9984(5)$
- $V_{ud}=0.9735(2)$, $V_{us}=0.2252(4)$
- “New Physics” in $V_{ud}$
Radiative Corrections to Neutron Decay

**Preliminary Update**

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<tbody>
<tr>
<td>RC</td>
<td>3.886(38)%</td>
<td>3.992(22)%</td>
<td>3.949(32)%</td>
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<tr>
<td>$\Delta V_R$</td>
<td>2.361(38)%</td>
<td>2.467(22)%</td>
<td>2.428(32)%</td>
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<tr>
<td>$V_{ud}$</td>
<td>0.97420(21)</td>
<td>0.97370(14)</td>
<td>0.97389(18)</td>
</tr>
</tbody>
</table>

*Seng, Gorchtein, Ramsey-Musolf*  
**Nuclear Quenching**  
(0.97392)*  
(0.97411)*

**My Guess**  
$V_{ud} = 0.97427(11)$,  
$\tau_n = 878.2(7)$ sec

Based on $V_{us}$ via $K_{\mu2}/\pi\mu2$  
*(very clean theory)*
The Infamous $\gamma W$ Box Diagram

Weak Axial-Vector Induced Radiative Corrections

$$AV\ Loop \rightarrow V \rightarrow \text{Superallowed } B\text{-decays}$$

$$\nu \rightarrow \bar{\nu} e^+ \gamma \rightarrow \bar{u} l \bar{d} e^+ \gamma \nu \nu$$

$$RC = \frac{\alpha}{4\pi} \int_0^{\infty} dQ^2 \frac{m_W^2}{Q^2 + m_W^2} F(Q^2)$$

Large $Q^2$: $F(Q^2) = \frac{1}{Q^2} [1 - \frac{\alpha}{\pi} + ...] + O(\frac{1}{Q^4})$

Small $Q^2$: Nucleon Form Factors

$$\nu \rightarrow \bar{\nu} e^+ \gamma \rightarrow \frac{\alpha}{\pi} (g_{\gamma} (H_p + M_n))$$

$$\frac{\alpha}{\pi} \{ \frac{\rho_n}{m_n} + \rho_p + 2c \}$$

$\sqrt{\text{QCD Long Distance}}$
Dispersion Relation Approach to Box Diagram

Seng, Gorchtein, Patel & Ramsey-Musolf PRL

<table>
<thead>
<tr>
<th></th>
<th>M&amp;S2006((x\alpha/\pi))</th>
<th>S,G,P&amp;R-M2018((x\alpha/\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perturbation</td>
<td>1.85</td>
<td>1.87</td>
</tr>
<tr>
<td>Born</td>
<td>0.83(8)</td>
<td>0.91(5)</td>
</tr>
<tr>
<td>Interpolator</td>
<td>0.14(14)</td>
<td>0.48(7)</td>
</tr>
<tr>
<td>Total</td>
<td>2.82(16)</td>
<td>3.26(9)</td>
</tr>
</tbody>
</table>

\(+\text{QED leading logs} + \text{Pure Vector Contribution}\)

<table>
<thead>
<tr>
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<th>M&amp;S2006((x\alpha/\pi))</th>
<th>S,G,P&amp;R-M2018((x\alpha/\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+RC</td>
<td>1.03886(38)</td>
<td>1.03992(22)</td>
</tr>
<tr>
<td>Universal (\Delta^V_R)</td>
<td>0.02361(38)</td>
<td>0.02467(22)</td>
</tr>
<tr>
<td>Difference ~ 0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V_{ud} = )</td>
<td>0.97420</td>
<td>0.97370</td>
</tr>
</tbody>
</table>
From: S,G,P,R-M
1.) Relate neutron beta decay to Bjorken Sum Rule \((N_F=3)\)

\[
1 - \frac{\alpha_s}{\pi} \rightarrow 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.583\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20.212\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3 \\
- 175.7 \left(\frac{\alpha_s(Q^2)}{\pi}\right)^4 \quad \text{(Baikov, Chetyrkin, Kuhn)} \text{ 4 loops!}
\]

\[= 1 - \frac{\alpha_{Bj}(Q^2)}{\pi} \quad \text{physical coupling} \]

perturbative

\[
F(Q^2) = \frac{1}{Q^2} \left(1 - \frac{\alpha_{Bj}(Q^2)}{\pi}\right) \quad \text{for } Q^2 > Q^2_0 = 1.08 \text{GeV}^2
\]

non-perturbative  Light Front Holography (LFH)

\[
\frac{\alpha_{Bj}(Q^2)}{\pi} = \exp\left(-\frac{Q^2}{Q^2_0}\right) \quad Q^2 \leq Q^2_0 \quad \frac{\alpha_{Bj}(0)}{\pi} = 1 \quad \text{AdS}
\]

\[
F(0) = \frac{1}{Q^2_0} = 0.93 \text{GeV}^{-2}
\]

The extra QCD correction leads to a matching between short and long distance couplings at about \(Q^2 = 1.08 \text{GeV}^2\)
Running QCD Coupling from Bj sum rule data

Brodsky, Duer et al.
Figure 2. Effective coupling $\alpha_{g1}(Q^2)$ as a function of $Q^2$. The exponential form of eq. (23) is used for low $Q^2$ (dashed red), and the perturbative QCD expression (22) for high $Q^2$ (solid blue). Note the remarkably smooth matching between the two regimes.

We take our largest corrections (line 3) as representative of this work and use it for discussing implications and differences with the DR approach.

The difference between line 3 in table I and the DR result [4] has been reduced to about 0.0005 which is less than $2 \times 10^{-4}$ (although combining error budgets is difficult due to correlations. At that level many small effects come into play. The DR approach is better theoretically grounded while the Bj function approach comes closer to CKM unitarity constraints and is based on better sum rule data. Further theoretical and experimental input is called for. In the meantime, we close this section with a comparison of the various $Q^2$ contributions for the DR and AdS based results in table I for the universal radiative correction.

At this point it is useful to compare our updated result with the DR result. At the lowest order we find axial-vector induced effects $0.07/\pi$ while the DR approach gets $0.26/\pi$. Including pure vector current effects and higher orders the comparison becomes $V_{R}=0.02419(30)$ vs. $0.02467(22)$ (45) and a difference of roughly $0.5 \times 10^{-4}$ about half of what a comparison with the 2006 result suggested. About $2 \times 10^{-4}$ of that difference stems from perturbative and Born contributions. The perturbative difference is likely due to different treatments of the QCD corrections while the Born difference is due to different upper integration limits. The remaining difference stems primarily from non-perturbative intermediate $Q^2$ loop effects. We believe that most of the remaining difference is due to our use of BjSR sum rule experimental guidance while the DR approach employed GLS neutrino data matching. In addition, we employed four loop QCD corrections to BjSR while the DR analysis used three loop corrections to GLS.
Running QCD Bj coupling
\[ \gamma W \text{ Box RC: Bj Function Approach} \]
\[ \alpha_s(m_Z) = 0.1181(10), \, m_c=1.3 \text{ GeV}, \, m_b=4.2 \text{ GeV} \]

<table>
<thead>
<tr>
<th>( Q^2 ) Domain</th>
<th>Integral (( x\alpha/\pi ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0&lt;Q^2&lt;Q^2_0 )</td>
<td>0.20 New Effect</td>
</tr>
<tr>
<td>( Q^2_0&lt;Q^2&lt;1.69\text{GeV}^2 )</td>
<td>0.079</td>
</tr>
<tr>
<td>( 1.69\text{GeV}^2&lt;Q^2&lt;\infty )</td>
<td>1.89</td>
</tr>
<tr>
<td>ZW Box</td>
<td></td>
</tr>
<tr>
<td>Born</td>
<td>0.06</td>
</tr>
<tr>
<td>( \Sigma ) not ( Q_0 ) sensitive</td>
<td><strong>Midway 3.085</strong> Total</td>
</tr>
<tr>
<td></td>
<td>3.32 DR (2018)</td>
</tr>
<tr>
<td></td>
<td>2.82 MS (2006)</td>
</tr>
</tbody>
</table>
2. Large $N_c$ QCD Approach (new)

Infinite sum of Vector and Axial-Vector Resonances
Use 3 Resonances + 3 Matching Conditions

$$F(Q^2) = \frac{A}{Q^2+m^2_\rho} + \frac{B}{Q^2+m^2_A} + \frac{C}{Q^2+m^2_{\rho'}}$$

- $m_\rho = 0.776\text{GeV}$
- $m_A = 1.230\text{GeV}$
- $m_{\rho'} = 1.465\text{GeV}$

1. Integral $Q_0^2 - \infty$ equals Bj Function Integral = 7.885
2. No $1/Q^4$ terms in large $Q^2$ limit
3. $F(0)$ Arbitrary
3 Resonance Solution for F(0)=Arbitrary

- A = -1.514 + 1.422F(0)
- B = +6.968 - 3.535F(0)
- C = -4.487 + 2.092F(0)

\[0 < Q^2 < Q^2_0 \rightarrow (0.092 + 0.102F(0))\alpha/\pi\]

\[\Delta V_R = 0.02398 + 2.4 \times 10^{-4}F(0)\]

F(Q_0^2) = 0.588(35) \rightarrow F(0) = 1.46(27)

\[\Delta V_R = 0.02433(6) \text{ 3 resonance result}\]

F(0) = 2.9 \rightarrow \Delta V_R = 0.02467 = DR result

off by 5.7 sigma see figure
F(Q^2) Interpolators
F(0) = 0.93 BjSR, 1.46(27) band, 2.1, 2.9

Figure 3. Examples of F(Q^2) at low Q^2:

- Q^2 \downarrow 1
- g_1(Q^2) \uparrow (solid blue line);
- three resonance interpolator based on F(0) = (1.46 \pm 0.27) / GeV^2 (grey band);
- two resonance interpolator discussed below eq. (44) (dash-dotted green line);
- and F(0) = 2.9 red dashed curve which reproduces numerically the DR result.

The different radiative corrections correspond to the changes in the areas under the curves.

Employing averages \bar{\tau}_n = 879.4(6) and g_A = 1.2762(5) leads to V_{ud} = 0.9735(5),
which is starting to become competitive with superallowed beta decay determinations. A return to unitarity for V_{ud} \approx 0.9742 would require a reduction in either \bar{\tau}_n or g_A. Given the recent precision of Perkeo III, we consider g_A fixed at the new post 2002 average 1.2762(5) which then suggests a \bar{\tau}_n not far above 878 s. Future experiments are aiming for ±0.1 sensitivity.

An alternate interpretation of the apparent violation of CKM unitarity in eq. (9) resulting from larger universal radiative corrections, consistent with |V_{us}|/|V_{ud}| = 0.2313 from K_{\mu}/\pi_{\mu}, suggests the solution V_{ud} = 0.9735 and V_{us} = 0.2252 which requires "new physics." Such an effect could be due to a 0.1\% increase in the muon decay rate from "new physics" which shifts G_{\mu} to a value larger than the real G_F. Alternatively, it could stem from an opposite sign effect in beta decay. That solution agrees with the current central value in eq. (48). Of course, such a scenario would be very exciting. It will also be well tested by the next generation of precise \bar{\tau}_n and g_A measurements.

Recently, we discussed a resolution of the neutron lifetime problem (the beam \bar{\tau}_n = 888.0(2.0) and trap \bar{\tau}_n = 879.4(0.6) sliced time discrepancy) based on a precise connection between \bar{\tau}_n and g_A, the axial coupling measured in neutron decay asymmetries. We note that a shift in the universal beta decay...
We make the following observations regarding the sensitivity of the three resonance interpolator:

The radiative corrections grow with $F(0)$ as illustrated by the areas under the curves in fig. 3. In that figure we plot four distinct $F(Q^2)$ cases. The solid curve at the bottom represents the BjSR function with $F(0) = 0.93 \text{ GeV}^2$ while the gray band corresponds to a 3 resonance interpolator with $F(0) = 1.46(27) \text{ GeV}^2$. Both curves overlap at the matching condition $F(Q^2_0) = 0.588(35)$ and represent the 2 solutions we average to get our radiative corrections. The other 2 curves corresponding to $F(0) = 1.1$ and $2.9 \text{ GeV}^2$ have the following properties:

- $V_R = 0.02448 F(0) = 1.1, F(Q^2_0) = 0.68(45)$
- $V_R = 0.02468 F(0) = 2.9, F(Q^2_0) = 0.78(46)$

They fail to respect the $F(Q^2_0) = 0.588(35)$ matching constraint at 2.5 and 5.4 sigma respectively and are therefore not used, although they illustrate interesting features that correspond to larger radiative corrections.

$F(0) = 1.1 \text{ GeV}^2$ corresponds to a two resonance interpolator. Requiring a parameter fit automatically leads to large radiative corrections. The $F(0) = 2.9 \text{ GeV}^2$ curve is constructed to reproduce the DR radiative corrections. Its $Q^2$ dependence appears to also follow the DR dependence illustrated in [4, 5]. On the basis of that observation, it may be that the remaining difference between our result and the larger DR radiative corrections may stem from differences in using 3 vs 4 QCD loops and the resulting matching condition.

In Table I we compare the universal and neutron specific radiative corrections obtained from a dispersion relation approach (line 1) with an earlier calculation from 2006 (line 2) as well as the AdS BjSR result (line 3), three resonance interpolator with $F(0)=1.46(27)$ (line 4), and the average of 3 and 4 in line 5. We take the average on line 5 as representative of our study and use it in discussing implications. Although it is somewhat smaller than the earlier DR result [4], they are fairly consistent. In fact our analysis can be viewed as validation of the LFHQCD and three resonance interpolator approaches.

### Table I. Universal and neutron specific radiative corrections.

<table>
<thead>
<tr>
<th>r</th>
<th>$\Delta^V_R$</th>
<th>RC</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Line 1</td>
</tr>
<tr>
<td>0.02467(22)</td>
<td>0.03992(22)</td>
<td>[4]</td>
<td></td>
</tr>
<tr>
<td>0.02361(38)</td>
<td>0.03886(38)</td>
<td>[3]</td>
<td></td>
</tr>
<tr>
<td>0.02423(32)</td>
<td>0.03944(32)</td>
<td>AdS BjSR Approach, eqs. (34) and (35)</td>
<td></td>
</tr>
<tr>
<td>0.02433(32)</td>
<td>0.03953(32)</td>
<td>Three Resonance Interpolation</td>
<td></td>
</tr>
<tr>
<td>0.02428(32)</td>
<td>0.03949(32)</td>
<td>Average of lines 3 and 4</td>
<td></td>
</tr>
</tbody>
</table>
Implications of larger RC for Neutron Lifetime

CMS 1+RC=1.03949(32)
$V_{ud} = 0.9741$
$g_A = 1.2762(5)$ (After PerkeoIII)

Predict Neutron Lifetime = 878s

Currently  **Neutron Lifetime Problem**

$\tau_{n}^{beam} = 888.0(2.0)s$  $\tau_{n}^{trap} = 879.4(6)s$

Could Both Be Right? Neither?

**Fornal-Grinstein Solution**

$BR(n \rightarrow dark\ particles) \sim 1\%$

We find $BR(n \rightarrow exotic\ decay) < 0.1\% (95\%CL)$
Final Comments

- Watch $g_A$ & $\tau_n$ future $10^{-4}$ sensitivity
- What is $V_{us}$?
- $F(Q^2)$ for $Q^2<1\text{GeV}^{-2}$ lattice QCD
- DR value of $F(0)$
- DR and 4 loop BjSR