Electroweak Radiative Corrections & CKM Unitarity

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UMASS Workshop

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with A. Czarnecki & A. Sirlin Update

BROOKHAVEN
NATIONAL LABORATORY

Outline

1. Early History at a Glance
2. Muon vs Neutron Decay
3. CKM Unitarity
4. Dispersion Relation (DR) Approach (2018)

Seng, Gorchtein, Patel, Ramsey-Musolf (PRL2018)

$\Delta^V_R = 0.02361(38)$ ➔ $\mathbf{0.02467(22)}$

Universal (Inner) Radiative Correction

5. CMS Update $\Delta^V_R = \mathbf{0.02428(32)}$
6. Implications for the Neutron Lifetime

EW Radiative Corrections
Some Pioneering Work

R. Behrends, R. Finkelstein & A. Sirlin (1956)

S. Berman (1958)

T. Kinoshita & A. Sirlin (1959) ***60th Anniversary***

S. Berman & A. Sirlin (1962)

J. D. Bjorken (1966)

E. Abers, R. Norton & D. Dicus (1967)

A. Sirlin (1967) ***Classic***

1967 EW Unification − Renormalizable (1972)

A. Sirlin (1974) RC to Neutron Beta Decay

Beginning of a new era

Muon vs Neutron Decay

Muon Lifetime=2.1969803(22)x10^{-6}s

$$G_\mu = 1.1663787(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

$SU(2)_L \times U(1)_Y$ Standard Model Electroweak Radiative Corrections to $\mu \rightarrow e\nu_e\nu_\mu$ and $n \rightarrow pe\nu_e$ both Infinite but renormalized using ($G_F^0 \rightarrow G_\mu$)

Quark mixing divergences absorbed in $V^0_{ud} \rightarrow V_{ud}$ maintaining Unitarity

The CKM Quark Mixing Matrix:

$$V^{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$ 3x3 Unitary Matrix

Unitarity $\rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Real "Apparent" Deviation from 1 Implies "New Physics"

Short-distance behavior of μ and β decays differ due to weak hypercharges of μ_L(Y=-1) and $d_{L.}$(Y=+1/3)

Nevertheless, the ratio of τ_n and τ_μ is finite !

- Muon Decay $\Gamma_0(\mu \to e\nu\nu) = F(m_e^2/m_\mu^2) G^0{}_F{}^2 m_\mu^5/192\pi^3 = 1/\tau_\mu{}^0$
- Neutron Decay $\Gamma_0(n \to pe\nu) = \mathbf{f} G^0{}_F{}^2 |V^0{}_{ud}|^2 m_e^5 (1+3g_A{}^2)/2\pi^3 = 1/\tau_n{}^0$

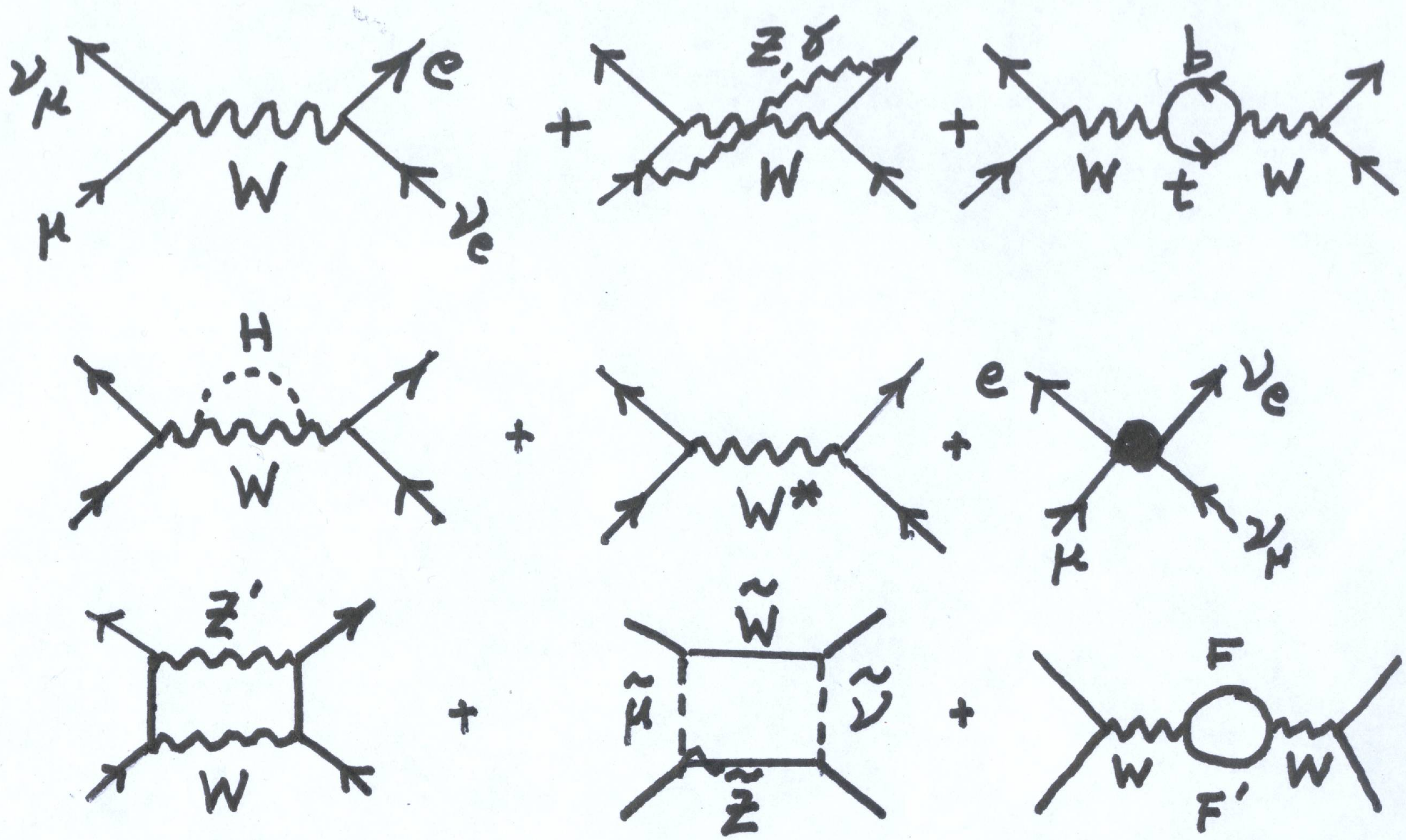
$F(x)=1-8x+8x^3-x^4-12x^2 \ln x$ Phase Space Factor

f=1.6887(1) phase space factor Fermi function(~3.6%), proton recoil, finite nucleon size… Uncertainty<10^{-4}

g_A and τ_n important for Unitarity test, solar neutrino flux, primordial abundances, spin content of proton, Goldberger-Treiman/Muon Capture, Bjorken Sum Rule, lattice benchmark…

Must be precisely determined!

Loop and Tree Level Corrections to Muon Decay



Z' Boson SUSY Technicolor

$+ \cdots$

Electroweak Radiative Corrections to Neutron Beta Decay

Include Virtual Corrections + Inclusive Bremsstrahlung

Normalize using G_μ from the muon lifetime

Absorbs Ultraviolet Divergences & some finite parts

$$1/\tau_n = fG_\mu^2|V_{ud}|^2 m_e^5(1+3g_A^2)(\underline{1+RC})/2\pi^3$$

f=1.6887(1) (Includes Fermi Function)

RC calculated for (Conserved) Vector Current since it is not renormalized by strong interaction at zero momentum transfer.

Same RC used to **define** g_A: $[A(g_A)=(1.001)A^{exp}]$

$$RC=\alpha/2\pi[\langle g(E_m)\rangle+3\ln(m_Z/m_p)+\ln(m_Z/m_A)+2C+A_{QCD}]$$

$$\Delta^V_R=\alpha/2\pi[3\ln(m_Z/m_p)+\ln(m_Z/m_A)+2C+A_{QCD}]$$ “Inner”

+ higher order

$g(E_e)$=Universal Function (1967 Sirlin))

$\alpha/2\pi\langle g(E_m=1.292581\text{MeV})\rangle=0.015035$ long distance loops and brem. averaged over the decay spectrum. Independent of Strong Int. up to $O(E_e/m_P)$

$g(E_e)$ also applies to Nuclei A. Sirlin (1967)

$3\alpha/2\pi\ln(m_Z/m_p)$ short-distance (Vector) log ***not*** renormalized by strong int.

$[\alpha/2\pi[\ln(m_Z/m_A)+2C+A_{QCD}]$ Induced by axial-current loop

Includes hadronic uncertainty

m_A=1.2GeV long/short distance matching scale (factor 2 unc.)

$C=0.8g_A(\mu_N+\mu_P)=0.891$ (long distance γW Box diagram) 1986

$A_{QCD}= -\alpha_s/\pi(\ln(m_Z/m_A)+\text{cons})=-0.34$ QCD Correction

$[\alpha/\pi\ln(m_Z/m)]^n$ leading logs summed via renormalization group,

Next to leading short distance logs~-0.0001,

and $-\alpha^2\ln(m_p/m_e)=-0.00043$ estimated (for neutron decay)

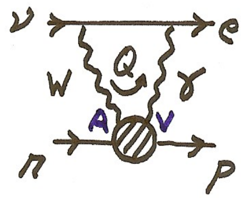
Czarnecki, WJM, Sirlin (2004) 1+RC=**1.0390(8)** main unc. from m_A matching short and long distance γW (VA) Box

DR Currently give I+RC =1.0399(2)

The Infamous γW Box Diagram

Weak Axial-Vector Induced Radiative Corrections

AV Loop → V → Superallowed β-decays

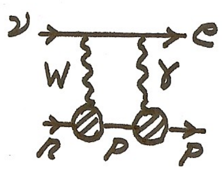


$$\longrightarrow \underbrace{\bar{u}\gamma_\mu d}_{\text{Vector}}\, \bar{e}_L \gamma^\mu \nu_L + \text{QCD}$$

$$RC = \frac{\alpha}{4\pi}\int_0^\infty dQ^2 \frac{m_W^2}{Q^2+m_W^2} F(Q^2)$$

Large Q^2 $\quad F(Q^2) = \frac{1}{Q^2}\left[1 - \frac{\alpha_s(Q^2)}{\pi} + \ldots\right] + O\left(\frac{1}{Q^4}\right)$

Small Q^2 → Nucleon Form Factors



$$\rightarrow \frac{\alpha}{2\pi}\left(0.8\, g_A (\mu_p + \mu_n)\right)$$

$$\frac{\alpha}{2\pi}\left\{\ln\frac{m_Z}{m_A} + \underbrace{A_g}_{\text{QCD}} + \underbrace{2C}_{\text{Long Distance}}\right\} \qquad m_A = \text{matching}$$

2006 Improvement WJM & A. Sirlin

1.) Use large N_{QCD} Interpolator to connect long-short distances

2.) Relate neutron beta decay to Bjorken Sum Rule (N_F=3)

$$1-\alpha_s/\pi \rightarrow 1-\alpha_s(Q^2)/\pi-3.583(\alpha_s(Q^2)/\pi)^2-20.212(\alpha_s(Q^2)/\pi)^3 -175.7\,(\alpha_s(Q^2)/\pi)^4$$

(Baikov,Chetyrkin and Kuhn)

The extra QCD corrections lead to a matching between short and long distance corrections at about Q^2=1.08GeV^2

1+RC= 1.0390(8)→<u>1.03886(39)</u> for Neutron Beta Decay

Reduction by 1.4×10^{-4} (Same for $0^+\rightarrow0^+$ beta decays)

RC Error Budget

1) Neglected Two Loop Effects: **±0.0001** conservative

2) Long Distance $\alpha/\pi C \sim \alpha/\pi\ (0.75 g_A(\mu_N+\mu_P)) = 0.0020$

Assumed Uncertainty ±10%→**±0.0002** reasonable?

3) Long-Short Distance Loop Matching: 0.8GeV<Q<1.5GeV

±*100*% → **±0.0003** conservative

Total RC Error **±0.00038** →ΔV_{ud}=**±0.00019**

More Aggressive Analysis →ΔV_{ud}=±0.00013

(1/2 conservative)

Superallowed ($0^+ \rightarrow 0^+$) Beta Decays & V_{ud}

RC same as in Neutron Decay but with $g(E_m)$ averaged Nuclear decay spectrum, C modified by Nucleon-Nucleon Interactions and $+Z\,\alpha^2 \ln(m_p/m_e)$ corrections (opposite sign from neutron)

$$ft = |V_{ud}|^2 (2984.5\text{s})(1+\Delta^V_R)(1+\text{NP corr.})$$

Nuclear Physics (NP) isospin breaking effects (Hardy & Towner Calculations)

ft values + RC for 13 precisely measured nuclei found to be consistent with CVC: Average $\rightarrow V_{ud}$

Superallowed Nuclear Beta Decays

RC Uncertainty-Same as Neutron Decay

Nuclear Unc. - Significantly Reduced (2006-08)

Nuclear Coulomb Corrections Improved

$\mathbf{|V_{ud}| = \underline{0.97425}(11)_{Nuc}(19)_{RC}}$

(2008 Hardy and Towner Update)

(0.97418((13)(14)(19) in PDG08)

(0.97377(11)(15)(19) in PDG06)

(0.97340(80) in 2004) Factor of 3 worse

2018 PDG $\mathbf{|V_{ud}| = \underline{0.97420}(10)_{Nuc}(18)_{RC}}$

2019 DR $\mathbf{|V_{ud}| = \underline{0.97370}(10)_{Nuc}(11)_{RC}}$

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The Kaon Revolution of 2004-2005

(Starting with BNL E865) +FNAL, Frascati & CERN

BR($K\to\pi e\nu$) increased by ≈6%!

All Major K_L BRs Changed! ε_K changed by 3.7σ!

Now Based on: $\Gamma(K\to\pi l\nu)_{exp}$ & $\Gamma(K\to\mu\nu)/\Gamma(\pi\to\mu\nu)_{exp}$

\+ Lattice Matrix Elements $f_+(0)=0.960(5)$ & $f_K/f_\pi=1.193(6)$

***2010 Flavianet Analysis*:**

$|V_{us}|=0.2253(13)$ from $K\to\pi l\nu$ Vector

$|V_{us}|=0.2252(13)$ from $K\to\mu\nu$ Axial-Vector

$|V_{us}|=0.2253(9)$ Kaon Average (was ~0.220 pre 2004)

(Watch for lattice updates)

2018 STATUS of CKM Unitarity

$V_{ud} = 0.97420(10)_{exp.,nucl.}(18)_{RC}$ (superallowed) ,

$V_{us} = 0.2238(4)_{exp+RC}(6)_{lattice}$ (K_{l3} decays)

$V_{us} = 0.2253(7)$ ($K_{\mu 2}/\pi_{\mu 2}$ decays)

Average $V_{us} = 0.2243(9)$ S=1.8

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9994(4)_{Vud}(4)_{Vus}$$

$$=0.9994(6)$$

Good Agreement With Unitarity

Seng, Gorchtein, Patel & Ramsey−Musolf
PRL (2018)

"Reduced Hadronic Uncertainties in the determination of V_{ud}"

Radiative Corrections: Axial Induced part via Dispersion Relation.

2006 Δ^V_R =0.02361(38) ➔ 0.02467(22))

$|V_{ud}|$=0.97420(21) ➔ 0.97370(14)

$|V_{ud}|^2+|V_{us}|^2$ =0.9994(4)(4)➔0.9984(3)(4)

3 sigma deviation from unitarity

"New Physics"?

Comment on CKM Unitarity

- Most believable: $K_{\mu 2}/\pi_{\mu 2}$ & F_K/F_π lattice
- $|V_{us}|/|V_{ud}|=0.2313(5)$ **plus**
- $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$
- $V_{ud}=0.97427(11)$ $V_{us}=0.2254(4)$

- $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9984(5)$
- $V_{ud}=0.9735(2)$ $V_{us}=0.2252(4)$
- **"New Physics" in V_{ud}**

Update with A. Czarnecki & A. Sirlin (CMS)

Radiative Corrections to Neutron Decay

Preliminary Update

	MS(2006)	SGPR-M(2018)	CMS(2019)
RC =	3.886(38)%	3.992(22)%	3.949(32)%
Δ^V_R =	2.361(38)%	2.467(22)%	2.428(32)%
V_{ud} =	0.97420(21)	0.97370(14)	0.97389(18)
		(0.97392)*	(0.97411)*

Nuclear Quenching

**Seng, Gorchtein, Ramsey-Musolf*

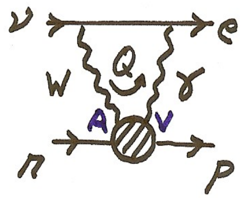
My Guess **V_{ud}=0.97427(11)**, τ_n = ***878.2(7)sec***

Based on V_{us} via $K_{\mu 2}/\pi_{\mu 2}$ (very clean theory)

The Infamous γW Box Diagram

Weak Axial-Vector Induced Radiative Corrections

AV Loop → V → Superallowed β-decays

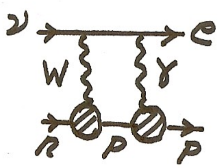


$$\longrightarrow \underbrace{\bar{u}\gamma_\mu d}_{\text{Vector}}\, \bar{e}_L \gamma^\mu \nu_L + QCD$$

$$RC = \frac{\alpha}{4\pi}\int_0^\infty dQ^2 \frac{m_W^2}{Q^2+m_W^2} F(Q^2)$$

Large Q^2 $\quad F(Q^2) = \frac{1}{Q^2}\left[1 - \frac{\alpha_s(Q^2)}{\pi} + \dots\right] + O\left(\frac{1}{Q^4}\right)$

Small Q^2 → Nucleon Form Factors



$$\rightarrow \frac{\alpha}{2\pi}\left(0.8 g_A (\mu_p + \mu_n)\right)$$

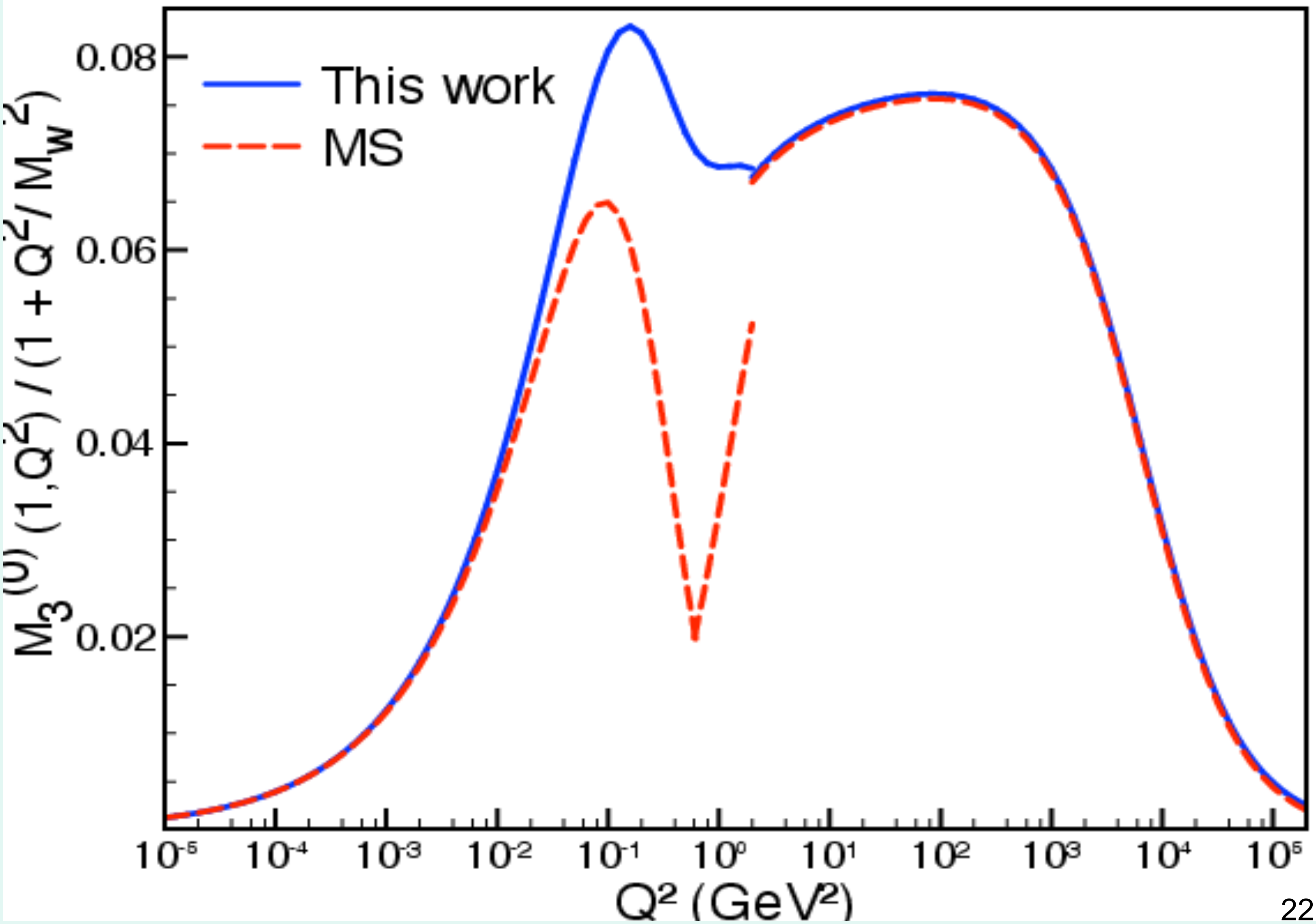
$$\frac{\alpha}{2\pi}\left\{ \ln\frac{m_Z}{m_A} + \underbrace{A_g}_{QCD} + \underbrace{2C}_{\text{Long Distance}} \right\} \qquad m_A = \text{matching}$$

Dispersion Relation Approach to Box Diagram

Seng, Gorchtein, Patel & Ramsey-Musolf PRL

	M&S2006($\times\alpha/\pi$)	S,G.P&R-M2018($\times\alpha/\pi$)
Perturbation	1.85	1.87
Born	0.83(8)	0.91(5)
Interpolator	0.14(14)	0.48(7)
Total	2.82(16)	3.26(9)
+QED leading logs + Pure Vector Contribution		
1+RC	1.03886(38)	1.03992(22)
Universal Δ^V_R	0.02361(38)	0.02467(22)
	Difference ~ 0.001	
V_{ud} =	0.97420	0.97370

From: S,G,P,R-M



This work
MS
0.08
0.06
0.04
0.02
$M_3^{(U)}(1,Q^2)\,/\,(1+Q^2/M_W^2)$
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
Q^2 (GeV^2)

2019 Update Czarnecki, WJM & Sirlin

1.) Relate neutron beta decay to Bjorken Sum Rule (N_F=3)

$1-\alpha_s/\pi \rightarrow 1-\alpha_s(Q^2)/\pi-3.583(\alpha_s(Q^2)/\pi)^2-20.212(\alpha_s(Q^2)/\pi)^3$

$-175.7\,(\alpha_s(Q^2)/\pi)^4$ (Baikov,Chetyrkin,Kuhn) 4 loops!

$= 1 - \alpha_{Bj}(Q^2)/\pi$ **physical coupling**

perturbative

$F(Q^2) = 1/Q^2(1-\alpha_{Bj}(Q^2)/\pi)$ for $Q^2>Q^2_0$ =1.08GeV^2

non-perturbative Light Front Holography (LFH)

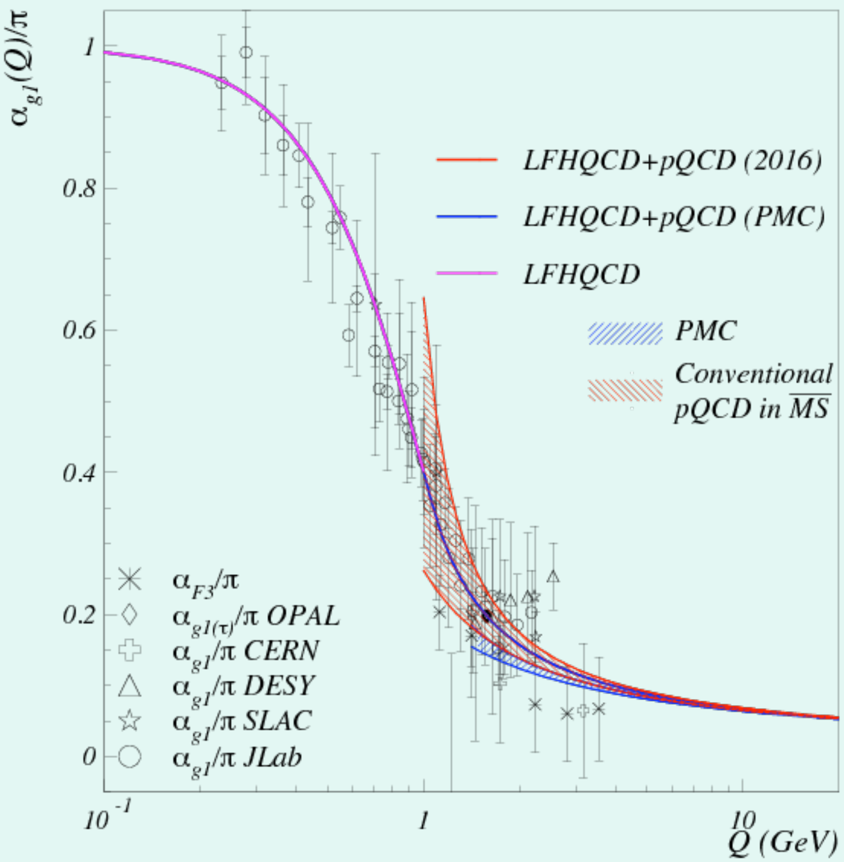
$\alpha_{Bj}(Q^2)/\pi = \exp(-Q^2/Q^2_0)$ $Q^2 \leq Q^2_0$ $\alpha_{Bj}(0)/\pi = 1$ AdS

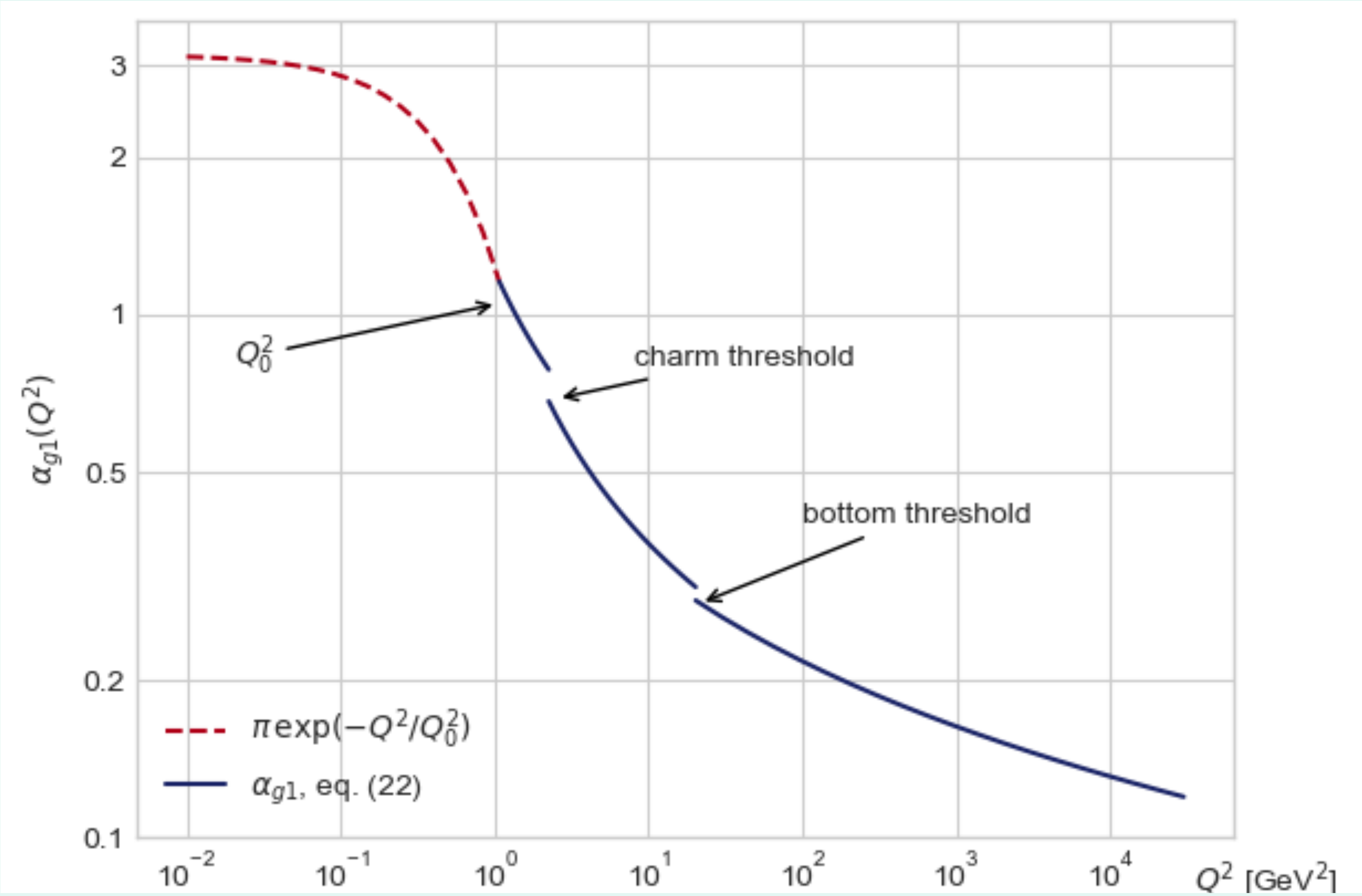
$F(0)=1/Q^2_0$ =0.93GeV^{-2}

The extra QCD correction leads to a matching between short and long distance couplings at about Q^2=1.08GeV^2

Running QCD Coupling from Bj sum rule data

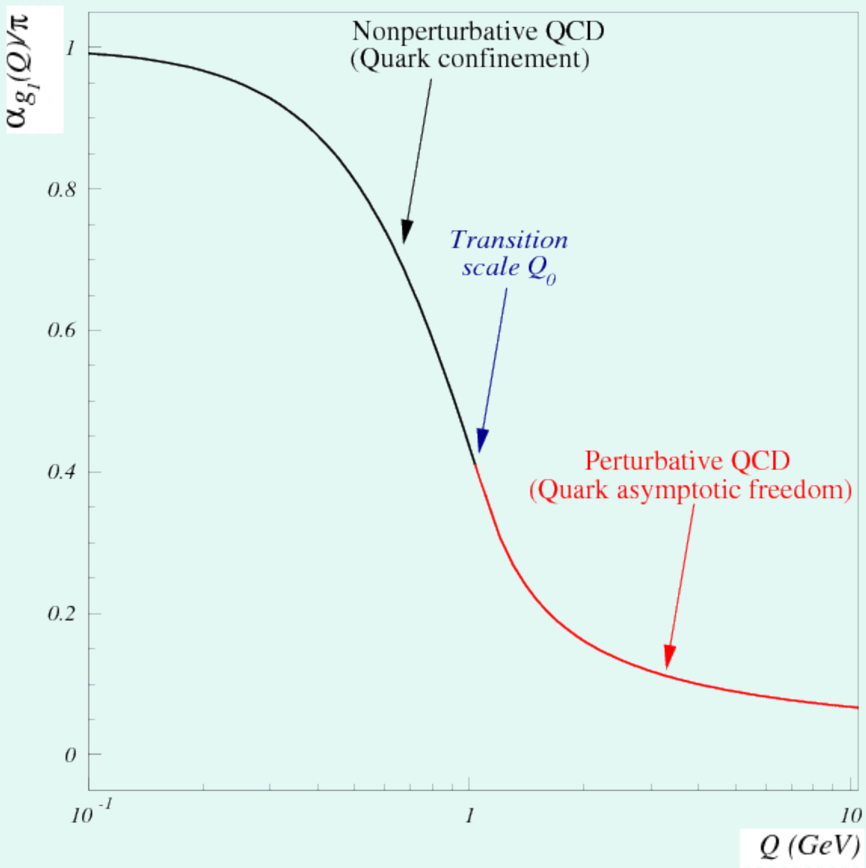
Brodsky, Duer et al.





3
2
1
0.5
0.2
0.1
$\alpha_{g1}(Q^2)$
Q_0^2
charm threshold
bottom threshold
$\pi \exp(-Q^2/Q_0^2)$
α_{g1}, eq. (22)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Q^2 [GeV^2]

Running QCD Bj coupling



γW Box RC: Bj Function Approach

$\alpha_s(m_Z) = 0.1181(10)$, $m_c=1.3$ GeV, $m_b=4.2$GeV

• <u>Q^2 Domain</u>	Integral ($\times\alpha/\pi$)
$0<Q^2<Q^2_0$	0.20 New Effect
$Q^2_0<Q^2<1.69\text{GeV}^2$	0.079
$1.69\text{GeV}^2<Q^2<\infty$	1.89
ZW Box	0.06
Born	<u>0.856</u>
Σ not Q_0 sensitive	**Midway 3.085 Total**
	3.32 DR (2018)
	2.82 MS (2006)

2. Large N_c QCD Approach (new)

Infinite sum of Vector and Axial-Vector Resonances

Use 3 Resonances + 3 Matching Conditions

$$F(Q^2) = \frac{A}{Q^2+m^2_\rho} + \frac{B}{Q^2+m^2_A} + \frac{C}{Q^2+m^2_{\rho'}}$$

$m_\rho = 0.776\,\mathrm{GeV}$ $m_A = 1.230\,\mathrm{GeV}$ $m_{\rho'} = 1.465\,\mathrm{GeV}$

1. Integral $Q_0^2 - \infty$ equals Bj Function Integral=7.885
2. No $1/Q^4$ terms in large Q^2 limit
3. F(0) Arbitrary

3 Resonance Solution for F(0)=Arbitrary

- **$A = -1.514 + 1.422F(0)$**
- **$B = +6.968 - 3.535F(0)$**
- **$C = -4.487 + 2.092F(0)$**

$0<Q^2<Q^2_0 \Rightarrow \underline{\mathbf{(0.092+0.102F(0))\alpha/\pi}}$

$\Delta^V_R = 0.02398 + 2.4\times10^{-4} F(0)$

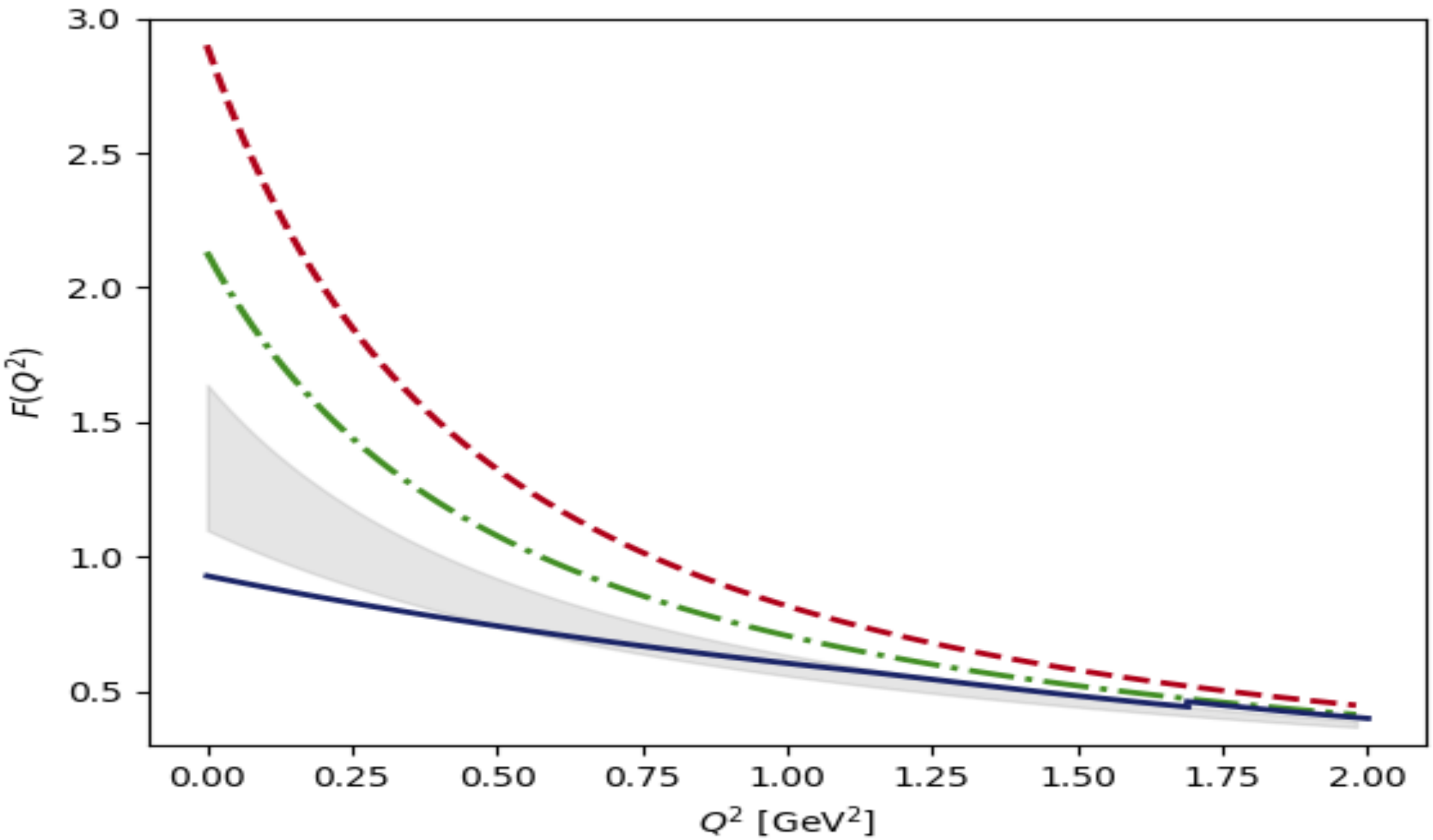
$F(Q_0^2)=0.588(35) \Rightarrow F(0)=1.46(27)$

$\Delta^V_R=0.02433(6)$ **3 resonace result**

$F(0)=2.9 \Rightarrow \Delta^V_R=0.02467$ **= DR result**

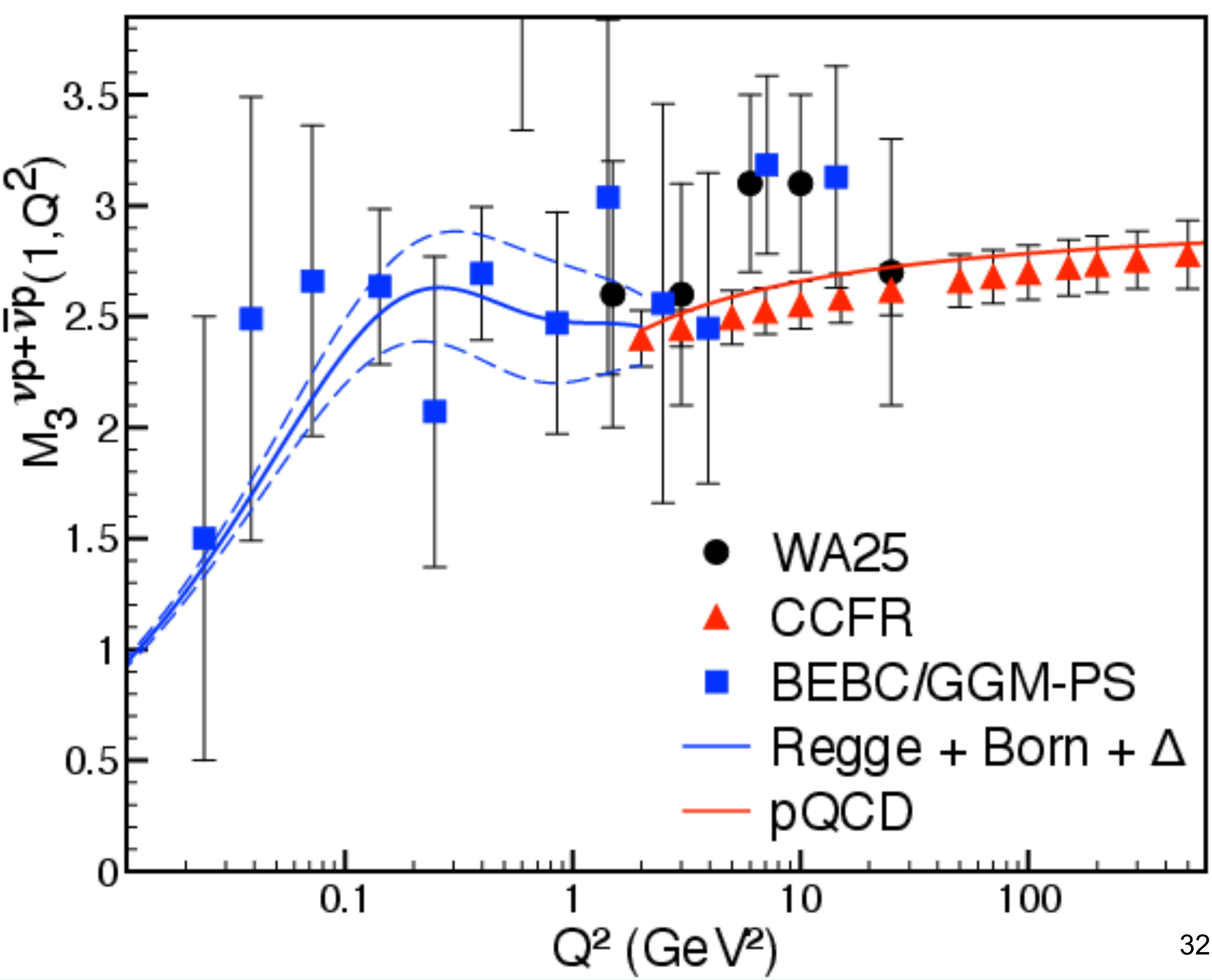
off by 5.7 sigma see figure

$F(Q^2)$ Interpolators
F(0)=0.93 BjSR, 1.46(27) band, 2.1, 2.9



ble I. Universal and neutron specific radiative correc

r	Δ_R^V	RC	Source
	0.02467(22)	0.03992(22)	[4]
	0.02361(38)	0.03886(38)	[3]
	0.02423(32)	0.03944(32)	AdS BjSR Approach, eq
	0.02433(32)	0.03953(32)	Three Resonance Interp
	0.02428(32)	0.03949(32)	Average of lines 3 and 4



$M_3^{\nu p+\bar{\nu}p}(1,Q^2)$
3.5
3
2.5
2
1.5
1
0.5
0
0.1
1
10
100
Q^2 (GeV^2)
WA25
CCFR
BEBC/GGM-PS
Regge + Born + Δ
pQCD

Implications of larger RC for Neutron Lifetime

CMS $1+RC=1.03949(32)$

$V_{\underline{ud}}=0.9741$

$g_A=1.2762(5)$ (After PerkeoIII)

Predict Neutron Lifetime =878s

Currently ***Neutron Lifetime Problem***

$\tau_n^{beam}=888.0(2.0)s$ $\tau_n^{trap}=879.4(6)s$

Could Both Be Right? Neither?

Fornal-Grinstein Solution

BR(n→dark particles) ~ 1%

We find BR(n→exotic decay)<0.1% (95%CL)

Final Comments

- **Watch g_A & τ_n future 10^{-4} sensitivity**
- **What is V_{us}?**
- **$F(Q^2)$ for Q^2<1GeV^{-2} lattice QCD**
- **DR value of F(0)**
- **DR and 4 loop BjSR**